# THE ILLUSION OF SKILL

## THE PROBLEM OF EVALUATING HEDGE FUNDS WITH SKEWED DISTRIBUTIONS

THE SHARPE RATIO: A BLUNT WEAPON – HIDING THE RISK OF ASYMMETRIC RETURN DISTRIBUTIONS

*by Wayne Himelsein CIO and David Taylor, Ph.D.*

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## Introduction

The big problem behind thoughtful capital allocation is that there are a wide range of approaches to choose from, where each method relies upon a unique—or purportedly unique—hypothesis to extract "allocation alpha" and where such hypotheses run the gamut of investment sophistication. Nonetheless, allocators welcome countless portfolio management strategies thanks to a wide variety of well established procedures that seek to enable effective comparing and contrasting of such diversity in strategies. More so, this toolbox supposedly results in wiser allocation decisions that should outperform a naïve approach. So with so much relying on these tools, it is critical to understand them. At their core, the vast array of decision-making tools boil down to the simplest of ideas, which is the accurate measurement of risk versus reward; collectively, the pinnacle of an ideal allocation is the most reward gained per unit of risk taken, or simply, the greatest risk adjusted return.

To achieve the most accuracy in the assessment of both the risk and reward, allocators utilize quantitative or statistical methods in hopes that such tools will better process, and intelligently interpret, expansive data. To this end, a crucial step in asset allocation is to have a robust mathematical model to sift through the data and analyze (or weigh) the risk relative to the reward for a given style or strategy. One of the most widely utilized models for accomplishing this task is the highly regarded Markowitz Mean-Variance [1] framework, which simply put, views the mean return relative to the variance inherent in generating such return as the ideal construct for "risk-adjusted" return. Following this revolutionary idea came the logical next step of summary ratios that dispersed throughout the financial community. One of the purest spin-offs was the Information Ratio, which is the mean relative to the volatility, and one of the most widely adopted ratios of them all, was the iteration to Sharpe Ratio [2]. In this modification, William Sharpe realized, quite appropriately, that if one were to propose a risk-adjusted return, one naturally had to subtract any return that could be generated *without* taking any risk, and hence the mean return was first reduced by the risk-free rate, and the net difference viewed relative to the volatility of those "net of riskless" returns.

While used extensively for portfolio assessment, in this article, we will thoroughly investigate how reliance on the ubiquitous Sharpe Ratio metric, as well as all similarly oriented summary statistics which depend only on the average rate of return and variance/volatility of an asset, do not adequately evaluate risk versus reward. The simple reason is that asset returns are not normally distributed.

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The fallibility of Sharpe Ratio, and all of its cousins, becomes evident upon recognizing that it relies solely on the mean and standard deviation, which, as summary statistics, cannot distinguish between processes that are symmetric and asymmetric. In order for the Sharpe ratio, and other related statistical tools, to accurately measure a risk reward profile, the underlying time series has to be symmetric. Most users of Sharpe and related tools assume this is the case. However, the standard across the vast majority of investment strategies is quite the opposite of this assumption; returns are most generally asymmetrically distributed, and particularly, negatively skewed. Moreover, erroneously assuming that an asymmetric distribution is symmetric when it is left skewed, will cause any model to *dramatically understate risk*. This is especially true when underlying distributions exhibit high levels of negative skewness, which lo and behold, is most often the case.

To this end, we will incorporate the necessary statistical properties of skewness and kurtosis into the distribution of returns for "standard" assets to investigate the extent of the shortcomings of Mean-Variance and Sharpe, along with other more rigorous investment assessment and risk measures such as value at risk (VaR) ([3], p. 494) and Conditional Tail Expectation (CTE) ([3], p. 496). Following that work, we will share examples that powerfully conclude that these incorrect evaluation methods, which rely only on mean and variance in the presence of non-normal distributions, are dramatically underestimating the presence of risk, and to a far greater degree than investors should be comfortable with. In fact, we posit that if investors actually observed the true risk levels, relative to the respective reward of a potential investment, they would alter their allocation behavior (e.g. allocate less, or perhaps not at all, see Table 1) . Finally, with this issue exposed, we will show the statistical methods necessary for thoroughly evaluating the prospects of an investment, as well as provide a general mechanism to make reasonable assessments on the fly, a function we have dubbed the **Skill Metric.**

## The "Normal" Framework

Most formal approaches to mathematical finance assume that the returns of an asset are *normally distributed* ([3], p. 314). For example, the grand theory on capital market behavior, the Efficient Market Hypothesis (EMH), assumes equity prices are generated by a geometric Brownian motion diffusion process, which results in normally distributed returns. In fact, the famous Black-Scholes partial differential equation, a quintessential model for evaluating the price of an option on an underlying security, relies on the same core premise that stock prices follow geometric Brownian motion ([3], p. 321) and that those stock returns are normally distributed. Furthermore, one of the most widely used investment analysis techniques – regression – assumes the underlying process is normally distributed. To this end, any technique that shares this mathematical structure, in computing, and relying upon, the deviations from the mean (from simple correlation analysis to more complex Principal Component Analysis, alongside a wide variety of related techniques), will provide inaccurate results if fed with asymmetric distributions. *At large, the gamut of industry accepted tools and techniques for investment analysis are based on a grand assumption that, ironically, is most often false.*

We therefore begin by asking, what is so special about a normal distribution that all the major models begin with this underlying assumption? The first key aspect, and the fundamental elegance of the normal distribution, is that only two quantities, mean, $\mu$, and variance, $\sigma^2$, are required to fully describe it. This notion is both fundamental and elegant because it naturally leads to how well we can understand (and therefore extrapolate) the future cumulative outcome of a normally distributed process.

When only two parameters (the mean and variance) are needed to describe some process, we can conclude that the outcome is stable; that is, cumulative returns will land at some expected level, plus/minus the expected error, and these two quantities suffice to describe the outcome probabilistically. Moreover, if the observed periodic returns are normally distributed, the cumulative return will also be normally distributed, and so normal distributions are stable in this sense as well.

