

MATHEMATICS

1. How much work done it take to slide a create for a distance of 25 m along a loading dock by pulling on it with a 180 N force where the dock is at angle 45° from the horizontal? [2024]

- (a) 3.18198×10^3 J (b) 3.18198×10^2 J
(c) 3.4341×10^3 J (d) 3.4341×10^4 J

Ans. (a)

Sol. Work done

$$\begin{aligned} &= \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta = 180 \times 25 \times \cos 45^\circ \\ &= 180 \times 25 \times \frac{1}{\sqrt{2}} = \frac{180 \times 25 \times \sqrt{2}}{2} = 90 \times 25 \times 1.414213 \\ &= 3181.979 = 3.18198 \times 10^3 \text{ J} \end{aligned}$$

2. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(0) = \frac{1}{\pi}$

and $f(x) = \frac{x}{e^{\pi x} - 1}$ for $x \neq 0$. Then, [2024]

- (a) $f(x)$ is not continuous at $x = 0$.
(b) $f(x)$ is continuous but not differentiable at $x = 0$.
(c) $f(x)$ is differentiable at $x = 0$ and $f'(0) = -\frac{\pi}{2}$.
(d) None of the above

Ans. (b)

Sol. $f(x) = \frac{x}{e^{\pi x} - 1}, x \neq 0$

For continuity, at $x = 0$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\ &= \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{0-h}{e^{\pi(0-h)} - 1} \\ &= \lim_{h \rightarrow 0} \frac{h \cdot e^{\pi h}}{e^{\pi h} - 1} = \lim_{h \rightarrow 0} \frac{h \cdot e^{\pi h}}{e^{\pi h} - 1} (\pi h) \\ &= \lim_{h \rightarrow 0} \frac{e^{\pi h}}{\pi \cdot e^{\pi h} - 1} = \frac{1}{\pi} \left[\because \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right] \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} \frac{0+h}{e^{\pi(0+h)} - 1} \\ &= \lim_{h \rightarrow 0} \frac{h}{e^{\pi h} - 1} (\pi h) = \lim_{h \rightarrow 0} \frac{1}{\pi \left(\frac{e^{\pi h} - 1}{\pi h} \right)} = \frac{1}{\pi} \end{aligned}$$

$$\therefore \text{LHL} = \text{RHL} = f(0)$$

Hence, $f(x)$ is continuous at $x = 0$

For differentiability, at $x = 0$

$$\begin{aligned} Lf'(0) &= \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \rightarrow 0} \frac{\frac{-h}{e^{-\pi h} - 1} - \frac{1}{\pi}}{-h} \\ &= \lim_{h \rightarrow 0} \left[\frac{1}{e^{-\pi h} - 1} - \frac{1}{\pi h} \right] = \lim_{h \rightarrow 0} \left[\frac{e^{\pi h}}{-(e^{\pi h} - 1)} - \frac{1}{\pi h} \right] \end{aligned}$$

$$= \lim_{h \rightarrow 0} \left[\frac{e^{\pi h}}{\pi h \cdot \frac{e^{\pi h} - 1}{\pi h}} - \frac{1}{\pi h} \right] = (\infty - \infty) \text{ (Not exists)}$$

$$Rf'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\frac{h}{e^{\pi h} - 1} - \frac{1}{\pi}}{h} = \lim_{h \rightarrow 0} \left[\frac{1}{e^{\pi h} - 1} - \frac{1}{\pi h} \right] \\ &= (\infty - \infty) \text{ (Not exists)} \end{aligned}$$

Hence, $f(x)$ is not differentiable at $x = 0$

3. The value of the limit

$$\lim_{x \rightarrow 0} \left(\frac{x^x + 2^x + 3^x + 4^x}{4} \right)^{1/x} \text{ is [2024]}$$

- (a) 1 (b) $3!^{1/3!}$ (c) $3!^{1/4}$ (d) $4!^{1/4}$

Ans. (d)

Sol. By the formula,

$$\begin{aligned} \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{1/x} &= (abcd)^{1/4} \\ \therefore \lim_{x \rightarrow 0} \left(\frac{1^x + 2^x + 3^x + 4^x}{4} \right)^{1/x} &= (4 \times 3 \times 2 \times 1)^{1/4} = (4!)^{1/4} \end{aligned}$$

4. The value of m for which volume of the parallelopiped is 4 cubic units whose three edges are represented by $a = m\mathbf{i} + \mathbf{j} + \mathbf{k}, b = \mathbf{i} - \mathbf{j} + \mathbf{k}, c = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ is [2024]

- (a) -1 (b) 1 (c) 0 (d) -2

Ans.

Sol. (b) Volume of parallelopiped = $[abc]$

$$\Rightarrow \begin{vmatrix} 4 & & \\ & a & b & c \\ & m & 1 & 1 \\ & 1 & -1 & 1 \\ & 1 & 2 & -1 \end{vmatrix}$$

$$\Rightarrow 4 = m(1-2) - 1(-1-1) + 1(2+1)$$

$$4 = m(1-2) - 1(-1-1) + 1(2+1)$$

$$\Rightarrow 4 = -m + 2 + 3$$

$$\Rightarrow m = 1$$

5. Consider the function $f(x) = x^{2/3}(6-x)^{1/3}$. Which of the following statement is false? [2024]

- (a) f is increasing in the interval $(0, 4)$.
 (b) f is decreasing in the interval $(6, \infty)$.
 (c) f is a point of inflection at $x = 0$.
 (d) f has a point of inflection at $x = 6$.

Ans. (d)

Sol. $f(x) = x^{2/3}(6-x)^{1/3}$

$$f'(x) = \frac{2}{3}x^{-1/3}(6-x)^{1/3} - \frac{1}{3}x^{2/3}(6-x)^{-2/3}$$

$$= \frac{2(6-x)^{1/3}}{3x^{1/3}} - \frac{x^{2/3}}{3(6-x)^{2/3}}$$

$$= \frac{2(6-x) - x}{3x^{1/3}(6-x)^{2/3}} = \frac{(4-x)}{x^{1/3}(6-x)^{2/3}}$$

For increase or decrease of $f(x)$ check the sign of $f'(x)$



Hence, it is increasing in the interval $(0, 4)$ and decreasing $(4, \infty)$.

Hence, option (a) and (b) are right.

For point of inflection ($f''(x) = 0$ or $f''(x)$ Does not exist)

$$f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}$$

$$f'(x) = \frac{4-x}{x^{1/3} \cdot x^{2/3} \left(\frac{6}{x} - 1\right)^{2/3}} \Rightarrow f'(x) = \frac{\left(\frac{4}{x} - 1\right)}{\left(\frac{6}{x} - 1\right)^{2/3}}$$

$$f''(x) = \frac{\left(\frac{6}{x} - 1\right)^{2/3} \cdot \left(-\frac{4}{x^2}\right) - \left(\frac{4}{x} - 1\right) \cdot \frac{2}{3} \left(\frac{6}{x} - 1\right)^{-1/3} \cdot \left(-\frac{6}{x^2}\right)}{\left(\frac{6}{x} - 1\right)^{4/3}}$$

$$= \frac{(6-x)^{2/3} \cdot \left(\frac{-4}{x^{8/3}}\right) + \frac{4}{x^{8/3}}(4-x)(6-x)^{-1/3}}{x^{-4/3} \cdot (6-x)^{4/3}}$$

When $f''(x) = 0$, then $6-x = 0 \Rightarrow x = 6$

Point of inflection at $x = 6$

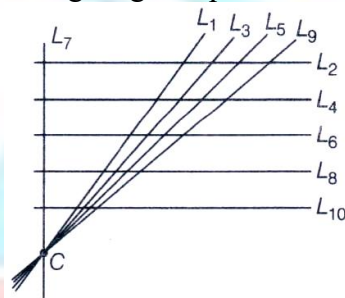
6. Lines $L_1, L_2, \dots, L_{10}$ are distinct among which the lines $L_2, L_4, L_6, L_8, L_{10}$ are parallel to each other and the lines L_1, L_3, L_5, L_7, L_9 pass through a given point C . The number of point of intersection of pairs of lines from the complete set $L_1, L_2, L_3, \dots, L_{10}$ is [2024]

- (a) 24 (b) 25 (c) 26 (d) 27

Ans. (c)

Sol. There are 10 lines $L_1, L_2, L_3, \dots, L_{10}$

$L_2, L_4, \dots, L_{10}$ are parallel, and L_1, L_3, L_5, L_7, L_9 pass through a given point C .



The number of point of intersection of pairs of lines $= 1 + 5 \times 5 = 26$

7. For an invertible matrix A , which of the following is not always true [2024]

- (a) $|\text{adj}(A)| \neq 0$ (b) $|A| \neq 0$
 (c) $|AA^{-1}| = 1$ (d) $|A(\text{adj}(A))| \neq 1$

Ans. (d)

Sol. For invertible matrix A , A should be non-singular i.e. $|A| \neq 0$

Hence, option (b) is always correct.

$$|\text{adj}(A)| = |A|^{n-1} \neq 0$$

Hence, option (a) is always correct.

$$|A^{-1}| = \frac{1}{|A|} \Rightarrow |AA^{-1}| = 1$$

Hence, option (c) is also correct.

$$A(\text{adj}A) = |A|I_n = (\text{adj}A)A = |A|^n \neq 0$$

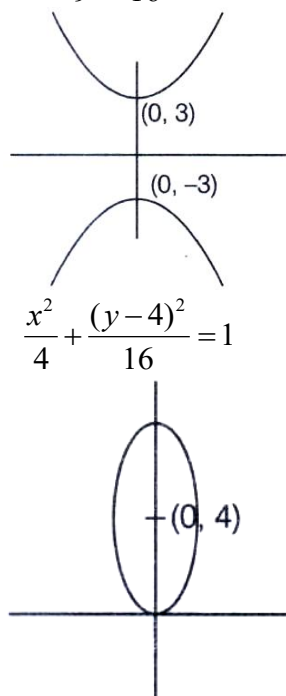
8. At how many points the following curves intersect

$$\frac{y^2}{9} - \frac{x^2}{16} = 1 \text{ and } \frac{x^2}{4} + \frac{(y-4)^4}{16} = 1 \quad [2024]$$

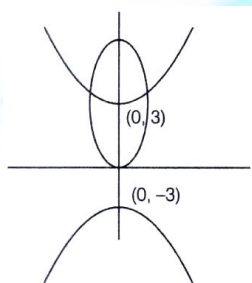
- (a) 0 (b) 1 (c) 2 (d) 4

Ans. (c)

Sol. $\frac{y^2}{9} - \frac{x^2}{16} = 1$



It is a vertical ellipses with centre (0, 4)



Hence, these curves are intersect at two points.

9. The value of $f(1)$ for $f\left(\frac{1-x}{1+x}\right) = x+2$ is [2024]

- (a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

Sol. $f\left(\frac{1-x}{1+x}\right) = x+2$

Put $x=0$

$\therefore f\left(\frac{1-0}{1+0}\right) = 0+2 \Rightarrow f(1) = 2$

10. A committee of 5 is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is [2024]

- (a) $5/9$ (b) $1/2$ (c) $2/3$ (d) $4/9$

Ans. (d)

Sol. Total number of ways to choose 5 peoples out of 9 peoples $= {}^9C_5 = 126$

Couple serves committee $= {}^7C_3 = 35$

Couple do not serves committee $= {}^7C_5 = 21$

Couple together serve or do not serve

So, favourable ways $= 21+35 = 56$

$\therefore \text{Probability} = \frac{56}{126} = \frac{4}{9}$

11. If $x = 1 + \sqrt[5]{2} + \sqrt[5]{4} + \sqrt[5]{8} + \sqrt[5]{16} + \sqrt[5]{32}$, then

$\left(1 + \frac{1}{x}\right)^{24} =$ [2024]

- (a) 1 (b) 4 (c) 16 (d) 24

Ans. (c)

Sol. $x = 1 + \sqrt[5]{2} + \sqrt[5]{4} + \sqrt[5]{8} + \sqrt[5]{16} + \sqrt[5]{32}$

$x = 1 + 2^{1/6} + (2^{1/6})^2 + (2^{1/6})^3 + (2^{1/6})^4 + (2^{1/6})^5$

$x = \frac{1(1 - (2^{1/6})^6)}{1 - 2^{1/6}} = \frac{1}{2^{1/6} - 1}$

Now, $\left(1 + \frac{1}{x}\right)^{24} = (1 + 2^{1/6} - 1)^{24} = 2^{24/6} = 2^4 = 16$

12. Among the given number below, the smallest number which will be divided by 9, 10, 15 and 20 leaves the remainders 4, 5, 10 and 15, respectively?

[2024]

- (a) 85 (b) 265 (c) 535 (d) 355

Ans. (d)

Sol. Find the smallest number which when divided by x, y, z leaves remainder a, b, c .

In this case always, $x-a = y-b = z-c = k$ (let)

LCM of x, y, z

LCM of 9, 10, 15, 20, = 180

$9-4 = 10-5 = 15-10 = 20-15 = 5(k)$

Numbers $= 180 - 5 = 175$

or $360 - 5 = 355$

13. Let A and B be two events defined on a sample space Ω . Suppose A^c denotes the complement of A relative to the sample space Ω . Then, the probability

$P((A \cap B^c) \cup (A^c \cap B))$ equals [2024]

- (a) $P(A) + P(B) + P(A \cap B)$
(b) $P(A) + P(B) - P(A \cap B)$
(c) $P(A) + P(B) + 2P(A \cap B)$
(d) $P(A) + P(B) - 2P(A \cap B)$

Ans. (d)

Sol. $P((A \cap B^c) \cup (A^c \cap B))$

$= P((A-B) \cup (B-A)) = P(A) + P(B) - 2P(A \cap B)$

$P(A \cap B^c) \cup (A^c \cap B) = P(A) + P(B) - 2P(A \cap B)$

14. A speaks truth in 40% and B in 50% of the cases. The probability that they contradict each other while narrating some incident is [2024]

- (a) $2/3$ (b) $1/4$ (c) $1/2$ (d) $1/3$

Ans. (c)

Sol. $P(A \text{ truth}) = 40\% = 0.4$

and $P(B \text{ truth}) = 50\% = 0.5$

$\therefore P(A \text{ false}) = 1 - 0.4 = 0.6$

and $P(B \text{ false}) = 1 - 0.5 = 0.5$

$\therefore$ Required probability $= P(A \text{ truth}) P(B \text{ truth})$

$+ P(A \text{ false}) P(B \text{ false})$

$$= 0.4 \times 0.5 + 0.6 \times 0.5 = 0.2 + 0.3$$

$$= 0.5 = \frac{5}{10} = \frac{1}{2}$$

15. The points $(1, 1/2)$ and $(-3, -1/2)$ are [2024]

(a) in between the lines $2x + 3y = 6$ and $2x + 3y = -6$

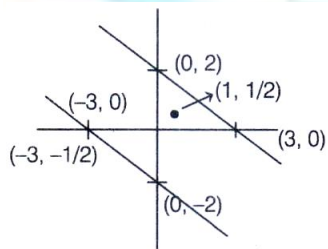
(b) on the opposite side of the line $2x + 3y = -6$

(c) on the same side of the line $2x + 3y = -6$

(d) on the same side of the line $2x + 3y = 6$

Ans. (a, c, d)

Sol. $L_1: \frac{x}{3} + \frac{y}{2} = 1, L_2: \frac{x}{-3} + \frac{y}{-2} = 1$



Hence, option (a), (c), (d) are correct.

16. If $(4, 3)$ and $(12, 5)$ are the two foci of an ellipse passing through the origin, then the eccentricity of the ellipse is [2024]

- (a) $\frac{\sqrt{13}}{9}$ (b) $\frac{\sqrt{13}}{18}$ (c) $\frac{\sqrt{17}}{18}$ (d) $\frac{\sqrt{17}}{9}$

Ans. (d)

Sol. If two foci are $S(4, 3)$ and $S'(12, 5)$ and the ellipse passes through origin O .

Then, $SO = \sqrt{16 + 9} = 5$

$$S'O = \sqrt{144 + 25} = 13$$

$$\text{And } SS' = \sqrt{68} = 2\sqrt{17}$$

As we know that for an ellipse $SO + S'O = 2a$ and

$$SS' = 2ae$$

$$\therefore e = \frac{SS'}{SO + S'O} = \frac{2\sqrt{17}}{18} = \frac{\sqrt{17}}{9}$$

17. For what values of λ does the equation [2024]

$6x^2 - xy + \lambda y^2 = 0$ represents two perpendicular lines and two lines inclined at angle of $\pi/4$.

- (a) -6 and -2 (b) 6 and 1
(c) -6 and -35 (d) -6 and 1

Ans. (c)

Sol. Given equation, $6x^2 - xy + \lambda y^2 = 0$

$$\therefore h = -\frac{1}{2}, a = 6, b = \lambda$$

For perpendicular lines $a + b = 0 \Rightarrow 6 + \lambda = 0 \Rightarrow \lambda = -6$

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{|a + b|}$$

$$\text{For angle } \theta = \frac{\pi}{4} \Rightarrow \tan \frac{\pi}{4} = \frac{2\sqrt{\frac{1}{4} - 6\lambda}}{|6 + \lambda|}$$

$$(6 + \lambda)^2 = 4 \left(\frac{1 - 24\lambda}{4} \right)$$

$$\left[\because \tan \frac{\pi}{4} = 1 \right]$$

$$\lambda^2 + 36 + 12\lambda = 1 - 24\lambda$$

$$\lambda^2 + 36\lambda + 35 = 0$$

$$(\lambda + 1)(\lambda + 35) = 0 \Rightarrow \lambda = -1, -35$$

Hence, $\lambda = -6$ and $\lambda = -35$

18. The value of $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{1 - \cos x}$ is equal to [2024]

- (a) 2 (b) 1 (c) 0 (d) -1

Ans. (c)

Sol. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{1 - \cos x} \left(\frac{0}{0} \text{ form} \right)$

Using L' Hospital Rule

$$= \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{\sin x}, \left(\text{again } \frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\cos x} = 0$$

19. The number of one-one functions

$f: \{1, 2, 3\} \rightarrow \{a, b, c, d, e\}$ is [2024]

- (a) 125 (b) 60 (c) 243 (d) None of these

Ans. (b)

Sol. Let f be a one-one function from a set A to the set B . Let $O(A) = m$ and $O(B) = n$ Here, $m \leq n$

$\therefore$ Total number of one-one functions from A to

$$B = n(n-1) \dots [n - (m-1)] = {}^n P_m$$

$$= {}^5 P_3 = \frac{5!}{2!} = 60$$

20. If one AM (Arithmetic Mean) 'a' and two GM's (Geometric Means) p and q be inserted between any two positive numbers, the value of $p^3 + q^3$ is [2024]
 (a) $2apq$ (b) pq/a (c) $2pq/a$ (d) $p+q+a$

Ans. (a)

Sol. Let the numbers be x and y. AM of x and y = a

$$\Rightarrow \frac{x+y}{2} = a \Rightarrow x+y = 2a \dots (i)$$

p and q be the GM's between x and y

$$\Rightarrow x, p, q, y \rightarrow GP \Rightarrow p^2 = xq \text{ and } q^2 = py$$

$$\Rightarrow p^3 = pxq \text{ and } q^3 = qpy$$

$$\text{Now, } p^3 + q^3 = pqx + pqy = pq(x+y) = 2apq$$

21. A coin is thrown 8 number of times. What is the probability of getting a head in an odd number of throw? [2024]

- (a) $3/4$ (b) $1/4$ (c) $1/2$ (d) $1/8$

Ans. (c)

Sol. In one throw of a coin, the number of possible ways is 2, since either head (H) or tail (T) may appear.

In two throws of a coin, the total number of ways is 2×2

i.e. 4, since corresponding to each way of the first coin there are 2 ways of second.

Similarly, in three throws of a coin, the number of ways is 2^3 and thus in n throws, the number of ways $N = 2^n$ and M = the favourable number of ways = the number of ways in which head will occur once or thrice or 5 times or 7 times and so on,

$$= {}^n C_1 + {}^n C_3 + {}^n C_5 + \dots = 2^{n-1}.$$

$$\text{Hence, required probability } p = \frac{M}{N} = \frac{2^{n-1}}{2^n} = \frac{1}{2}$$

22. The value of $\tan\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{3\pi}{4} + \theta\right)$ is [2024]

- (a) -2 (b) 2 (c) 1 (d) -1

Ans. (d)

$$\text{Sol. } \tan\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{3\pi}{4} + \theta\right)$$

$$\begin{aligned} &= \frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} \times \frac{\tan \frac{3\pi}{4} + \tan \theta}{1 - \tan \frac{3\pi}{4} \tan \theta} \\ &= \frac{1 + \tan \theta}{1 - \tan \theta} \times \frac{-1 + \tan \theta}{1 + \tan \theta} = -1 \end{aligned}$$

23. The value of $\sum_{r=1}^n \frac{1}{2^n} \frac{{}^n P_r}{r!}$ is [2024]

- (a) 2^n (b) $1 - 2^{-n}$ (c) $2^n - 1$ (d) $2^{2n} - 1$

Ans. (b)

$$\text{Sol. } \frac{{}^n P_r}{r!} = {}^n C_r$$

$$\begin{aligned} \sum_{r=1}^n \frac{1}{2^n} {}^n C_r &= \frac{1}{2^n} \sum_{r=1}^n {}^n C_r \\ &= \frac{1}{2^n} [{}^n C_1 + {}^n C_2 + {}^n C_3 + \dots + {}^n C_n] = \frac{2^n - 1}{2^n} = 1 - 2^{-n} \end{aligned}$$

24. Let C denote the set of all tuples (x, y) which satisfy $x^2 = 2^y$, where x and y are natural numbers. What is the cardinality of C? [2024]

- (a) 0 (b) 1 (c) 2 (d) 3

Ans. (c)

$$\text{Sol. } \because x^2 = 2^y \Rightarrow x, y \in N$$

there will be two solutions

$$x = y = 2 \text{ and } x = y = 4$$

$$(2, 2) \text{ and } (4, 4)$$

$$\therefore \text{The cardinality of } c = 2$$

25. The value of series $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \dots$, is [2024]

- (a) $2e^{-2}$ (b) e^{-2} (c) e^{-1} (d) $2e^{-1}$

Ans. (c)

Sol. We know that,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$$

$$S = \frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \dots \infty = \frac{3-1}{3!} + \frac{5-1}{5!} + \frac{7-1}{7!} + \dots \infty$$

$$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots \infty = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots \infty$$

$$= 1 + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!} + \frac{(-1)^5}{5!} + \dots \infty$$

$$= e^{-1}$$

26. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3 is [2024]

- (a) $4/33$ (b) $4/25$
 (c) $4/1155$ (d) $4/35$

Ans. (c)

Sol. The numbers are divisible by both 2 and 3 means numbers divisible by 6.