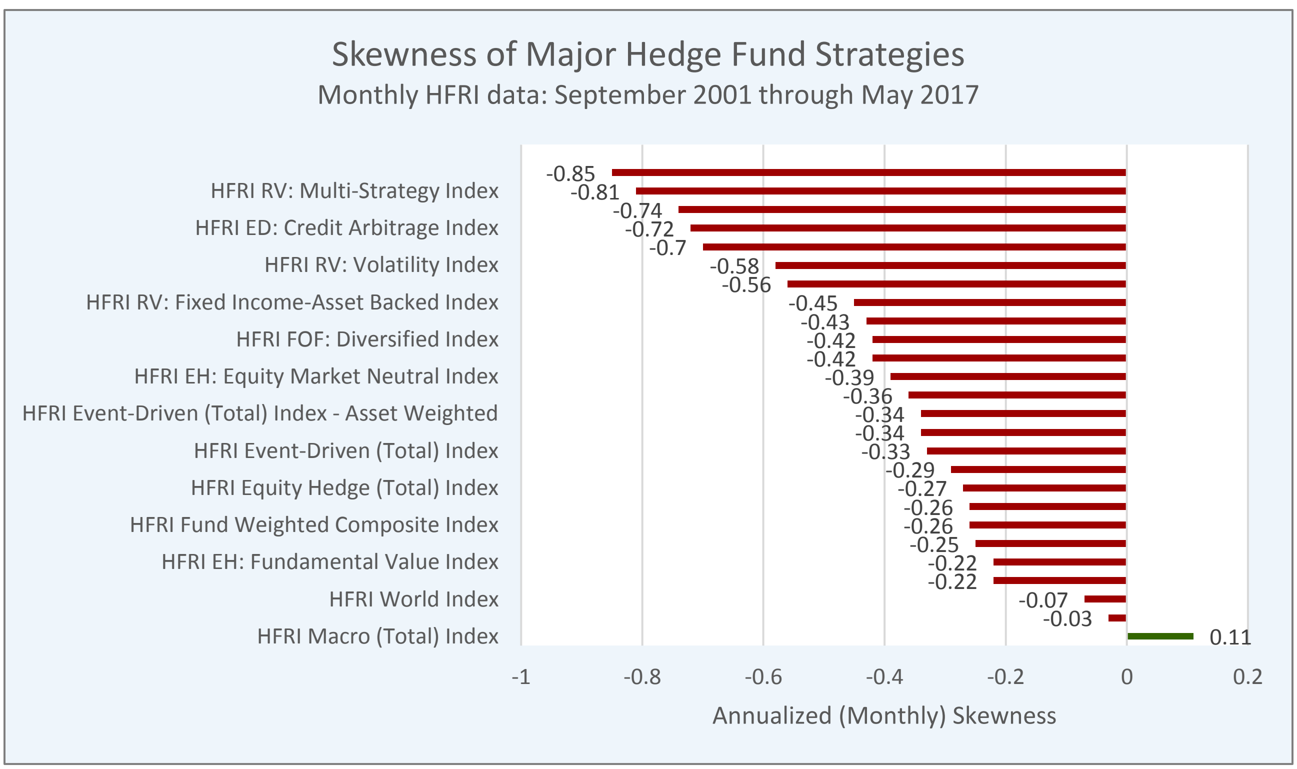
However, if a greater number of statistical parameters are necessary to explain what might occur, then quite obviously, there may be a more extreme range of possible outcomes, i.e. distributions one must consider. Derived from this fundamental definition, there are some additional laws that follow organically given sufficiently many normally, *identically, independently distributed events* (iid):

1. The *law of large numbers* [4] guarantees that the sample mean and sample variance will converge to the true mean and variance of the underlying process;
2. The *central limit theorem* [4] guarantees that the distribution of the sample mean will converge to in probability to a normal distribution;
3. The rate of this convergence is controlled by the *Berry-Esseen theorem* [5], which estimates the distribution of the error between both the sample mean and the true mean, and the sample variance and the true variance. The distribution of these errors is a function of the number of observations. (That is, the confidence in your statistical guess increases with the number of prior observations).

These fundamental laws establish what is truly significant and important about the mean-variance framework based upon the normal distribution. More so, it is likely that for reasons like these (along with the fact that normally distributed events explain many processes in the natural world) that the mean-variance framework has risen to reign supreme in the natural and social sciences. Similarly, and perhaps due to its pedagogical prominence in the natural and social sciences, it has been established as a primary tool for analysis of returns in the financial world. However, observation of the returns of many investments quickly demonstrate that they deviate from the assumption that they are:

1. Independent events;
2. Arise from a normally distributed process.

In fact, it has been observed that hedge fund returns, which are the outcome of specific stock picking strategies alongside complex portfolio management disciplines, and other return drivers, almost never exhibit a normal distribution [6], and to the contrary, most often demonstrate a negatively skewed one.



Therefore, analytics based solely upon mean and variance (or standard deviation), including all the related ratios that utilize them, will fail to accurately differentiate investments, and more importantly, will fail to properly characterize important phenomena they exhibit. Moreover, they will not provide the core stability of a normally distributed asset, and thus not follow its fundamental bylaws; the law of large numbers and the central limit theorem. Above all, ignoring skewness will underestimate risk, and more importantly, more extreme tail risk as measured by CTE. Likewise, the esteemed Sharpe ratio will drastically both overstate reward and understate risk.

## Facing Non-normality

The theory of non-normal distributions (for example sums of non-independent observations), is highly nuanced. Additional quantities must be calculated and more rigorous analysis performed to properly assess the true risk relative to the reward of a strategy whose returns are modeled by non-normal distributions. In contrast to the elegance of the normal distribution that merely requires two variables, the mean and the standard deviation, to project future outcomes, non-normal distributions may require numerous additional parameters to properly model the risk. Even once calculated, such models cannot, by definition, be as accurate or reliable as those determined with fewer parameters, given the same number of observations. To accurately estimate higher-order statistical quantities such as skewness, one needs an increasing number of observation and higher quality data.

Obviously, one can start with a far easier position and just avoid this issue altogether with a very simple approach: search for optimal investment opportunities which best adhere to the normal distribution, thereby minimizing the failure of models which all rely on this basic assumption. But because such investments are incredibly hard to find, the alternative solution is to develop new tools that better explain, both in a theoretical and statistical sense, the observed non-normal phenomena. Further, and beyond mere explanation, such tools become crucial to accurately assess the inherent risk, both "on average" and more importantly, at the tails.

To this end, we will rigorously investigate returns which clearly deviate from the normal hypothesis in a way that captures their important features, inclusive of traditional assets, such as the S&P 500, as well as alternative assets, such as hedge-fund returns. In doing so, we will propose a means to more precisely quantify the degree to which the mean-variance framework may mislead investor's preferences.

# Risk/Reward Metrics

Starting from our core concept, the weighing of the reward of possible profits against the risk of possible losses, it is first useful to quantify each "side" independently. However, before we can even do this, we need to step even further back and find a means of evaluating what we mean by "risk" and what we mean by "reward", i.e. a mathematically pure definition of their standing and their relationship. Therefore, in this first section, we will introduce and review some relevant concepts from game theory and behavioral economics, which help to define our core concept, and then relate them to quantities one can calculate in terms of asset returns.

## Utility Theory

Before one can decide how to evaluate reward, one must have concrete goals to identify what exactly they are seeking. In this context, the most basic goal is to maximize the growth of one's wealth over some interval of time, or $W(t)$. We express this need by maximizing the quantity with respect to time:

$$\frac{\Delta W}{W(t_0)} = \frac{W(t_f)}{W(t_0)} - 1.$$

Here, $t_0$ is the initial time of investment and $t_f$ is the final time of investment.

Since returns over any arbitrary time period are subject to chance variation, one cannot expect any strategy to guarantee any particular value of the above quantity. However, over the longer course of time, one can endeavor to maximize this quantity in a probabilistic sense.

The relevant concept from game theory is that of a *utility function*, $U(x)$ (sometimes called a Neumann-Morgenstern utility function), which assigns to any particular investment scenario, $x$, a numerical value based on the possible outcomes. The most important, and intuitive, observation regarding the quantification of utility amongst a wide range of uncertain alternatives is that the average return of an investment is certainly not the only relevant consideration. Naturally, ***extreme losses beyond a certain threshold are relatively more painful than moderately small losses***. Conversely, ***extreme gains beyond a certain level are of less relative utility than moderate gains***. A wonderful quotation that illuminates this concept is: "A man drowned crossing a lake with an average depth of 12 inches."

A practical class of such utility functions which captures real-world investor behavior is the class of non-increasing absolute risk aversion (NIARA) utility functions [7], which amongst other aspects, demonstrate how such behavior modifies as wealth changes.