Number divisible by 6 in 1 to 100 are

$$\{6, 12, 18, 24, \dots, 96\} = 16 \text{ numbers}$$

Number of ways of selecting 3 of them are ${}^{16}C_3$.

$$\text{Required probability} = \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{16 \times 15 \times 14}{100 \times 99 \times 98} = \frac{4}{1155}$$

27. If the line $a^2x + ay + 1 = 0$, for some real number a , is normal to the curve $xy = 1$, then [2024]

- (a) $a < 0$ (b) $0 < a < 1$
(c) $a > 0$ (d) $-1 < a < 1$

Ans. (a)

Sol. $xy = 1$

$\Rightarrow$ Either x , and $y > 0$ or x and $y < 0$

$$x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} < 0$$

$$\text{Slope of normal} = \frac{x}{y} > 0$$

$$\text{Slope of line } ay = -a^2x \cdot 1$$

$$y = -ax - \frac{1}{a}$$

$$\text{Hence, } -a > 0 \Rightarrow a < 0$$

28. Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then, which of the

following is true

[2024]

- (a) $f(x)$ is not continuous at $x = 0$
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f'(x)$ is not continuous at $x = 0$
(d) $f'(x)$ is continuous at $x = 0$

Ans. (c)

Sol. For $x \neq 0$, we have $f(x) = x^2 \sin \left(\frac{1}{x} \right)$

For continuity at $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h)$$

$$= \lim_{h \rightarrow 0} (0 - h)^2 \sin \left(\frac{1}{0 - h} \right)$$

$$= \lim_{h \rightarrow 0} h^2 \cdot \sin \left(\frac{1}{h} \right) = \lim_{h \rightarrow 0} h \cdot \frac{\sin \left(\frac{1}{h} \right)}{\frac{1}{h}} = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h)$$

$$= \lim_{h \rightarrow 0} (0 + h)^2 \sin \left(\frac{1}{0 + h} \right)$$

$$= \lim_{h \rightarrow 0} h^2 \cdot \sin \left(\frac{1}{h} \right) = \lim_{h \rightarrow 0} h \cdot \left(\frac{\sin \frac{1}{h}}{\frac{1}{h}} \right) = 0$$

$$\therefore \text{LHL} = \text{RHL} = f(0)$$

$\therefore f(x)$ is continuous at $x = 0$

For differentiability at $x = 0$

$$\begin{aligned} \text{Lf}'(0) &= \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{-h} \\ &= \lim_{h \rightarrow 0} \frac{(-h)^2 \sin \left(\frac{1}{-h} \right)}{(-h)} = \lim_{h \rightarrow 0} \frac{\sin \left(\frac{1}{h} \right)}{\left(\frac{1}{h} \right)} = 0 \end{aligned}$$

$$\text{Rf}'(0) = \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{h} = \frac{h^2 \sin \left(\frac{1}{h} \right)}{h} = 0$$

$$\text{Lf}'(0) = \text{Rf}'(0)$$

Hence, it is differentiable at $x = 0$

$$f'(x) = 2x \sin \frac{1}{x} + x^2 \cos \left(\frac{1}{x} \right) \left(-\frac{1}{x^2} \right) = 2x \sin \frac{1}{x} - \cos \frac{1}{x}$$

$$\text{Now, } f'(x) = \begin{cases} 2x \sin \frac{1}{x} - \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f'(x) = \lim_{h \rightarrow 0} \left[-2h \sin \frac{1}{-h} - \cos \frac{1}{-h} \right] \\ &= \lim_{h \rightarrow 0} 2 \cdot \frac{\sin \frac{1}{h}}{\frac{1}{h}} - \lim_{h \rightarrow 0} \cos \frac{1}{h} = \text{Not exists} \end{aligned}$$

Similarly RHL = Not exists.

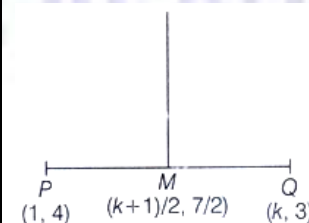
Hence, f' is not continuous at $x = 0$.

29. If the perpendicular bisector of the line segment joining $p(1, 4)$ and $q(k, 3)$ has y -intercept -4 , then the possible values of k are [2024]

- (a) -2 and 2 (b) -1 and 1
(c) -3 and 3 (d) -4 and 4

Ans. (d)

Sol.



$$\text{Slope of line } PQ = m_{PQ} = \frac{3 - 4}{k - 1} = \frac{-1}{k - 1}$$

$$\text{Slope of line perpendicular to } PQ = m_{\perp} = k - 1$$

Equation of perpendicular bisector of line PQ

$$y - \frac{7}{2} = (k-1) \left(x - \frac{k+1}{2} \right)$$

$$\frac{2y-7}{2} = \frac{(k-1)}{2} (2x-k-1)$$

For Y - axis intercept put $x=0$

$$2y-7 = -(k-1)(k+1) \Rightarrow 2y = 7 - (k^2 - 1)$$

$$\begin{aligned} -8 &= 8 - k^2 \text{ put } y = -4 \\ k^2 &= 16 \Rightarrow k = \pm 4 \end{aligned}$$

30. The equation $3x^2 + 10xy + 11y^2 + 14x + 12y + 5 = 0$ represents **[2024]**

- (a) a circle (b) an ellipse
(c) a hyperbola (d) a parabola

Ans. (b)

Sol. Given equation,

$$3x^2 + 10xy + 11y^2 + 14x + 12y + 5 = 0$$

Comparing this equation by,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$a=3, h=5, b=11, g=7, f=6, c=5$$

$$\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2$$

$$\begin{aligned} &= (3 \times 11 \times 5) + 2(6)(7)(5) - 3 \times 36 - 11 \times 49 - 5 \times 25 \\ &= 165 + 420 - 108 - 539 - 125 \end{aligned}$$

$$= 585 - 772 = -187 \neq 0 \text{ and } h^2 - ab = 25 - 33 = -8$$

$$\therefore \Delta \neq 0 \text{ and } h^2 - ab < 0$$

So, the given equation represents an ellipse.

31. Out of a group of 50 students taking examinations in Mathematics, Physics and Chemistry, 37 students passed Mathematics, 24 passed Physics, and 43 passed Chemistry. Additionally, no more than 19 students passed both Mathematics and Physics, no more than 29 passed both Mathematics and Chemistry and no more than 20 passed both Physics and Chemistry. What is the maximum number of students who could have passed all three examinations? **[2024]**

- (a) 12 (b) 9 (c) 14 (d) 10

Ans. (c)

Sol. Let M be the set of students passing in Mathematics, P be the set of students passing in Physics and C be the set of students passing in Chemistry.

$$\text{Given, } n(M \cup P \cup C) = 50, n(M) = 37,$$

$$n(P) = 24, n(C) = 43, n(M \cap C) \leq 29, n(M \cap P) \leq 19,$$

$$n(P \cap C) \leq 20$$

Now,

$$n(M \cup P \cup C) = n(M) + n(P) + n(C)$$

$$-n(M \cap P) - n(P \cap C) + n(M \cap P \cap C) \leq 50$$

$$\Rightarrow 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50$$

$$\Rightarrow n(M \cap P \cap C) \leq 50 - 36 \Rightarrow n(M \cap P \cap C) \leq 14$$

Thus, the largest possible number of students that could have passed in all the three examinations is 14.

32. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where $[\cdot]$

stands for the greatest integer function, then $f\left(\frac{\pi}{2}\right) =$

[2024]

- (a) -1 (b) 0 (c) 1 (d) 2

Ans. (a)

$$f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$$

$$\text{Sol. } = \cos 9x + \cos(-10x) \quad (\because \pi^2 = 9.85 \text{ app.})$$

$$= \cos 9x + \cos 10x$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \cos 9\left(\frac{\pi}{2}\right) + \cos 10\left(\frac{\pi}{2}\right) = \cos \frac{9\pi}{2} + \cos 5\pi = 0 - 1 = -1$$

33. If for non-zero x , $cf(x) + df\left(\frac{1}{x}\right) = |\log|x|| + 3$,

where $c \neq d$, then $\int^e f(x) dx =$ **[2024]**

- (a) $\frac{(c^1-d)(2e-1)}{c^2-d^2}$ (b) $\frac{(c-d)(3e-2)}{c^2-d^2}$
(c) $\frac{(c-d)(3e+2)}{c^2-d^2}$ (d) $\frac{(c-d)(2e+1)}{c^2-d^2}$

Ans. (b)

$$\text{Sol. } cf(x) + df\left(\frac{1}{x}\right) = |\log|x|| + 3 \quad \dots(i)$$

$$x \rightarrow \frac{1}{x}$$

$$cf\left(\frac{1}{x}\right) + df(x) = \left|\log\left|\frac{1}{x}\right|\right| + 3$$

$$df(x) + cf\left(\frac{1}{x}\right) = |\log|x|| + 3 \quad \dots(ii)$$

We have to calculate, $\int_1^e f(x) dx$

$$\text{Hence, } 1 < x < e \Rightarrow |\ln|x|| = \ln x$$

Multiplying Eq. (i) by c and Eq. (ii) by d and subtract each other.

$$c^2 f(x) + cdf\left(\frac{1}{x}\right)$$

$$-d^2 f(x) - cdf\left(\frac{1}{x}\right) = c \log|x| + 3c - d |\log|x|| + 3d$$

$$f(x) = \frac{(c-d)(\log x + 3)}{(c^2-d^2)}$$

$$c \neq d$$

$$f(x) = \frac{\ln x + 3}{(c+d)}$$

$$\int_1^e f(x) dx = \int_1^e \frac{(\ln x + 3) dx}{(c+d)} = \frac{1}{c+d} \left\{ \int_1^e \ln x dx + 3 \int_1^e dx \right\}$$

$$(x \ln x - x)_1^e + 3(x)_1^e$$

$$= \frac{1}{c+d} \left[(e \ln e - e - (\ln 1 - 1)) + 3(e - 1) \right]$$

$$= \frac{1}{c+d} [3e - 2] = \frac{(c-d)(3e-2)}{(c^2-d^2)}$$

34. Find the cardinality of the set C which is defined

$$\text{as } C = \left\{ x : \sin 4x = \frac{1}{2} \text{ for } x \in (-9\pi, 3\pi) \right\} \quad [2024]$$

$$(a) 24 \quad (b) 48 \quad (c) 36 \quad (d) 12$$

Ans. (b)

$$\text{Sol. } C = \left\{ x : \sin 4x = \frac{1}{2}, x \in (-9\pi, 3\pi) \right\}$$

$$\text{As, } -9\pi < x < 3\pi \Rightarrow -36\pi < 4x < 12\pi$$

We know that, for $\theta \in (0, 2\pi); \sin \theta = \frac{1}{2}$ for only 2

values of θ

$$\theta = \pi/6 \text{ and } 5\pi/6$$

Hence, for $4x \in (-36, 12\pi)$ there are total 24 intervals of $(0, 2\pi)$.

Hence, total number of solutions = 48.

35. The number of distinct values of λ for which the

vectors $\lambda^2 \mathbf{i} + \mathbf{j} + \mathbf{k}, \mathbf{i} + \lambda^2 \mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} + \lambda^2 \mathbf{k}$ are

coplanar is [2024]

$$(a) 1 \quad (b) 2 \quad (c) 3 \quad (d) 6$$

Ans. (b)

Sol. For coplanar vectors $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 0$

$$\Rightarrow \begin{vmatrix} \lambda^2 & 1 & 1 \\ 1 & \lambda^2 & 1 \\ 1 & 1 & \lambda^2 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$\begin{vmatrix} \lambda^2 + 2 & 1 & 1 \\ \lambda^2 + 2 & \lambda^2 & 1 \\ \lambda^2 + 2 & 1 & \lambda^2 \end{vmatrix} = 0 \Rightarrow (\lambda^2 + 2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & 1 \\ 1 & 1 & \lambda^2 \end{vmatrix} = 0$$

$$\lambda^2 + 2 \neq 0$$

$$R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$$

$$\begin{vmatrix} 0 & 1-\lambda^2 & 0 \\ 0 & \lambda^2-1 & 1-\lambda^2 \\ 1 & 1 & \lambda^2 \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda^2)^2 = 0 \Rightarrow \lambda = \pm 1$$

$\therefore$ The number of distinct value of $\lambda = 2$

36. The number of solution of $5^{1+|\sin x|+|\sin x|^2+\dots} = 25$ of $x \in (-\pi, \pi)$ is [2024]

$$(a) 2 \quad (b) 0 \quad (c) 4 \quad (d) \text{infinite}$$

Ans. (c)

$$\text{Sol. } 5^{1+|\sin x|+|\sin x|^2+\dots=25} \Rightarrow 5^{\frac{1}{1-|\sin x|}} = 5^2$$

$$\Rightarrow \frac{1}{1-|\sin x|} = 2 \Rightarrow \sin x = \pm \frac{1}{2}$$

Hence, number of solutions for $x \in (-\pi, \pi) = 4$

37. Let Z be the set of all integers and consider the set

$$X = \{(x, y) : x^2 + 2y^2 = 3, x, y \in Z\} \text{ and}$$

$$Y = \{(x, y) : x > y, x, y \in Z\}. \text{ Then, the number of}$$

elements in $X \cap Y$ is [2024]

$$(a) 2 \quad (b) 1 \quad (c) 3 \quad (d) 4$$

Ans. (b)

$$\text{Sol. } X = \{(x, y) : x^2 + 2y^2 = 3, x, y \in Z\}$$

$$\text{and } Y = \{(x, y) : x > y, x, y \in Z\}$$

$$\therefore X = \{(1, 1), (1, -1), (-1, 1), (-1, -1)\} \text{ and } Y = \{(1, -1)\}$$

$$\therefore X \cap Y = \{(1, -1)\}$$

The number of elements of $X \cap Y = 1$

38. If $\sin x = \sin y$ and $\cos x = \cos y$, then the value of $x - y$ is [2024]

$$(a) \frac{\pi}{4} \quad (b) \frac{n\pi}{2} \quad (c) n\pi \quad (d) 2n\pi$$

Ans. (d)

Sol. We know that,

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$= \cos x \cos x + \sin x \sin x = \cos^2 x + \sin^2 x = 1$$

$$\Rightarrow \cos(x - y) = 1 \Rightarrow x - y = 2n\pi$$

39. Which of the following is true? [2024]

(a) If f is continuous on $[a, b]$, then

$$\int_a^b x f(x) dx = x \int_a^b f(x) dx$$

$$(b) \int_0^3 e^{x^2} dx = \int_0^5 e^{x^2} dx + \int_5^3 e^{x^2} dx$$

(c) If f is continuous on $[a, b]$, then

$$\frac{d}{dx} \left(\int_a^b f(x) dx \right) = f(x)$$

(d) Both (a) and (b)

Ans. (b)

Sol. Option (a) is clearly wrong option (b).

$$\begin{aligned} \int_0^3 e^{x^2} dx &= \int_0^5 e^{x^2} dx + \int_5^3 e^{x^2} dx \\ &= \int_0^3 e^{x^2} dx + \int_3^5 e^{x^2} dx - \int_3^5 e^{x^2} dx = \int_0^3 e^{x^2} dx \end{aligned}$$

Hence, option (b) is right.

40. The vector $\mathbf{A} = (2x+1)\hat{i} + (x^2-6y)\hat{j} + (xy^2+3z)\hat{k}$ is a [2024]

- (a) sink field (b) solenoidal field
(c) source field (d) None of these

Ans. (a)

Sol. $\mathbf{A} = (2x+1)\hat{i} + (x^2-6y)\hat{j} + (xy^2+3z)\hat{k}$

Here $X = 2x+1, Y = x^2-6y$ and $Z = xy^2+3z$

$$X' = 2, Y' = -6, Z' = 3$$

There are X', Y' and Z' are constant.

So, it is a sink field.

41. Given a set A with median $m_1 = 2$ and set B with median $m_2 = 4$. What can we say about the median of the combined set? [2024]

- (a) atmost 1 (b) atmost 2
(c) atleast 1 (d) atleast 2

Ans. (d)

Sol. Let m_1 be the median of n_1 data and m_2 be the median of n_2 data. Then $(n_1 + n_2)$ data combined mean

$$m_1 < m < m_2.$$

Hence, median of combined set lies between $(2, 4)$.

42. Consider the function [2024]

$$f(x) = \begin{cases} -x^3 + 3x^2 + 1 & \text{if } x \leq 2 \\ \cos(x), & \text{if } 2 < x \leq 4 \\ e^{-x}, & \text{if } x > 4 \end{cases}$$

Which of the following statement about $f(x)$ is true?

- (a) $f(x)$ has a local maximum at $x = 1$, which is also the global maximum.
(b) $f(x)$ has a local maximum at $x = 2$, which is not the global maximum.
(c) $f(x)$ has a local maximum at $x = \pi$, but it is not the global maximum.
(d) $f(x)$ has a global maximum at $x = 0$

Ans. (b)

Sol. For $x \leq 2$

$$\begin{aligned} f''(x) &= -3x^2 + 6x = 0 \\ -3x(x-2) &= 0 \Rightarrow x = 0, 2 \\ f''(x) &= -6x + 6 \Rightarrow f''(2) < 0 \end{aligned}$$

Hence, at $x = 0$ function has local maxima.

43. The two parabolas $y^2 = 4a(x+c)$ and $y^2 = 4bx, a > b > 0$ cannot has a common normal unless [2024]

- (a) $c > 2(a+b)$ (b) $c > 2(a-b)$
(c) $c < 2(a-b)$ (d) $c < \frac{2}{a-b}$

Ans. (b)

Sol. Equation of normal to

$$y^2 = 4bx \text{ is } y = mx - 2bm - bm^3 \dots(i)$$

Equation of normal to $y^2 = 4a(x+c)$ is

$$y = m(x+c) - 2am - am^3 \dots(ii)$$

Condition for common normal

$$\begin{aligned} mx - y - (2bm + bm^3) &= 0 \\ mx - y - (2am + am^3 - mc) &= 0 \text{ both are identical} \\ 2bm + b^3 &= 2am + am^3 - mc \\ \Rightarrow 2(a-b)m + (a-b)m^3 - mc &= 0 \\ \Rightarrow m[2(a-b) + (a-b)m^2 - c] &= 0 \\ \Rightarrow m^2 = \frac{c-2(a-b)}{(a-b)} > 0 \\ \Rightarrow [a > b] \Rightarrow c - 2(a-b) > 0 &\Rightarrow c > 2(a-b) \end{aligned}$$

44. The system of equations

$x+2y+2z=5, x+2y+3z=6, x+2y+\lambda z=\mu$ has infinitely many solutions if [2024]

- (a) $\lambda \neq 2$ (b) $\lambda \neq 2, \mu \neq 5$
(c) $\lambda = 2, \mu = 5$ (d) $\mu \neq 5$

Ans. (c)

Sol. Given equations, $x+2y+2z=5 \dots(i)$

$$x+2y+3z=6 \dots(ii)$$

$$\text{and } x+2y+\lambda z=\mu \dots(iii)$$

$$[A:B] = \begin{bmatrix} 1 & 2 & 2 & : & 5 \\ 1 & 2 & 3 & : & 6 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 & : & 5 \\ 0 & 0 & 1 & : & 1 \\ 0 & 0 & \lambda-2 & : & \mu-5 \end{bmatrix}$$

$$(R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1)$$

$\therefore$ The system of equations has infinitely many solutions then rank of $[A] = \text{Rank of } [A:B]$

$$\therefore \lambda - 2 = 0 \text{ and } \mu - 5 = 0 \Rightarrow \lambda = 2, \mu = 5$$

45. It is given that the mean, median and mode of a data set is $1, 3^x$ and 9^x respectively. The possible values of the mode is [2024]

- (a) 1,4 (b) 1,9 (c) 3,9 (d) 9,8

Ans. (a)

Sol. $3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$

$$\Rightarrow 3 \times 3^x = 9^x + 2 \Rightarrow 9^x - 3 \times 3^x + 2 = 0 \Rightarrow 3^x = 2, 1$$

Hence, mode $= 9^x = 4, 1$

46. If $|F| = 40 \text{ N}$ (Newton), $|D| = 3 \text{ m}$, and $\theta = 60^\circ$, then the work done by F acting from P to Q is [2024]

- (a) $60\sqrt{3} \text{ J}$ (b) 120 J (c) $60\sqrt{2} \text{ J}$ (d) 60 J

Ans. (d)

Sol. $W = Fd \cos 60^\circ = 40 \times 3 \times \frac{1}{2} = 60 \text{ Joule}$

47. A man starts at the origin O and walks a distance of 3 units in the North-East direction and then walks a distance of 4 units in the North-West direction to reach the point P . Then, $\overrightarrow{OP}$ is equal to [2024]

- (a) $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$ (b) $\frac{1}{2}(\hat{i} + \hat{j})$
(c) $\frac{1}{\sqrt{2}}(\hat{i} - 7\hat{j})$ (d) $\frac{1}{\sqrt{2}}(-\hat{i} + 7\hat{j})$

Ans. (d)

Sol. Unit vector in the direction of North- East

$$\hat{v}_{NE} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

Unit vector along North-West direction $\hat{v}_{NW} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}}$

A man starts at the origin O and walks a distance of 3

units in the North-East direction $\overrightarrow{OA} = 3 \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right)$

Then, walks a distance of 4 units in the North-West

direction $\overrightarrow{AP} = 4 \left(\frac{-\hat{i} + \hat{j}}{\sqrt{2}} \right)$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \frac{-\hat{i} + 7\hat{j}}{\sqrt{2}}$$

48. There are 9 bottles labelled $1, 2, 3, \dots, 9$ and 9 boxes labelled $1, 2, 3, \dots, 9$. The number of ways one can put these bottles in the boxes, so that each box gets one bottle and exactly 5 bottles go in their corresponding numbered boxes is [2024]

- (a) $9 \times {}^9C_5$ (b) $5 \times {}^9C_5$
(c) $25 \times {}^9C_5$ (d) $4 \times {}^9C_5$

Ans. (a)

Sol. The number of ways to put exactly 5 bottles to in their corresponding numbered boxes $= {}^9C_5 \times 1$

Now, remaining 4 bottles can not go to their corresponding boxes.