## Higher Moments and Utility

To more deeply assess the utility associated with a given reward, we must account for the observation that extreme losses are far more painful than moderate ones, and that extreme gains are not multiple times more exciting than moderate ones. Since we cannot allow ourselves to drown in an "on-average" shallow lake, we come to further appreciate that the selected reward is not normally distributed. Thus, we arrive at the need to add the additional variables that are required to explain the non-normal distribution, e.g. the higher moments of the distribution. Fortunately, NAIRA functions infuse those higher moments, as illustrated below.

Let $r$ denote the return over a time interval and $\boldsymbol{E(r)}$ denote the average value of the return over that time interval. The probability distribution which gives rise to the various returns of an investment relates to the utility function as follows: If the utility function, $\boldsymbol{U(r)}$, is differentiable to all orders then Taylor's theorem with remainder implies,

$$U(r) = \sum_{k=0}^{n} U^{(n)}\big(E(r)\big)\big(r - E(r)\big)^k + R_n\big(r - E(r)\big).$$

Here $\boldsymbol{R_n(x)}$ is the remainder term from the Taylor's expansion, and the symbol $U^{(n)} = \frac{d^n}{dr^n}U$ is the $n^{\text{th}}$ derivative of the utility function.

Since the investor seeks to maximize average utility, he or she wants to choose amongst alternatives for which there is as large as possible of a quantity derived by the equation:

$$\begin{aligned} E\big(U(r)\big) &= E\left(\sum_{k=0}^{n} \frac{U^{(n)}}{n!}\big(E(r)\big)\big(r - E(r)\big)^k + R_n\big(r - E(r)\big)\right) \\ &= \sum_{k=0}^{n} \frac{U^{(n)}}{n!}\big(E(r)\big)\, E\left(\big(r - E(r)\big)^k\right) + E\left(R_n\big(r - E(r)\big)\right) \end{aligned}$$

Fittingly, each of the terms $\boldsymbol{E\left(\big(r - E(r)\big)^k\right)}$ is exactly the expression for the ***k****th* *central moment* of the distribution of $\boldsymbol{r}$. Accordingly, maximizing a (NIARA) utility function involves choices which consider higher moments.

To illustrate further, and following our prior convention, we denote the average return by $\boldsymbol{\mu} = \boldsymbol{E}(\boldsymbol{r})$. Accordingly, the second central moment, the *variance*, has equation:

$$\sigma^2 = E((r-\mu)^2).$$

Wherein the quantity, $\boldsymbol{\sigma}$, is the *standard deviation*.

The third central moment, $\boldsymbol{m_3}$, is related to the *skewness*, $\boldsymbol{\gamma_1}$, of the distribution via the formula:

$$\gamma = \frac{E((r-\mu)^3)}{\sigma^3} = \frac{m_3}{\sigma^3}.$$

Similarly, the *fourth central moment*, $\boldsymbol{m_4}$, is related to the *kurtosis* by the formula:

$$\gamma_2 = \frac{E((r-\mu)^4)}{\sigma^4} = \frac{m_4}{\sigma^4}.$$

In practice, one often uses the *excess kurtosis*, which subtracts 3 from the kurtosis. This represents the difference between the kurtosis of the process in question from the "normal" kurtosis of a standard normal distribution (which equals 3).

## Conclusions of the NIARA Hypothesis

Under the natural hypothesis that investor preferences are of the NIARA type, the signs attributed to the first three derivatives of the Neumann-Morgenstern utility function satisfy the following inequalities [7]:

$$U'(r) \geq 0, \;\; U''(r) \leq 0, \;\; U^{(3)}(r) \geq 0.$$

When we impose these conditions on the expression for expected utility and evaluate at $\boldsymbol{E}(\boldsymbol{r})$, we see that,

$$\frac{dE\big(U(r)\big)}{dE(r)} = U'\big(E(r)\big) \geq 0, \frac{d^2E\big(U(r)\big)}{dE(r)^2} = U'\big(E(r)\big) \leq 0, \frac{d^3E\big(U(r)\big)}{dE(r)^3} = U^{(3)}\big(E(r)\big) \geq 0.$$

Taking the signs of these coefficients into account implies that:

1. As the average return, $\boldsymbol{\mu}$, grows, the constant term increases and thus utility increases,
2. As the variance, $\boldsymbol{\sigma^2}$, grows, the second order term decreases and thus utility decreases,
3. As the skewness, $\boldsymbol{\gamma_1}$, grows (in many real-world cases this is simply becoming less negative), the third order term increases and thus utility increases.
4. Though not explicitly illustrated, albeit under the same premise, as kurtosis, $\boldsymbol{\gamma_2}$, grows, the fourth order term increases and thus utility decreases.

Considering these, we determine that amongst all possible investments, given equal average returns and equal variance, a higher (or more typically, less negative) value of the skewness is more attractive, as is a lower value of kurtosis. We summarize these outcomes in the following grid:

| ***High Return*** | ***High Variance*** | ***Higher Skewness*** | ***Higher Kurtosis*** |
|---|---|---|---|
| **Good** + | Bad − | Good + | Bad − |

## Returns as Reward

With a deeper understanding of ideal reward goals, and the associated utility functions in mind, we can now come back to looking more directly at the high-level concept of reward. In its simplest form, economic reward associated with an investment may be expressed as the active return on that investment. This is defined as the expectation of the return of the portfolio, $r_p$, minus the benchmark return, $r_b$,

$$E(r_p - r_b).$$

For further simplicity, we define $r = r_p - r_b$ (where for Sharpe ratio, $r_b$ is generally the Risk-Free Rate). We can then define the reward benchmark (as done previously) by,

$$\mu = E(r).$$

With the basic assumption that any investment is a series of returns, this quantity will either lead to the expected excess return or the expected excess rate of return (depending on the context, we will use $\mu$ also). If one has no other constraints on one's investments (an unrealistic assumption, but ideal for illustration purposes), then the size of the quantity $\mu$ would be the only consideration necessary to determine the superiority of one investment over another. However, since losses are undesirable, and large losses are unacceptable (beyond a certain level, dependent upon the individual investor), one must seriously account for the expected level of losses. Moreover, as we learned from NAIRA utility functions, the more extreme the losses, the more one *must* consider them.

The summary lesson here is that rational, risk-averse investors are not just focused on reward, or the return of an investment. They are necessarily sensitive to the deviations of those returns, starting with the average, or expected deviation, and followed by the outliers that might lean the distribution, i.e. the skewness, and in general, by the higher moments of the return distribution that will more extremely effect their anticipated reward.

## Deviations as Risk

Wholly distinct from reward, we may now come back to look more deeply into the other "side" of the performance equation, which is the embedded risk of an investment. Most often, we see risk defined as the volatility of an investment. First and foremost, it is important to understand the inherent limitations when ascribing the variance (or volatility) of an asset as its risk. In doing so, the main aspect to consider is the relationship between the variability of a process and its other statistical quantities, such as central moments, and more specifically, how the variability in and of itself can mask the higher moments (which we care most about!).

At its most basic level, it is interesting to observe that ***losses, or returns below a benchmark return, naturally occur within the variation of returns***. By definition, not all returns can be above the average return! And of course, a fair portion must be below. Consider this fact with applied to volatility. Recall the standard deviation of a return, $r$, is denoted by,

$$\sigma = \sqrt{E(r^2) - E(r)^2}.$$

Looking at this equation, we immediately see that this derived quantity is the average deviation of each return from its mean (e.g., the average error off the average outcome). ***It is crucial to realize that this formula does not distinguish how the returns are distributed;*** since the average is a summary statistic, it will inevitably smooth out the extremes – and as is often the case, it is the extreme deviations that will unavoidably, and perhaps materially, alter the skewness.