Here we use formula of derangement

$$n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \dots + (-1)^n \frac{1}{n!} \right]$$

$$4! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right] = 9$$

Hence, total number of ways $= 9 \times {}^9C_5$.

49. A critical orthopedic surgery is performed on 3 patients. The probability of recovering a patient is 0.6. Then the probability that after surgery, exactly two of them will recover is [2024]

- (a) 0.321 (b) 0.234 (c) 0.432 (d) 0.123

Ans. (c)

Sol. Probability that patient will recover $= 0.6$

Probability that patient will die $= 0.4$

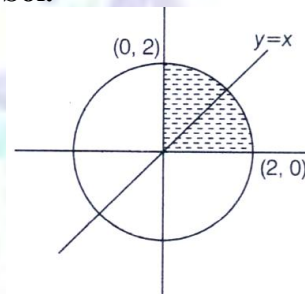
Probability that after surgery, exactly two of them will recover $= {}^3C_2 (0.6)^2 (0.4)^1 = 0.432$

50. Region R is defined as region in first quadrant satisfying the condition $x^2 + y^2 < 4$. Given that a point $p = (r, s)$ lies in R , what is the probability that $r > s$? [2024]

- (a) 1 (b) 0 (c) $1/2$ (d) $1/3$

Ans.

Sol.



Region R is defined as region in first quadrant satisfying the condition $x^2 + y^2 < 4$ is the shaded region of diagram.

$y = x$ line divide this region into two equal parts

Given that a point $p = (r, s)$ lies in R , probability that

$$r > s = \frac{(\pi 2^2)/4}{(\pi 2^2)/8} = \frac{1}{2}$$

Because favourable region is the part of graph below

the line $y = x$.

And sample space area of quarter circle.

Analytical Ability & Logical Reasoning

51. Aryan bought 100 shares of a company at ₹ 50 per share. He paid a brokerage fee of 2% on the purchase. Later, he sold all the shares at ₹ 55 per share and paid a brokerage fee of 2% on the sale. What is Aryan's net profit percentage on his investment? **[2024]**

- (a) 6% (b) 5.5% (c) 6.1% (d) 5.69%

Ans. (d)

Sol. According to the question,

$$\text{Total CP of shares} = 100 \times 50 + (100 \times 50) \times 2\%$$

$$= 5000 + 100$$

$$= ₹ 5100$$

$$\text{And Total SP of shares} = 100 \times 55 - (100 \times 55) \times 2\%$$

$$= 5500 - 110 = ₹ 5390$$

$$\therefore \text{Net profit} = \frac{5390 - 5100}{5100} \times 100 = 5.69\%$$

52. Four friends, Aditi, Bharat, Chandan and Deepika went to a restaurant for dinner. Each of them ordered a different dish from the menu: pizza, pasta, burger and salad. Additionally, each friend ordered a different drink: cola, lemonade, orange juice and water. Based on the following clues, determine the combination of friend, dish and drink: **[2024]**

- Aditi didn't order pizza or cola.
- Bharat ordered salad but not lemonade.
- Chandan ordered pasta.
- Deepika didn't order burger or orange juice.
- Aditi ordered orange juice.

Who ordered the burger and what drink did they order?

- (a) Aditi, orange juice (b) Bharat, water
(c) Chandan, lemonade (d) Deepika, cola

Ans. (a)

Sol. According to the question,

	Pas ta	Piz za	Bu rge r	S al ad	Co la	Le mo nad e	Or ang e Jui ce	Wat er
Aditi	×	×	√	×	×	×	√	×
Bhara t	×	×	×	√		×	×	
Chand an	√	×	×	×			×	
Deepi ka	×		×	×			×	

Clearly, Aditi ordered burger and she ordered orange juice

53. Odometer is to mileage as Compass is to

- (a) Needle (b) Speed (c) Direction (d) Hiking

Ans. (c)

Sol. As, an odometer is an instrument used to measure mileage. Similarly, a compass is an instrument used to determine the direction.

54. The mean of consecutive positive integers from 2 to n is

- (a) $\frac{n+2}{2}$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{n+1}{2}$ (d) $\frac{n-1}{2}$

Ans. (a)

Sol. We know that, sum of 1 to n terms $= \frac{n(n+1)}{2}$

$$\text{Now, sum of 2 to } n \text{ terms} = \frac{n(n+1)}{2} - 1$$

$$\therefore \text{Mean of 2 to } n \text{ terms} = \frac{\frac{n(n+1)}{2} - 1}{n-1}$$

$$= \frac{n^2 + n - 2}{2(n-1)} = \frac{n^2 - 1 + n - 1}{2(n-1)}$$

$$= \frac{(n+1)(n-1) + (n-1)}{2(n-1)}$$

$$= \frac{(n-1)[n+1+1]}{(n-1) \cdot 2} = \frac{n+2}{2}$$

55. If 30th September, 1991 was a Wednesday, then what was the day on 14th March 1992?

- (a) Sunday (b) Saturday
(c) Wednesday (d) Monday

Ans. (d)

Sol. Given, on 30th September 1991 = Wednesday

Now, actual day on 30th September 1991

$$\Rightarrow 91 \leftarrow \text{Year}$$

$$30 \leftarrow \text{Date}$$

$$22 \leftarrow \text{Leap year}$$

$$\frac{+6}{149} \leftarrow \text{Code}$$

$$\frac{14}{7} = 2 \text{ (odd days)}$$

Now, actual day on 30th September 1991 = Monday (2 code) Then, Monday + 2 = Wednesday

$\therefore$ Actual day on 14th March 1992 $\Rightarrow 92 \leftarrow \text{Year}$

$$14 \quad \leftarrow \text{Date}$$

$$23 \quad \leftarrow \text{Leap year}$$

$$+4 \quad \leftarrow \text{Code}$$

$$\frac{133}{7} = 0 \text{ (odd day)}$$

So, Actual day on 14th March 1992
= Saturday(0 code)+2 = Monday

56. In the following question, three statements and three conclusions are given. [2024]

Statements

All students are intelligent.
No intelligent person is lazy.
Some lazy people are poor

Conclusions

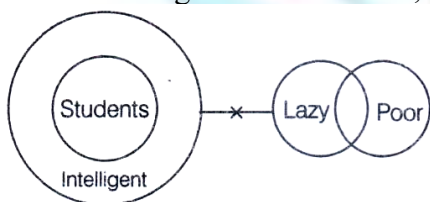
1. No student is lazy.
2. Some poor people are not intelligent.
3. All poor people are lazy.

Find out the most appropriate conclusion(s) from the following options.

- (a) Only Conclusions 1 and 2 follow
- (b) Only Conclusion 1 follows
- (c) Only Conclusion 2 follows
- (d) Only Conclusions 2 and 3 follow

Ans. (a)

Sol. According to the statements,



Conclusions

1. (✓) 2. (✓) 3. (x)

Hence, only Conclusions 1 and 2 follow.

57. You are on an island with two tribes. One tribes always tells the truth, and the other tribe always lies. You meet three individuals from the island: A, B and C. Each individual belongs to one of the tribes. You ask each of them the same question: "Is B a truth-teller?" Here are their responses:

A says, "Yes, B is a truth-teller."

B says, "No, I am not a truth-teller"

C says, " B is a liar."

Given that each individual is either a truth-teller or a liar, who is telling the truth? [2024]

- (a) Both B and C (b) A only
- (c) C only (d) B only

Ans. (c)

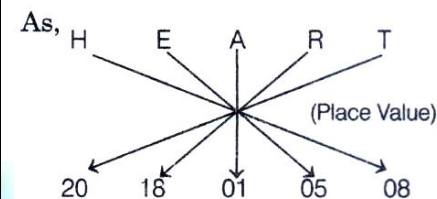
Sol. According to the given situation. Only (C) is telling the truth. Hence, option (c) is correct.

58. In certain languages, HEART is written as 2018010508, and LUNGS is written as 1907142112. If BRAIN is written in that language, what will be the last number? [2024]

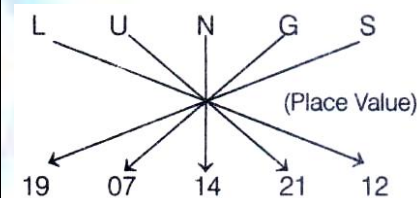
- (a) 5 (b) 9 (c) 4 (d) 2

Ans. (d)

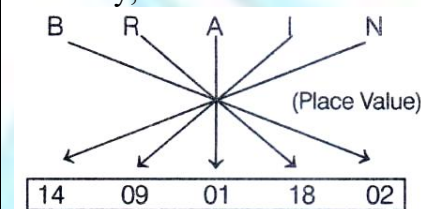
Sol.



and



Similarly,



Clearly, last number in the code is 2

59. Study the following information carefully and answer the given question:

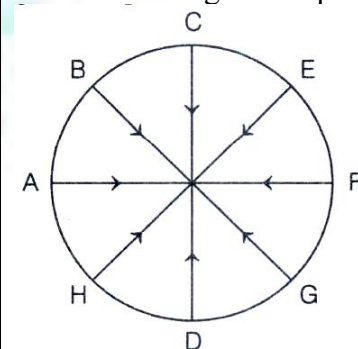
Eight friends A, B, C, D, E, F, G, and H are sitting on a round table facing the centre. A sits second to the left of D, who sits third to the left of E. C sits third to the right of G, who is not an immediate neighbour of E. H sits opposite to the E. B is between A and C.

Who sits opposite to A? [2024]

- (a) E (b) G (c) D (d) F

Ans. (d)

Sol. According to the question,



Clearly, F sits opposite to A.

60. Select the pair of words, which are related in the same way as the capitalized words are related to each other. [2024]

DATA : GRAPH

- (a) Mother : Father (b) Milk : Butter
(c) Water : Glass (d) Plant : Leaf

Ans. (b)

Sol. As, data is represented by graph. Similarly, butter is made by milk.

61. After allowing 20% cash discount, a trader still earns a profit of 11.11%. How much above the cost price, the trader marks his goods? [2024]

- (a) 40% (b) 30.33% (c) 28% (d) 38.89%

Ans. (d)

Sol. Let the CP of goods = ₹100

$$\text{Then, SP} = 100 \times \left(\frac{100 + 11.11}{100} \right) = ₹ 111.11$$

$$\text{We know that, SP} = \text{MP} \left(\frac{100 - d\%}{100} \right)$$

$$\Rightarrow 111.11 = \text{MP} \times \frac{80}{100} \Rightarrow \text{MP} = ₹ 138.89$$

$$\therefore \text{Above percentage} = \frac{138.89 - 100}{100} \times 100 = 38.89\%$$

62. Select the one which is different from the other three. [2024]

- (a) HEM (b) NKS (c) JGP (d) OLT

Ans. (c)

Sol. Here,

$$\text{H}^{-3}\text{E}^{+8} \rightarrow \text{M}; \quad \text{N}^{-3}\text{K}^{+8} \rightarrow \text{S};$$

$$\text{J}^{-3}\text{G}^{+9} \rightarrow \text{P}; \quad \text{O}^{-3}\text{L}^{+8} \rightarrow \text{T};$$

Hence, JGP is odd one.

63. Ramu visits Delhi on every 15 days and Samu goes to Delhi every 20 days. They met at Delhi 5 days back. After how many days, from today, they will meet at Delhi next time? [2024]

- (a) 35 (b) 60 (c) 55 (d) 65

Ans. (c)

Sol. Required number of days = LCM of (15 and 20) = 60

$$\therefore \text{From today they will meet at Delhi at next time} = 60 - 5 = 55 \text{ days}$$

64. Which pairs of bits can be joined together to form two words that have opposite meanings? [2024]

ERT UCE DES END EXP EAR AND AND SIP RED GOS

- (a) (9,2), (5,7) (b) (1,3), (8,10)
(c) (1,5), (10,8) (d) (4,2), (7,8)

Ans. (a)

Sol. From option (a), we get

9 2 5 7
↓ ↓ ↓ ↓
REDUCE EXPAND

Clearly, both letters are opposite to each other. Hence, option (a) is correct.

65. At what time between 2 pm and 3 pm, will the hour and minute hands of a clock in opposite directions (diametrically opposite)? [2024]

- (a) 2:45pm (b) 2:44pm
(c) $2:43\frac{9}{11}$ pm (d) $2:43\frac{7}{11}$ pm

Ans. (d)

Sol. At 2 pm it is 10 minute space between hands of clock. For the hands opposite to each other should be 40 minute space between the hands of clock.

We know that, In 60 min it is 55 min space.

$$\text{Then, in 40 min space} = 40 \times \frac{60}{55} = 43\frac{7}{11}$$

So, $43\frac{7}{11}$ minutes from 2, the hands will be opposite to each other.

$$\therefore \text{Required time} = 2:43\frac{7}{11} \text{pm}$$

66. In which year was Arjun born?

Arjun at present is 25 years younger to his mother.

Arjun's brother, who was born in 1964, is 35 years younger to his mother. [2024]

- (a) 1964 (b) 1944 (c) 1954 (d) 1974

Ans. (c)

Sol. According to the question,

$$\text{Arjun's mother was born in} = 1964 - 35 = 1929$$

$$\therefore \text{Arjun was born in} = 1929 + 25 = 1954$$

67. Rajesh will not go to the concert, if Rakesh goes. Rakesh will go to the concert if his dog barks three times. Based only on the information above, which of the following must be true? [2024]

- (a) Rakesh will not go to the concert unless Rajesh goes.
(b) If Rajesh doesn't go to the concert, then Rakesh will go.
(c) If Rakesh's dog barks three times, then Rajesh will not go to the concert.
(d) If Rakesh's dog does not bark three times, then Rakesh will not go to the concert.

Ans. (c)

Sol. According to the given conditions, if Rakesh's dog barks three times, then Rajesh will not go to the concert must be true.

68. In a tournament, many teams participated. All teams in the tournament have 5 to 15 players. If a team has more than 10 players, then they have reversible t-shirts. Based only on the information above, which of the following must be true? [2024]

- (a) Teams that have 13 players have reversible t-shirts.
 (b) Teams that have 12 players do not have reversible t-shirts.
 (c) Teams with 8 players do not have reversible t-shirts.
 (d) Only people on teams can have reversible t-shirts.

Ans. (a)

Sol. According to the given condition, teams that have 13 players have reversible t-shirts must be true.

69. A cat climbs a 21-m pole. In the first minute it climbs 3 m and in the second minute it descends one meter. In how minutes the cat would reach the top of the pole? **[2024]**

- (a) 21 min (b) 18 min (c) 19 min (d) 20 min

Ans. (c)

Sol. According to the question,

In 2 min cat climb = 2 m

Then, in 18 min cat climb = 18 m pole

Now, in next min cat climb = 3 m pole

So, total time to climb = 18 + 1 = 19 min

70. Which out of the following words will appear last in the dictionary **[2024]**

- (a) Compliment (b) Compline
 (c) Complete (d) Complicit

Ans. (b)

Sol. The correct dictionary order is as follow,

Complete → Complicit → Compliment → Compline

Clearly, compline appear last in the dictionary order.

71. Arrange the words given below in a meaningful sequence. **[2024]**

1. Software 2. Code 3. Data
 4. Analysis 5. Report

- (a) 3, 1, 2, 4, 5 (b) 5, 4, 3, 1, 2
 (c) 2, 1, 5, 3, 4 (d) 3, 1, 2, 5, 4

Ans. (a)

Sol. The correct meaningful order is as follows,

Data → Software → Code → Analysis → Report

- (3) (1) (2) (4) (5)

72. From the given options, find the pair which is like the given pair 8 : 4 **[2024]**

- (a) 45 : 5 (b) 216 : 32 (c) 72 : 24 (d) 27 : 9

Ans. (d)

Sol. As, $(8 : 4) \Rightarrow 8 \Rightarrow (2)^3$ and $4 \Rightarrow (2)^2$

Similarly, $(27 : 9) \Rightarrow 27 \Rightarrow (3)^3$ and $9 \Rightarrow (3)^2$

73. Which one of the following is the odd one from the given alternative? **[2024]**

- (a) Highest education (b) Salary
 (c) Years of experience (d) Age

Ans. (a)

Sol. Except highest education, all other terms are related to job recruitment.

74. What is the value of $x^2 + y^2 = ?$ **[2024]**

Statement I $xy = 5$

Statement II $x + y = 10$

(a) Choose this option, if the question can be answered by using one of the statements alone, but cannot be answered using the other statement.

(b) Choose this option, if the question can be answered by using both the statements together, but cannot be answered using the other statement.

(c) Choose this option, if the question can be answered by using either statement alone.

(d) Choose this option, if the question cannot be answered even by using both the statements together.

Ans. (b)

Sol. From both statements,

$$x + y = 10 \quad \dots (i)$$

$$\text{and } xy = 5 \quad \dots (ii)$$

Squaring on both sides in equation (i), we get

$$(x + y)^2 = (10)^2$$

$$\Rightarrow x^2 + y^2 + 2xy = 100$$

$$\Rightarrow x^2 + y^2 + 2 \times 5 = 100 \quad [\text{from (ii)}]$$

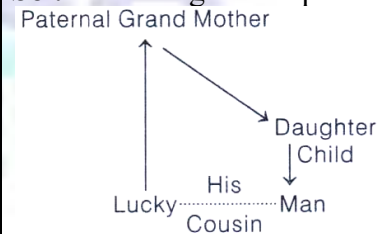
$$\therefore x^2 + y^2 = 90$$

75. Looking at the portrait of a man, Lucky (male) said, "This person is the only child of my paternal grandmother's daughter." Whose portrait was Lucky looking at? **[2024]**

- (a) His cousin (b) His uncle
 (c) His brother (d) Himself

Ans. (a)

Sol. According to the question



Clearly, lucky looking at his cousin portrait.

76. This question contains six statements followed by four sets of combinations of three. Choose the set in which the combinations are most logically related:

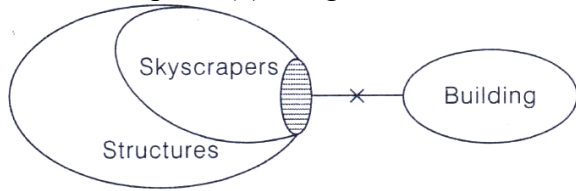
- A. Some buildings are not skyscrapers.
 B. Some skyscrapers are not buildings.
 C. No structure is a skyscraper.
 D. All skyscrapers are structures.
 E. Some skyscrapers are buildings.

F. Some structures are not buildings. [2024]

- (a) ACF (b) BDF (c) ACE (d) FDA

Ans. (b)

Sol. From option (b), we get



Hence, combination given in option (b) are logically related.

77. In a reality show, two judges independently provided marks based on the performance of the participants. If the marks provided by the second judge are given by $Y = 10.5 + 2X$, where X is the marks provided by the first judge. If the variance of the marks provided by the second judge is 100, then the variance of the marks provided by the first judge is: [2024]

- (a) 50 (b) 25 (c) 99 (d) 49.5

Ans. (b)

Sol. Given, the variance of the marks provided by 2 nd judge $\text{Var}(Y) = 100$

We have $Y = 10.5 + 2X$

We know that, Y is a linear transformation of X .

In general, if $Y = a + bX$

Then variance of Y is given by

$$\text{Var}(Y) = b^2 \cdot \text{Var}(X)$$

Here, $a = 10.5$ and $b = 2$

Then, $\text{Var}(Y) = (2)^2 \cdot \text{Var}(X)$

$$\Rightarrow \frac{100}{4} = \text{Var}(X)$$

$$\therefore \text{Var}(X) = 25$$

78. If by rearranging the letters of the word NABMODINT, a name of a game is formed. What would be the first and last letter of the mirror image of the name of the game? [2024]

- (a) B, T (b) N, B (c) T, B (d) B, N

Ans. (b)

Sol. Given word, NABMODINT

After arrangement, BADMINTON

Mirror image, ИОТИИМГАЯ

Clearly, 1st and last letter of mirror image is N and B.

79. This question contains six statements followed by four sets of combinations of three. Choose the set in which the combinations are most logically related:

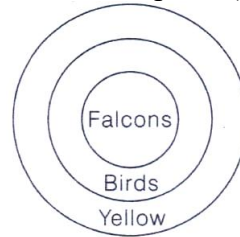
- A. All falcons fly high.
B. All falcons are blind.
C. All falcons are birds.
D. All birds are yellow.
E. All birds are thirsty.

F. All falcons are yellow. [2024]

- (a) CDF (b) BCA (c) ABC (d) DEF

Ans. (a)

Sol. From option (a), we get



Hence, combination given in option (a) is logically related.