Moreover, because the summary nature of expected variability necessarily combines deviations above and below the average, it doesn't distinguish between good return deviations (profits) and bad return deviations (losses), nor the ratio of profit deviations to loss deviations, and in general, how frequently loss/profit deviations past a certain level occur.

Accordingly, when one considers investments that change value with time, e.g. that vary, this approach can be adequately utilized to determine the quantity of volatility, but cannot be accepted to more completely determine risk.

***Therefore, risk metrics which rely only on the first two moments of a return distribution cannot reliably distinguish between investments, especially given the reality that the returns of most investments are not normally distributed.***

## Measures of Risk versus Reward

Given the nuances we have learned about reward, and the limitations we have learned about risk, we can now look more closely at weighing risk against reward by the celebrated, albeit ultimately flawed, Sharpe ratio. It is defined as the ratio of excess return with respect to the inherent volatility, that is,

$$(\text{Sharpe ratio}) := \frac{\mu}{\sigma}$$

Between investments with normal-like return distributions (which are rare, but do exist), one would always prefer those with the highest Sharpe ratio. If, however, one compares an investment whose returns are not normally distributed, whether against other non-normal, or even against one which is normal, other factors ***must*** be taken into account; apples must be compared to apples.

For instance, the generally higher Sharpe ratio prevalent amongst hedge funds, on its surface, suggests that investors should consider them less risky investment alternatives. However, a greater Sharpe ratio with a non-normally distributed investment (most often, negative skewness) may actually be of less preference than a lower Sharpe ratio with a normally distributed profile. The pervasive negative skewness, or excess left tail risk, in hedge funds implies that hedge fund investors bear significant "masked" downside risk, as skewness is not captured by the first two moments of returns, and thereby excluded from the apparently acceptable Sharpe ratio.

To further illustrate the above point, we shall consider a family of distributions which differ from the normal distribution with the introduction of parameters that directly affect skewness, and consequently, kurtosis. These are the, aptly called, *skew-normal distributions*. As we will see clearly in the next section, the introduction of skewness allows us to more closely model the behavior of real-world investments, and better yet, to establish a framework for identifying preferences between differently distributed investments, even when they have very similar mean and variance.

# Normal and Skew-normal Distributions

The skew-normal distribution is an extension of the normal distribution in the sense that when one sets the skewness parameter equal to zero, it becomes a normal distribution. That said, the normal and skew-normal distributions differ by single parameter which directly controls skewness. To understand this nuance more deeply, we introduce some formal definitions.

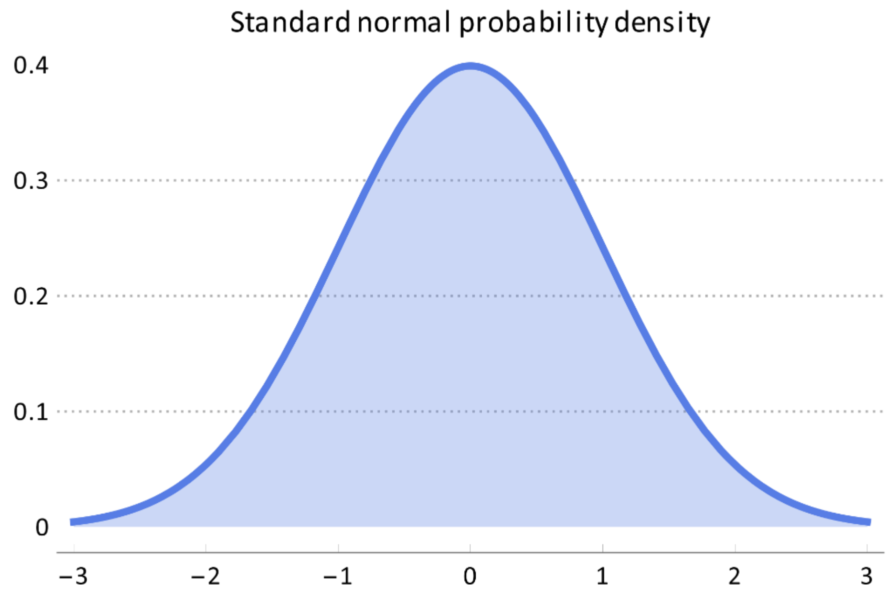
## Standard Normal Distribution

The *standard normal distribution* is characterized by the probability density function,

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

This is the classic "bell-curve" with mean zero and variance equal to one (Figure 1).

**Figure 1**




## Normal Distribution

As remarked earlier, the probability density function for the normal distribution depends on only two parameters, mean and variance. For a mean of $\mu$ and variance $\boldsymbol{\sigma}$, the following probability density function characterizes the *normal distribution*:

$$\phi_{\mu,\sigma^2}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

## Skew-normal Distribution

The probability density function for the skewed-normal distribution (see [8]) depends on three parameters,

1. $\boldsymbol{\xi}$, which is analogous to the mean, $\boldsymbol{\mu}$, for a normal distribution,
2. $\boldsymbol{\omega}$, which is analogous to the standard deviation, $\boldsymbol{\sigma}$, for the normal distribution, and
3. $\boldsymbol{\eta}$, which controls the distribution's skewness.

The probability density which determines the *skewed-normal distribution* has the equation:

$$\phi_{\xi,\omega,\eta}(x) = \frac{1}{\omega}\sqrt{\frac{1}{2\pi}} e^{-\frac{(x-\xi)^2}{2\omega^2}} \int_{-\infty}^{\eta\left(\frac{x-\xi}{\omega}\right)} e^{\frac{-t^2}{2}} dt$$

The mean for a skewed-normally distributed random variable is given by:

$$\mu = \xi + \omega\alpha\left(\frac{2}{\pi(1+\eta^2)}\right)^{1/2},$$

and the standard deviation is:

$$\sigma = \left(1 - \frac{2\eta^2}{\pi(1+\eta^2)}\right)^{1/2} \omega.$$

It is interesting to note that the probability density function may be described succinctly by the expression,

$$f(x) = \frac{2}{\omega}\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\eta\left(\frac{x-\xi}{\omega}\right)\right),$$

where $\phi$ is the density of the standard normal distribution and $\Phi$ is the cumulative distribution function of a standard normally distributed random variable. This expression suggests that one can parametrically skew any symmetric probability density and create a family of skewed distributions with similar tail behavior.

# Comparison of Normal and Skew-normal Distribution Returns

In comparing the normal and skewed-normal distributions, it is most interesting to notice that for the limit where $\boldsymbol{\eta} \to 0$, the skewed normal distribution becomes the normal distribution with mean $\boldsymbol{\xi}$ and variance $\boldsymbol{\omega}^{\mathbf{2}}$. However, when $\boldsymbol{\eta} \neq 0$, the mean and variance of the skewed normal distribution increases or decreases, depending on the sign of $\eta$, and hence, the standard deviation will change. Accordingly, direct comparison between these two functions requires a re-scaling of the mean and variance of the distribution so that they can stay the same. Said differently, ***a non-zero skew explicitly generates an adjusted mean and variance***. With this in mind, in the following section, we shall explore the even broader consequences of skew, as it affects our investment preferences. To achieve this, we will:

1. assume returns to be either skewed-normally or normally distributed;
2. fix a mean and variance for each choice of skewness (starting at zero, or no skew);
3. compute Conditional Tail Expectation (CTE) for these various levels of skewness.