80. In a recent survey of 500 employees in a company, it was found that 60% of the employees prefer coffee over tea, 25% prefer tea over coffee, and the remaining 15% have no preference. If 20% of the employees who prefer coffee are also tea drinkers, how many employees prefer only tea? [2024]

- (a) 75 (b) 65 (c) 50 (d) 55

Ans. (b)

Sol. According to the question,

Number of employee who like coffee

$$= 500 \times \frac{60}{100} = 300$$

$$\text{Number of employee who like tea} = 500 \times \frac{25}{100} = 125$$

$$\text{Number of employee who like coffee and also a tea drinker} = 300 \times \frac{20}{100} = 60$$

$$\therefore \text{Number of employee who like only tea} = 125 - 60 = 65$$

81. Two cars, Car A and Car B, are traveling on a highway. Car A starts from point X and travels at a constant speed of 60 km/h, while Car B starts from the same point X but travels at a constant speed of 80 km/h. If both cars travel for 1.5 hours, what is the difference in distance covered by Car B compared to Car A? [2024]

- (a) 35 km (b) 30 km (c) 20 km (d) 25 km

Ans. (b)

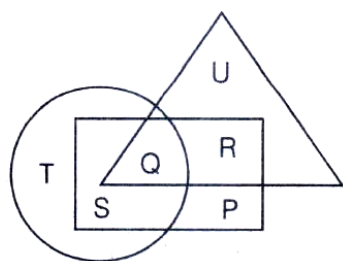
Sol. According to the question,

$$\text{Distance covered by car A} = 60 \times 1.5 = 90 \text{ km}$$

$$\text{Distance covered by car B} = 80 \times 1.5 = 120 \text{ km}$$

$$\therefore \text{Required difference} = 120 - 90 = 30 \text{ km}$$

82. Study the following diagram and answer the following question



→ Married people



→ People who live in joint family



→ School teachers

By which letter, the married teachers who do not live in joint family are represented? [2024]

- (a) P (b) S (c) Q (d) R

Ans. (d)

Sol. Clearly, 'R' represent the married teachers who do not live in joint family.

83. In the half yearly exam only 60% of the students were passed. Out of these (passed in half yearly) only 70% students are passed in annual exam, out of remaining students (who fail in half-yearly exam) 80% passed in annual exam. What per cent of the students passed the annual exam? [2024]

- (a) 72% (b) 76% (c) 65% (d) 74%

Ans. (d)

Sol. Let the total number of statements = $100x$

Then, number of students who pass in half-yearly

$$= 100x \times \frac{60}{100} = 60x$$

Now, number of students who pass in annual exam

$$= 60x \times \frac{70}{100} = 42x$$

and number of students who pass in annual exam

$$= (100 - 60)x \times \frac{80}{100} = 32x$$

Total students pass in annual exam = $(42 + 32)x = 74x$

$$\therefore \text{Required percentage} = \frac{74x}{100x} \times 100 = 74\%$$

Sol. (Q.Nos. 84-86) According to the question,

Swift	Creta	Nexon
B ⁺ (Engineer)	A ⁻ (Teacher)	E ⁻ (Teacher)
G ⁻ (Teacher)	F ⁻ (Engineer)	C ⁻ (Doctor)
	D ⁺ (Doctor)	

Where '+' $\Rightarrow$ Male and '-' $\Rightarrow$ Female

COMPREHENSION

Directions (Q. Nos. 84-86) A, B, C, D, E, F and G are travelling in three different vehicles. There are at least two passengers in each vehicle-Swift, Creta, Nexon and only two of them are a male. There are two engineers, two doctors and three teachers among them.

(i) C is a lady doctor and she does not travel with the pair of sisters A and F.

(ii) B a male engineer, travels with only G, a teacher in a Swift.

(iii) D is a male doctor.

(iv) Two persons belonging to the same profession do not travel in the same vehicle.

(v) A is not an engineer and travels in a Creta.

(vi) The pair of sisters A and F travels in the same vehicle.

84. What is F's profession?

[2024]

- (a) Doctor (b) Data inadequate
(c) Engineer (d) Teacher

Ans. (c)

Sol. F is an engineer.

85. In which vehicle does C travel?

[2024]

- (a) Swift (b) Data inadequate
(c) Nexon (d) Creta

Ans. (c)

Sol. C travels in Nexon

86. Which of the following represents the three teachers? [2024]

- (a) Data inadequate (b) GBF
(c) GEA (d) GEF

Ans. (c)

Sol. G, E and A represents the three teachers.

Directions (Q. Nos. 87-90) A, B, C, D and E are five different integer. When written in the ascending order of values, the difference between any two adjacent integers is 8. D is the greatest and A the least. B is greater than E, but less than C. The sum of the integers is equal to E.

87. The value of A is:

[2024]

- (a) -18 (b) -17 (c) -15 (d) None of these

Ans. (a)

Sol. According to the question,

$$A < E < B < C < D$$

Let the value of A = x, then

$$E = x + 8, B = x + 16, C = x + 24, \text{ and } D = x + 32$$

$$\text{Now, } A + B + C + D + E = E \Rightarrow A + B + C + D = 0$$

$$\Rightarrow x + x + 16 + x + 24 + x + 32 = 0$$

$$\Rightarrow 4x = -72 \Rightarrow x = -18$$

The value of A = -18

88. The sum of A and B is: [2024]

- (a) -15 (b) -30 (c) -20 (d) None of these

Ans. (c)

Sol. According to the question,

$$A < E < B < C < D$$

Let the value of $A = x$, then

$$E = x + 8, B = x + 16, C = x + 24, \text{ and } D = x + 32$$

$$\text{Now, } A + B + C + D + E = E \Rightarrow A + B + C + D = 0$$

$$\Rightarrow x + x + 16 + x + 24 + x + 32 = 0$$

$$\Rightarrow 4x = -72 \Rightarrow x = -18$$

$$\text{Sum} = 2x + 16 = -36 + 16 = -20$$

89. The greatest number has the value: [2024]

- (a) 14 (b) 15 (c) 12 (d) 17

Ans. (a)

Sol. According to the question,

$$A < E < B < C < D$$

Let the value of $A = x$, then

$$E = x + 8, B = x + 16, C = x + 24, \text{ and } D = x + 32$$

$$\text{Now, } A + B + C + D + E = E \Rightarrow A + B + C + D = 0$$

$$\Rightarrow x + x + 16 + x + 24 + x + 32 = 0$$

$$\Rightarrow 4x = -72 \Rightarrow x = -18$$

$$\text{Greatest value} = x + 32 = -18 + 32 = 14$$

90. The sum of the integers is: [2024]

- (a) -6 (b) -10 (c) -8 (d) None of these

Ans. (b)

Sol. According to the question,

$$A < E < B < C < D$$

Let the value of $A = x$, then

$$E = x + 8, B = x + 16, C = x + 24, \text{ and } D = x + 32$$

$$\text{Now, } A + B + C + D + E = E \Rightarrow A + B + C + D = 0$$

$$\Rightarrow x + x + 16 + x + 24 + x + 32 = 0$$

$$\Rightarrow 4x = -72 \Rightarrow x = -18$$

$$\text{Sum} = 5x + 80 = -90 + 80 = -10$$

Computer Awareness

91. Given that numbers A and B are two 8 bit 2's

Complement numbers with $A = 11111111$;

$B = 11111111$. Then, sum $A + B$ is _____ [2024]

- (a) 00000010 (b) 11111100

- (c) 11111110 (d) 00000000

Ans. (c)

Sol. $A = 11111111$: In 8-bit 2's complement, this represents -1. Similarly $B = 11111111$ represents -1.

Add the Numbers: $(-1) + (-1) = -2$

2 in binary representation is 00000010

Now 1's complement is 11111101

And 2's complement is adding 1 to 1's complement so solution is $11111101 + 1 = 11111110$.

92. Consider an arbitrary number system with independent digits as 0, 1 and A . If we generate first few numbers in sequence as 00, 01, 0A, 10, 11, 1A and if this process is continued to generate the numbers, then the position of 10A is _____ [2024]

- (a) 15 (b) 12 (c) 9 (d) 10

Ans. (b)

Sol. Series can be think as 00, 01, 0A, 10, 11, 1A,

A0, A1, AA, 100, 101, 10A

So, the positions can be 00 on 1st position

01 on 2nd position

0A on 3rd position and so on

That's why answer is 10A is on 12th position.

93. The Boolean expression for the following truth table is _____ [2024]

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

(a) $F = x'yz' + xy'z + x'y'z'$ (b) $F = x'y'z' + xy'z + xyz'$

(c) $F = x'yz' + xy'z + xyz$ (d) None of these

Ans. (c)

Sol. To create Boolean Expression we write the terms where $F = 1$

Which are $x = 0, y = 1, z = 0$

$x = 1, y = 0, z = 1 \Rightarrow x = 1, y = 1, z = 1$

So, expression will be $x'yz' + xy'z + xyz$

94. Consider the following 4-bit binary numbers represented in the 2's complement form: 1101 and 0100. What would be the result when we add them? [2024]

(a) 0001 and an overflow (b) 1001 and no overflow

(c) 1001 and an overflow (d) 0001 and no overflow

Ans. (d)

Sol. Let $A = 1101$ which is a negative number as it is starting with 1, according to question it is represented in 2's complement.

2's complement of A is 0011 so the number is -3

Let $B = 0100$ which is a positive number as it is starting with 0 so the number is 4.

Now add $A + B$ then result will be $(-3) + 4 = 1$ i.e. 0001 and no overflow in 4 bit binary representation .

95. Which of the following interfaces perform the transfer of data between the memory and the I/O peripheral without involving the CPU? [2024]

- (a) Branch Interface (b) Serial Interface
(c) DMA (d) DDA

Ans. (c)

Sol. DMA (Direct Memory Access) allows peripheral devices (like hard drives or network cards) to directly transfer data to/from memory without involving the CPU. This speeds up data transfer and reduces CPU load.

96. Which of the following is the smallest unit of data in a computer? [2024]

- (a) Byte (b) Bit (c) Nibble (d) KB

Ans. (b)

Sol. It is the smallest unit

1 bit = 0,1 $\Rightarrow$ 1 byte = 8 bits

97. Consider the program below which uses six temporary variables a, b, c, d, e and f .

$a = 10 \Rightarrow b = 20$

$c = 30 \Rightarrow d = a + c$

$e = b + d \Rightarrow f = c + e$

$b = c + e \Rightarrow e = b + f$

$d = 5 + e$

return $d + f$

Assuming that all the above operations take their operands from registers, the minimum number of registers needed to execute this program without spilling is _____. [2024]

- (a) 5 (b) 6 (c) 3 (d) 4

Ans. (c)

Sol. Without spilling means the value of original value should not be changed. Also we try to reduce the variables used.