## Equivalent Mean and Standard Deviation Parameters

For a skewed-normal distribution to have the same mean and variance as a normal distribution with mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$, we can algebraically solve for a scaled mean and standard deviation, $\boldsymbol{\xi}$ and $\boldsymbol{\omega}$ respectively. This will then allow us to vary the scaling quantity $\eta$ while keeping the mean and variance (and hence, Sharpe ratio) constant. Thus, we derive:

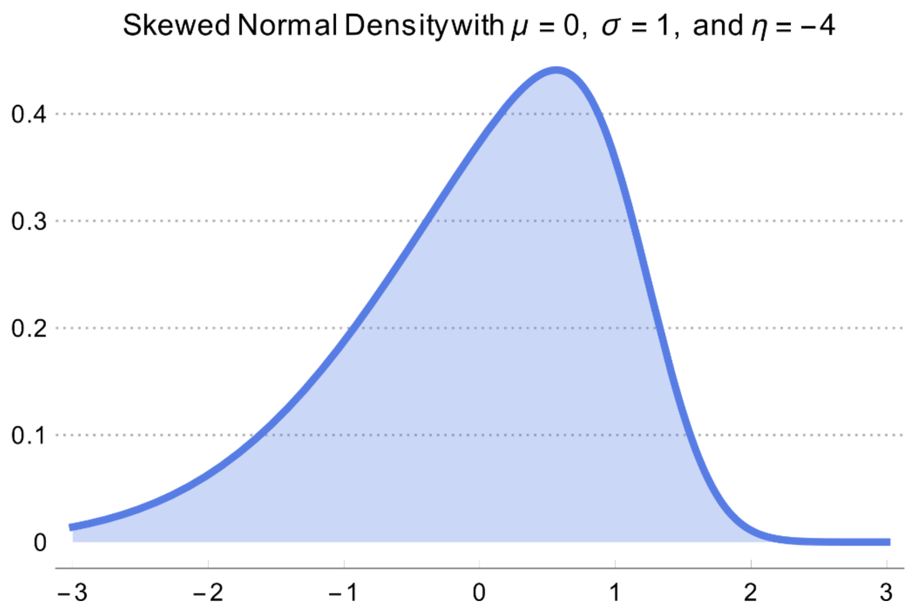
$$\xi = \mu - \left(\frac{\sqrt{2}\eta}{\sqrt{\pi - 2\kappa\eta^2 + \pi\eta^2}}\right)\sigma$$

**and**

$$\omega = \frac{\sigma}{\sqrt{\frac{1 + (1 - 2/\pi)\eta^2}{1 + \eta^2}}}.$$

If we then set the mean to zero, the standard deviation to one, but concurrently, set $\boldsymbol{\eta} = -2$, the resulting density function looks like this [see Figure 2]:

Figure 2




As intended, infusing $\boldsymbol{\eta}$ into the equation affects the skew of the distribution. In fact, we will soon see that setting this scaling variable to -2 corresponds to a skewness of $-0.78$. In fact, by design, there is a monotonically increasing relationship between our scaling variable and the skewness of the distribution. In the following sections, we will investigate exactly how this variable changes our assessment of the higher moments, and therefore the greater elements of risk when straying from the normal distribution.

## Skewness in Terms of $\eta$

Given a skewed-normal distribution with the above mean and standard deviation, we compute its skew, $\boldsymbol{\gamma}$, as a function of $\boldsymbol{\eta}$ (for negative values of $\boldsymbol{\kappa}$ this will generate a negative skew):

$$\gamma = \frac{\sqrt{2}(4-\pi)\eta^3}{(\pi + (-2+\pi)\eta^2)^{3/2}}$$

## Kurtosis in Terms of $\eta$

Similarly, the function relating $\boldsymbol{\eta}$ to the kurtosis, $\boldsymbol{\kappa}$, is:

$$\kappa = 3 + \frac{8(-3+\pi)\eta^4}{(\pi + (-2+\pi)\kappa\eta^2)^2}$$

And the excess kurtosis is then:

$$\kappa - 3 = \frac{8(-3+\pi)\eta^4}{(\pi + (-2+\pi)\eta^2)^2}$$

## Kurtosis in Terms of Skewness

While we are primarily focusing on skew, we gain further insight in seeing how skew, in turn, affects the greater moments of the distribution. As revealed above, our sensitivity to extreme losses are exponentially greater than to moderate ones. To explore these effects, we solve for the kurtosis as a function of skewness, obtaining:

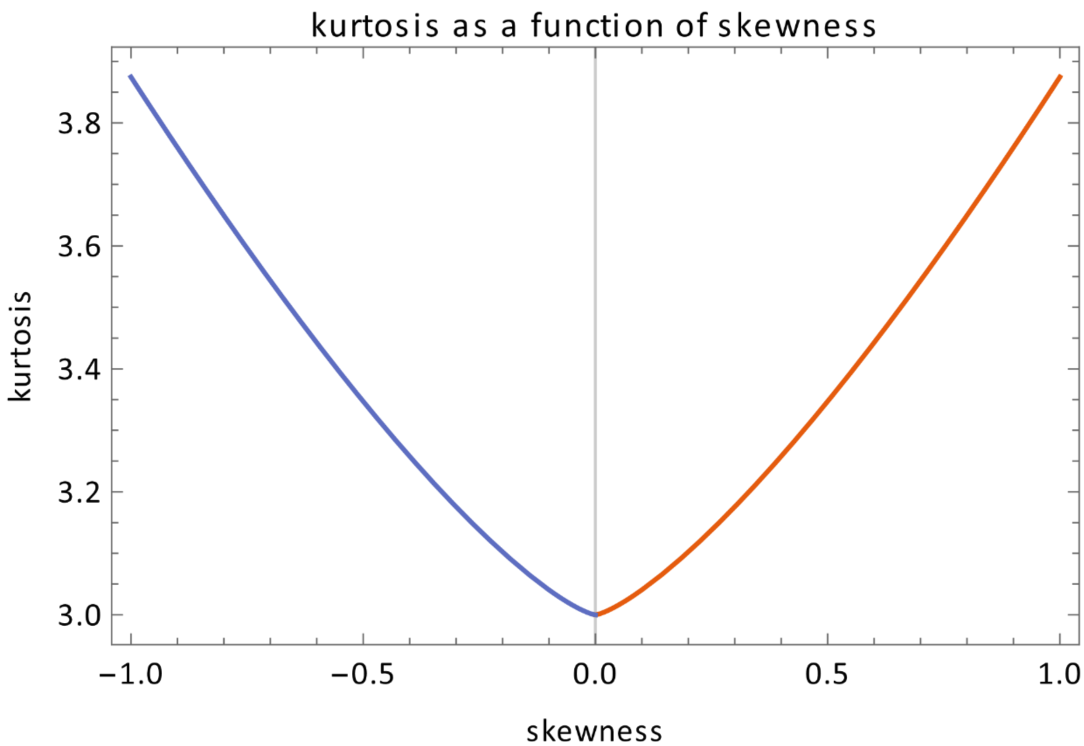
$$\kappa = 3 + \frac{2^{7/3}(\pi - 3)}{(4-\pi)^{4/3}}\gamma^{4/3}$$

Evaluating the expression for the coefficient of $\gamma_1$ in the above expression gives us:

$$\kappa \approx 3 + 0.87\gamma^{4/3}$$

If we then graph kurtosis as a function of skewness (note $-1 \leq \gamma \leq 1$), we derive the following [see Figure 3]:

Figure 3




Interestingly, while the mean and variance stay the same, the skewness precisely affects the kurtosis, and to boot, in a significant way. In the above calculation, and in its resulting graph, we see faster than linear growth of the kurtosis with respect to change in skewness, demonstrating that the increase in tail events (i.e. tail risk) is directly associated with the increase in skew. As noted earlier, rational NAIRA investors would be averse to the increased kurtosis. Consequently, even though it may seem that increasing positive skewness is preferable to these investors, they will eventually reject such an increase due to the inevitable increase in time to payoff.

To understand the further implications of this relationship, we need to recall two distinct premises; one, that kurtosis is higher when a greater proportion of the mass of the probability density function falls within the tails of the distribution, and two, the prevalence of negatively skewed returns in the hedge fund space. With this references in mind, we conclude that this relatively simple and natural modification of the normal distribution to the skewed normal produces significant ramifications with the hedge fund evaluation.

The key problem is that the negative skew also drives the increased kurtosis, effectively, hitting the risk profile with a "double-whammy".

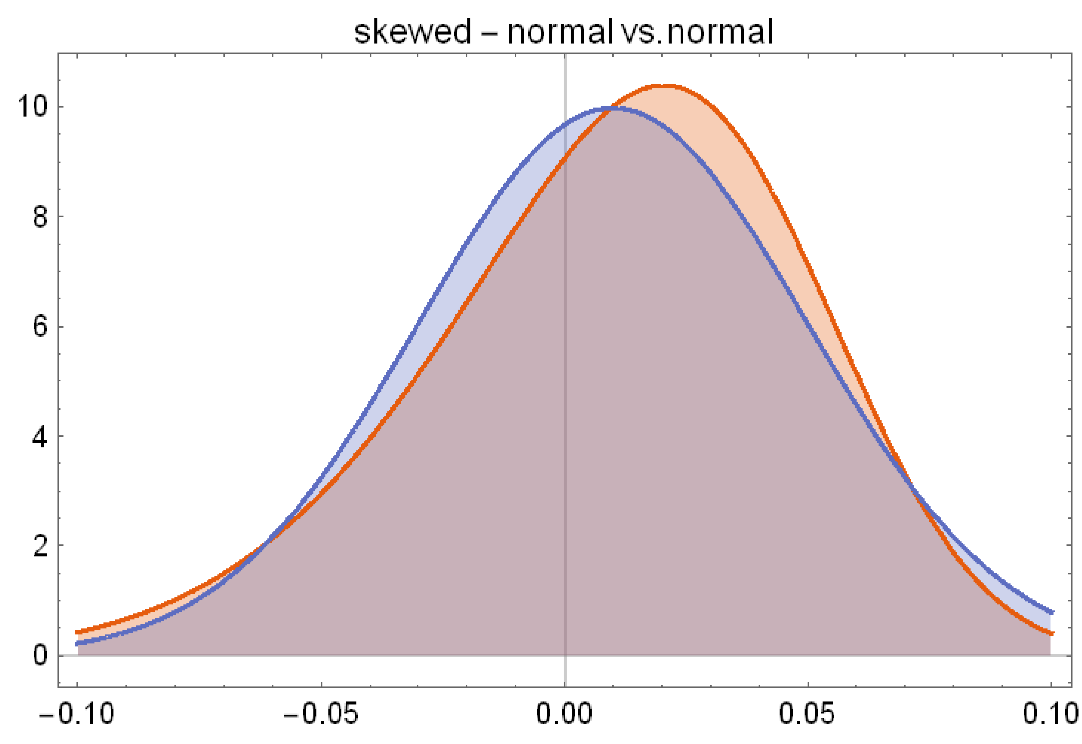
***To state the obvious, this "double-whammy" is the last thing one wants in an investment portfolio — it implies that increased risk comes in the form of not only more frequent losses (negative skew), but more frequent extreme losses (excess kurtosis).***

# Tail Risk Measurement

To test the extent to which the left tail of the (negatively) skewed normal distribution contains more frequent "extreme" events than a normally distributed process of the same mean and variance, we simulate 1,000,000 observations to compute quantile losses and conditional tail expectation (CTE) of both distributions, where the mean and standard deviation of each distribution is $\mu = 0.01$ and $\sigma = 0.04$. (If we interpret these as monthly returns, then they correspond to an annual return of $12\%$ and a volatility of 13.85% respectively.) The graphs of the density functions arising from these two are visualized in the following diagram (see

Figure 4):

Figure 4




Immediately, one can see that the skewed distribution (in orange) will lead to more frequent moderate returns above the mean as well as fewer moderate losses below the mean, giving the more regular appearance of "better" returns. However, where the rubber meets the road, i.e. where rational investors are most sensitive, the points on both sides where the two densities cross show that the ***normally distributed process (in blue) will generate both a***

***higher rate of excellent returns AND a lower rate of extreme losses***. In short, the impactful outliers unwind what otherwise appears to be a favorable frequency around the middle.

## Quantile Losses

To translate the relevant impact of these distributions into economic terms, we consider the hypothetical returns of $1,000,000 invested in a portfolio that exhibits a normal distribution of returns versus the same amount invested in a portfolio exhibiting a negatively skewed-normal distribution with $\boldsymbol{\eta} = -2.0$, both with $\boldsymbol{\mu} = 0.01$ and $\boldsymbol{\sigma} = 0.04$ (see Figure 4 above).

### Normally Distributed Portfolio

| *Return Percentile* | *Percentage Loss* | *Dollar Loss on 1 Million USD* |
|---|---|---|
| **0.1%** | -11.31% | $113,113 |
| **1%** | -8.31% | $83,114 |
| **10%** | -4.17% | $41,730 |

### Negatively Skewed-Normal Portfolio

| *Return Percentile* | *Percentage Loss* | *Dollar Loss on 1 Million USD* |
|---|---|---|
| **0.1%** | -13.58% | $135,810 |
| **1%** | -9.59% | $95,856 |
| **10%** | -4.35% | $43,450 |

With these individual profiles in hand, we can now compare the two to show the difference in losses at 0.1%, 1%, and 10% (where the losses are the percentiles of the distribution in which those losses fall, i.e. what percentage of the time they will occur).

### Normal vs. Negatively Skewed-Normal Portfolio

| *Return Percentile* | *Normal vs. Negative Skew* | *Excess % Loss* | *Excess Dollar Loss* |
|---|---|---|---|
| **0.1%** | 20% worse with negative skew | 2.27% | $22,698 |
| **1%** | 15% worse with negative skew | 1.27% | $12,742 |
| **10%** | 4% worse with negative skew | 1.72% | $1720 |

Considering a daily distribution of returns, we can now see that when a 0.1% (1:1000, or once-every-four-year) event happens, ***the normally distributed portfolio performs 120% better than the negatively skewed one***, protecting nearly $23,000 per $1,000,000 invested, or 2.3% of loss on the day it occurs. For a 1% event, or the worst return out of every 100 trading days (which should happen 2.5 times per year, on average), ***the normally distributed portfolio performs 115% better than the negatively skewed one*** and protects nearly $13,000 per $1,000,000 invested, or 1.3% of additional loss each time it happens. As is clearly illustrated, the presence of skew will have noticeable and meaningful effects at frequencies which matter most to investors.