We replace d -variable with a variable as a is not used further so we can change the value of a

$a = a + c$ (earlier $d = a + c$) $\Rightarrow e = b + a$ (earlier

$e = b + d$) $a = 5 + e$ (earlier $d = 5 + e$)

again we can replace e with a so

$a = b + a$ (earlier $e = b + a$) $\Rightarrow f = c + a$ (earlier $f = c + e$)

we can replace f with b

so the statement will be $b = c + a$

We have 2 same statements $f = c + e$

$b = c + e$

so we can remove 1 statement $f = c + e$ and write b everywhere f is used so the statements will be

$a = b + b$ (earlier it was $e = b + f$, also earlier e is replaced by a)

In short we have replaced d with a , e with a and f with b so by using only 3 variable we can do the calculations.

98. The quotient, if the binary number 11010111 is divided by 101, is _____. [2024]

- (a) 101011 (b) 101010
(c) 101101 (d) 111001

Ans. (a)

Sol. Dividing 11010111 by 101:

$$\begin{array}{r} 101 \overline{) 11010111} \quad (101011 \leftarrow \text{Quotient}) \\ \underline{101} \\ 110 \\ \underline{101} \\ 111 \\ \underline{101} \\ 101 \\ \underline{101} \\ 000 \text{ (Remainder)} \end{array}$$

Hence, the quotient is 101011 in binary and the remainder is 000 in binary.

99. Which of the following components is used to establish a communication link between a CPU and the peripheral devices to transfer data? [2024]

- (a) Memory address register (b) Instruction register
(c) Memory data register (d) Index register

Ans. (c)

Sol. Memory Data Register

To transfer data between CPU and Peripheral devices we use data register

100. A computer system has 16-bit wide address/data bus that uses RAM chips of $4 \text{ K} \times 8$ -bit capacity. The number of RAM chips are needed to provide a memory capacity of 64 Kbytes memory is _____. [2024]

- (a) 32 (b) 16 (c) 64 (d) 8

Ans. (b)

Sol. RAM needed = 64 K Byte

RAM exists = $4 \text{ K} \times 8 \text{ bit}$

8 bit = 1 Byte

So RAM exists = $4 \text{ K} \times \text{Byte}$

Total chips required = $64 \text{ K Byte} / 4 \text{ K} \times \text{Byte} = 16$ chips.

101. The primary purpose of cache memory in a computer system is _____. [2024]

- (a) to manage input and output operations between the CPU and peripherals
(b) to temporarily store frequently accessed data and instructions for faster access by the CPU

- (c) to permanently store data and programs
(d) to provide additional storage space when the main memory is full

Ans. (b)

Sol. By storing frequently used data in cache, the CPU can access this data much faster than if it had to retrieve it from the main memory or the hard drive.

102. Which of the following do not affects CPU performance? [2024]

- (a) Cache size (b) Number of cores
(c) Amount of RAM (d) Clock speed

Ans. (c)

Sol. Amount of RAM

103. A CPU generates 32 bits virtual addresses. The page size is 4 KB. The processor has a translation look-aside buffer (TLB) which can hold a total of 128-page table entries and is 4 way set associate. The minimum size of the TLB tag is [2024]

- (a) 11 bits (b) 15 bits (c) 13 bits (d) 20 bits

Ans. (b)

Sol. Total memory = 2^{32}

Page size = 4 kb = 2^{12} bit

So total page = $2^{32} / 2^{12} = 2^{20}$

Now, 1 entry = 128 page i.e. = 2^7 bit

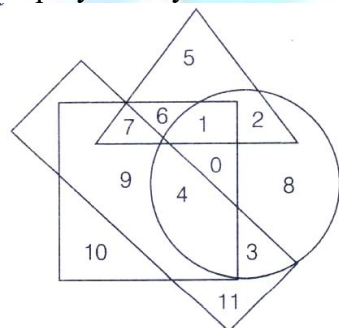
1 set = $2^7 / 2^4 = 2^5$

(because it is 4 way set associated).

So Total entry = $2^{20} / 2^5 = 2^{15}$

The answer is 15 bits.

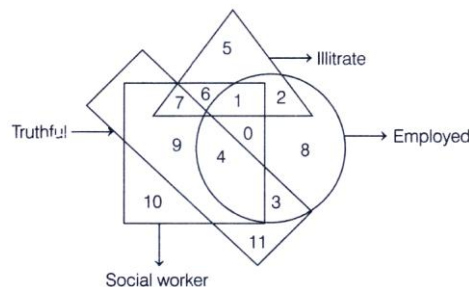
104. In the figure, the circle stands for employed, the square stands for a social worker, the triangle stands for illiterate, and the rectangle stands for truthful. Study the figure with its regions and find the number of neither truthful nor illiterate people among the employed only. [2024]



- (a) 4 (b) 8 (c) 1 (d) 11

Ans. (b)

Sol. According to the question,



Clearly, '8' are neither truthful nor illiterate people among the employed only.

105. Cache memory functions as an intermediary between [2024]

- (a) RAM and ROM (b) CPU and RAM
(c) CPU and Hard Disk (d) None of the above

Ans. (b)

Sol. Cache memory functions as a high-speed intermediary between the CPU and RAM. It stores frequently accessed data and instructions so that the CPU can access them more quickly than it could from RAM. This helps improve system performance by reducing the time the CPU spends waiting for data from the slower main memory (RAM).

Cache memory is not involved in the direct interaction between the CPU and hard disk or between RAM and ROM.

106. Let the given numbers 11001, 1001 and 111001 be correspond to the 2's complement representation. Then, with which one of the following decimal numbers, the given numbers match? [2024]

- (a) -25, -9 and -57, respectively
(b) -7, -7, and -7, respectively
(c) -6, -6, and -6, respectively
(d) 25, 9 and 57, respectively

Ans. (b)

Sol. -7, -7, -7 respectively

Because 11001 is a negative number as first bit is 1 and it's

2's complement is 00111 which is equal to -7

Similarly 1001 = 0111 (2's complement) = -7

Also 111001 = 000111 (2's complement) = -7

107. The range of the exponent E in the IEEE754 double precision (Binary 64) format is ____ [2024]

- (a) $-1023 \leq E \leq 1023$ (b) $-1022 \leq E \leq 1022$
(c) $-1023 \leq E \leq 1022$ (d) $-1022 \leq E \leq 1023$

Ans. (c)

Sol. $-1023 \leq E \leq 1022$

108. Which of the following components is not a part of an instruction format in CPU processing? [2024]

- (a) Source operand (b) Register file
(c) Destination operand (d) Opcode

Ans. (b)

Sol. Register file

we need other 3 options in instruction format in CPU processing.

109. Any given truth table can be represented by [2024]

- (a) a product of sum Boolean expression
- (b) a Karnaugh map
- (c) a sum of product Boolean expression
- (d) All of the options

Ans. (d).

Sol. Sum of Products (SOP): This is a form of Boolean expression where the output is expressed as the OR of multiple AND terms. It can represent a truth table by summing the minterms (combinations of inputs that produce a true output).

• **Product of Sums (POS)** : This is a Boolean expression where the output is the AND of multiple OR terms. It can represent a truth table by multiplying the maxterms (combinations of inputs that produce a false output).

Karnaugh map (K-map) : A graphical tool used for simplifying Boolean expressions. Any truth table can be mapped onto a K-map to minimize the expression. Since any truth table can be represented by both SOP and POS forms, and it can also be simplified using a K-map, the correct answer is "All of the options".

110. The expression $P + QR$ is the reduced form of [2024]

- (a) $(P + Q)R$
- (b) $(P + R)Q$
- (c) $(P + Q)(P + R)$
- (d) $PQ + QR$

Ans. (c)

Sol. $(P + Q)(P + R)$

Expand using distributive property:

$$(P + Q)(P + R) = P(P + R) + Q(P + R)$$

$$= PP + PR + QP + QR = P + PR + PQ + QR$$

$$= P(1 + R + Q) + QR = P \cdot 1 + QR \text{ (since } 1 + R + Q$$

reduce to 1)

$$= P + QR$$

General English

111. Choose the correct combination of prepositions to complete the sentence. [2024]

"The cat jumped _____ the table _____ the chair."

- (a) on, from
- (b) off, in
- (c) into, beside
- (d) onto, towards

Ans. (a)

Sol. 'On and from' are correct preposition to complete the sentence appropriately. (Reason) because 'onto' is

used with respect to position and direction both at the same time but 'on' is used for position only.

112. The company's _____ growth in revenue surprised analysts. [2024]

- (a) erratic
- (b) gradual
- (c) stagnant
- (d) exponential

Ans. (d)

Sol. 'Exponential' is the correct filler as it states that company's rapid growth surprised analysts.

113. Identify the word that means the same as 'Ostentatious'. [2024]

- (a) Lavish
- (b) Simple
- (c) Modest
- (d) Unassuming

Ans. (a)

Sol. 'Ostentatious' means expensive or impressive. Hence, lavish is the word which carries same meaning.

114. Write the antonym for 'Inscrutable'. [2024]

- (a) Comprehensible
- (b) Mysterious
- (c) Opaque
- (d) Obscure

Ans. (a)

Sol. 'Inscrutable' means vague or difficult to understand. Hence, 'comprehensible' is its antonym which means 'easy to understand'.

115. Choose the best option that indicates the change of voice for the sentence given below
Did Alice invite you? [2024]

- (a) Were you invited by Alice?
- (b) Was Alice invited you?
- (c) Had you invited Alice?
- (d) Did you invited by Alice?

Ans. (a)

Sol. 'Were you invited by Alice?' is the correct passive voice of the sentence. 'Did' is changed to 'were' in passive voice.

116. Which of the following is an essential element of a technical report? [2024]

- (a) Anecdotes and personal opinions
- (b) Statistical data and analysis
- (c) Creative storytelling
- (d) Emotional appeals

Ans. (b)

Sol. 'Statistical data and analysis' is an essential element of technical report because technical report writing contains formal approach and all other options are part of informal writing.

117. Select the correct meaning of 'Peruse'. [2024]

- (a) Continue
- (b) Pursue
- (c) Examine
- (d) Rescue

Ans. (c)

Sol. 'Peruse' means to read and analyse something in a careful way. Hence, 'examine' means the same.

118. Which sentence demonstrates correct preposition usage? **[2024]**

- (a) I prefer coffee over tea.
- (b) He is interested on learning new languages.
- (c) They were surprised of the sudden announcement.
- (d) She arrived to the party at 8 PM .

Ans. (a)

Sol. The verb 'prefer' is always followed by preposition 'to'. Hence, this sentence is correct.

⇒ Arrived is followed by 'at'.

⇒ 'Interested' is followed by 'in'.

⇒ 'Surprise' is followed by 'by/at' according to the need of sentence.

119. Select the appropriate synonym for 'Coercive'. **[2024]**

- (a) Gentle
- (b) Forceful
- (c) Corrective
- (d) Merciful

Ans. (b)

Sol. 'Coercive' means to forcefully persuade someone to do something. Hence, 'forceful' is its synonym.

120. What does the idiom 'Jump on the bandwagon' mean? **[2024]**

- (a) To join a popular trend or activity
- (b) To criticise something unfairly
- (c) To repair a vehicle
- (d) To start a business

Ans. (a)

Sol. 'Jump on the bandwagon' means to join an activity that has become very popular.