More importantly, it should be noted that both distributions share the exact same mean and volatility, at with $\mu = 0.01$ and $\sigma = 0.04$, and as such, will on average generate the same end result. The crucial difference, however, will be that the skewed distribution will do so with intermittent shocks that should cause an investor greater discomfort, whereas the normally distributed portfolio will do so symmetrically. Symmetry, by its nature, has a more dependable outcome than lack of symmetry does[1]. ***This differentiation is powerful because it effectively means that the normal distribution is more reliable, as the expected deviation will not surprise, i.e. it consistently generates what is expected.*** For any rational investor, reliability of future return expectations is paramount.

Further to this point on reliability, given the timing of shocks that might occur within a portfolio that exhibits a skewed distribution, there are numerous conceivable trajectories that result in such a shock arriving later in time, which may be after the compounding of capital has generated a higher capital base, and therefore, where the percentage move affects a larger absolute drawdown in cash terms.

# Conditional Tail Expectation (a.k.a. CVaR)

For further insight, and along the same lines, we now make the same calculations for the risk metric known as conditional tail expectation (CTE) (often referred to as expected shortfall (ES), or conditional value at risk (CVaR)). What CTE (and its companions) computes is the average loss given that a loss below a certain likelihood occurs.

If one is concerned with what the average loss will be on the worst 10% of days one computes CTE at 90%. If one wants to test the average outcome of a worst-case event which happens 2.5 times per year, then one computes CTE at the level[2] of 99%. We output these in the following table:

| *Distribution Type* | *CTE at 0.999* | *CTE at 0.99* | *CTE at 0.90* | *CTE at 0.80* |
|---|---|---|---|---|
| *Normal* | -12.50% | -9.70% | -6.06% | -4.63% |
| *Negative Skew* | -15.08% | -11.40% | -6.70% | -4.94% |

In terms of dollar amount loss on a portfolio of $1,000,000, this gives:

| *Distribution Type* | *Avg. Dollar Loss at 0.999 CTE on 1 Million USD* | *Avg. Dollar Loss at 0.99 CTE on 1 Million USD* | *Avg. Dollar Loss at 0.90 CTE on 1 Million USD* | *Avg. Dollar Loss at 0.80 CTE on 1 Million USD* |
|---|---|---|---|---|
| *Normal* | $124,957 | $96,995 | $60,585 | $46,346 |
| *Negative Skew* | $150,807 | $114,039 | $67,023 | $49,423 |

This demonstrates that the worst two to three days every year in a portfolio with a skewed normal distribution will be, on average, 18% worse than if the portfolio were normally distributed, generating a 1.7% greater loss per

[1] A symmetric process necessarily has fewer parameters than an asymmetric one, as the asymmetry introduces a need to quantify the higher moments with additional parameters such as skew. Furthermore, to effectively calibrate the added parameters requires a larger number of observations. Given an inadequate sample size, one cannot be confident that the model is aptly fit to the underlying process – and to boot, additional consideration must also be given to which parameter each next observation may fit, and the associated interaction such fit will have on the other parameters. In stark contrast, in a symmetric process, each next observation simply accumulates mass and increases confidence in the existing symmetry.

[2] When a return is at or above a level, $p$, this means that the return is *lower* than $100 \times p\ \%$ of returns.

occurrence, or $17,044 per $1mm invested. Again, the skewed-normal distribution has $\eta = -2.0$, and both the normal and skewed-normal have $\mu = 0.01$, and $\sigma = 0.04$. Summarizing, we see:

| ***CTE Level*** | ***0.999*** | ***0.99*** | ***0.90*** | ***0.80*** |
|---|---|---|---|---|
| ***Normal vs. Negative Skew*** | 120% worse at stress level | 118% worse at stress level | 111% worse at stress level | 107% worse at stress level |
| ***Excess % Loss*** | 2.51% | 1.70% | 0.64% | 0.31% |
| ***Excess Dollar Loss*** | $25,123 | $17,044 | $6,438 | $3,077 |

In sum, we see similar levels of excess loss when evaluating our two portfolios with different types of risk metrics. The take away point is that regardless of what model or method we use to assess risk, and while there might be slight differences to the outcomes of each method, the skewed distribution will inevitably be worse performing than the normally distributed portfolio, and more so, less reliable regardless of the method used.

## Extra Return needed to Balance Skewness-induced CTE increase

Having established the inherent problems in variance as a stand-alone risk evaluation tool, the intrinsic "excess" risk characteristic to a skewed distribution, as well as derived the specific levels of that excess through different risk measurement tools, we come to understand that, by definition, any risk measurement that relies solely on variance while assessing processes that depend on additional parameters, is outputting a seriously flawed result. Sharpe ratio falls directly into this camp.

A crucial observation to be taken out of the above examples is that not only does Sharpe ratio fail to accurately assess such investments, as a result, it in turn does not correctly differentiate between, or even worse, incorrectly ranks a range of potential investments, which to boot, investors have distinct preferences for. Sadly, a higher Sharpe ratio may actually correspond to a less desirable investment when skewness (and the resulting conditional tail expectation) are taken into account.

As a thought experiment, suppose that one would like to rank returns solely on the basis of what theoretically matters most to investors, Conditional Tail Expectation (CTE). For instance, one would consider the skewed normal distribution equivalent to its normal counterpart if their CTE's are at the same at a level of $p < 1$. To keep the CTE's the same, and concurrently, ensure that the variance/volatility of the two distributions remain the same, we can allow the mean to vary, as required. Solving for the difference in value of the average return, $\mu$, at each given level $p$, we get:

| *CTE Value* | $\boldsymbol{p = 0.999}$ | $\boldsymbol{p = 0.99}$ | $\boldsymbol{p = 0.90}$ | $\boldsymbol{p = 0.80}$ |
|---|---|---|---|---|
| *Required mean return,* $\boldsymbol{\mu}$ | 3.75% | 2.75% | 1.7% | 1.3% |

In other words, given like variance, in order to have an equivalent CTE at $p = 0.8$, our skew-normal distribution would have to generate a mean return of 1.3% higher. The situation gets even worse when one is more risk averse and requires that the CTE's at $p = 0.90$ are equal. At this level, the skew-normal distribution would require a mean of 1.7% higher than a normally distributed process with same volatility. For CTE's with $p = 0.99$ and $0.999$, the mean requirements are dramatically higher, as shown above, reaching an almost implausible level.

What this illustrates is that under highly reasonable requirements for acceptable tail risk (equivalent CTE between $p = 0.90$ and $0.99$ ), given like volatility, one must demand 170% to 275% higher "average" returns, and hence similarly higher Sharpe ratios, for investments whose returns follow a skew-normal distribution.

# The Cost of Sharpe Ratio When Skew is Present – A Bold Solution

In light of the examples provided above, we find that it becomes absolutely necessary to dig deeper into the real "cost" to Sharpe Ratio when skew is present. In order to estimate this cost, and in general, to better understand the impact of skew on metrics that ignore it, we need to step outside of the mean-variance framework and engineer a robust solution from the ground up. To achieve this, our solution must thoroughly address the impactful higher moment of skewness, and concurrently, enable a side-by-side comparison to the classical Sharpe Ratio. That is, the output should be in the same proportions as a Sharpe Ratio normalized for skew, such that a zero skew return distribution will produce output precisely equal to a Sharpe Ratio, and any skewed return profile will output in a range that resembles a "skew-adjusted" Sharpe Ratio.

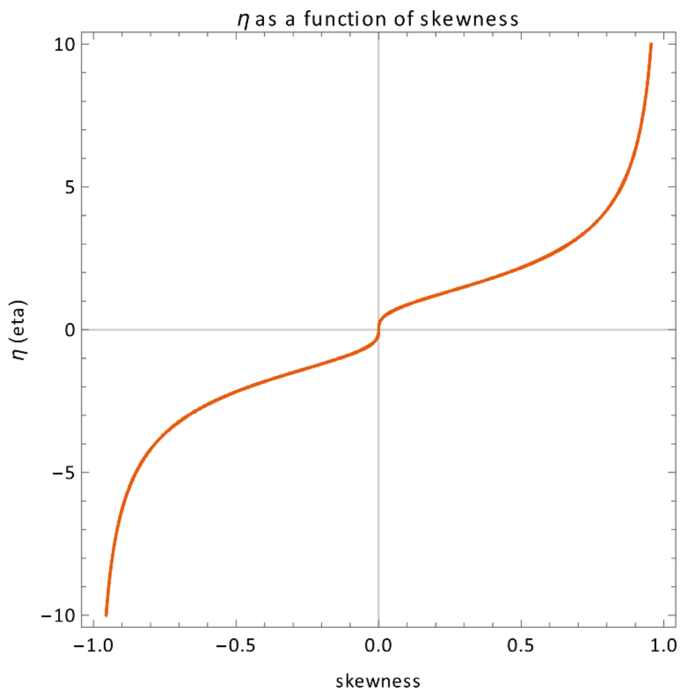
We took three broad steps to achieve this:

1. Determine a general class of distributions that have a shape parameter with which to appropriately model empirical observations that exhibit a fair range of skewness.
2. Quantify how changing the shape parameter, $\eta$, affects the mean-variance relationship (and thus the Sharpe Ratio) of an investment whose distribution fits within this class.
3. Appropriately apply our shape parameter to our model to efficiently compute a new risk metric whose output closely resembles a skew-adjusted Sharpe.

To build our model in a way that best illustrates our concept, as well as assesses a wide range of underlying distributions, the approach taken in step 1 assumes that our data and their skewness arise by sampling the de facto skew-normal distribution, which generalizes the normal distribution by allowing for non-zero skewness. Determining the actual stochastic process for specific returns data is a far more elaborate task, and an area of ongoing research that will not be addressed in this paper.

Utilizing the skew-normal distribution, we may numerically solve for the values of $\eta, \xi$, and $\omega$ which give rise to the sample estimates of $\mu, \sigma$ and $\gamma$. The algebraic expression for $\eta$ in terms of $\gamma$ is rather messy, but solvable. The algebraic solution enables step 2 of our procedure. To illustrate, the following graph depicts the relationship between $\eta$ and the skewness (see Figure 5). Also, to aid the reader in computing a specific $\eta$ from the annualized skewness, we have provided a table of values of $\eta$ in terms of skewness, $\gamma$ (see Appendix B).

<u>Figure 5</u>



Solving for $\xi/\omega$ in terms of $\eta, \mu$, and $\sigma$, we obtain,

$$\frac{\xi}{\omega} = \frac{\mu}{\sigma}\left(1 - \frac{2}{\pi}\frac{\eta^2}{1+\eta}\right)^{\frac{1}{2}} + \mathrm{sgn}(\eta)\left(\frac{2}{\pi}\frac{\eta^2}{1+\eta^2}\right)^{\frac{1}{2}},$$

where $\mathrm{sgn}(\kappa)$ equals $1$ if $\eta > 0$, $-1$ if $\eta < 0$, and $0$ if $\eta = 0$. Notice that as our shape parameter,$\eta$, approaches 0, $\xi/\omega$ approaches $\mu/\sigma$; that is, $\xi/\omega$ tends to the Sharpe Ratio as the skewness tends to zero. We will use this quantity as a basis for constructing a new model (concurrently, an adjustment to the classical Sharpe Ratio) to account for, more so penalize, negative skewness.

# Our New Risk/Reward Assessment Tool: The Skill Metric

The results from the previous section allow us to concisely state our principal result:

> Definition*: When analyzing returns compatible with the skew-normal distribution, we define the Skill Metric that appropriately penalizes for negative skewness,*
>
> $$\tau = \frac{\mu}{\sigma}\left(1 - \frac{2}{\pi}\frac{\eta^2}{1+\eta^2}\right)^{\frac{1}{2}} + \mathrm{sgn}(\eta)\frac{2}{\pi}\arctan\left(\frac{2\mu}{\pi\sigma}\right)\left(\frac{2}{\pi}\frac{\eta^2}{1+\eta^2}\right)^{\frac{1}{2}},$$
>
> *where $\eta$ is the shape parameter which we have computed as a function of skewness, $\gamma$.*[3]

We have thus captured the undeniable impact of skewness on mean-variance analysis, and to boot, have done so with a function that generates output in the same proportions as Sharpe Ratio, wherein a zero skew return profile will be precisely equal to the investment's Sharpe Ratio, and where a skewed return profile will produce a value that equates to a "skew-adjusted" Sharpe Ratio. This proportionality enables side-by-side comparison with Sharpe Ratio, so as to allow for re-ranking, and more importantly, providing an accurate assessment of the "cost" when skew is present.

That said, the obvious question surfaces as to why we dubbed our new function the "Skill Metric". As it stands, there are a wide range of investment statistics/metrics that are used to evaluate hedge funds, most of which aim to achieve two key things; one, to accurately assess the risk/return trade-off that the investment should provide, and two, to uncover manager skill. Per our nomenclature, it is of course the latter achievement that we are most focused on.

Since the overwhelming majority of hedge funds produce negatively skewed (non-symmetric) return distributions, and since most conventional metrics, including the ubiquitous Sharpe Ratio, assume normal (symmetric) distributions, as detailed in this paper, there is a tremendous amount of distortion surrounding investor's efforts to uncover alpha generation, or true skill. More precisely -- and more worrisome -- with the assumption of normality, risk is often understated and Sharpe Ratio is often overstated.

As we demonstrated above, since upside and downside volatility are asymmetric with skewed distributions, the heavier but infrequent downside volatility is blurred by the smaller but more frequent upside volatility in the

[3] Of note, because the prior function above (solving for $\xi/\omega$ in terms of $\eta, \mu$, and $\sigma$) would cause small positive mean-variance ratios (i.e. very low Sharpe Ratios) to output with negative values, we infused the inverse arc-tangent function, suitably normalized, to eliminate these counterintuitive outcomes. As a result, the output of our Skill Metric, when faced with a small positive mean-variance, will generate an intuitive result, as well as maintain its resemblance, and thus comparability, to a skew-adjusted Sharpe Ratio.

summarized "average" variance. This leads investors to mistakenly accept risk-adjusted performance where the hidden risk has merely been transferred to a later date, at which time the larger drawdowns inevitably occur. This is analogous to selling insurance, which in terms of return, can appear to generate current profit, but in terms of risk, must reserve for the inevitable event that will offset that current profit.

Thus, we propose a brand new risk-adjusted return tool, the Skill Metric, which unlike many others that only account for mean and variance, or that assume symmetry where there is not, the Skill Metric infuses mean, variance, and skew. And in doing so, we re-introduce the hidden risk back to the forefront and account for it in the here and now, and thus highlight return that does not rely on borrowed risk – i.e. skill. Effectively, where mean-variance calculations on asymmetric distributions transfers risk to a later date, we bring it back to today, and re-compute the risk/reward. Ideally, all metrics should dilute down to the key elements of potential risk vs. reward, not just backward-looking risk vs. reward. In this way, the Skill Metric truly illuminates skill.

Whether investors utilize our tool to assess the true Sharpe, to estimate the error in the Sharpe, or to further rank otherwise similar Sharpe's, the end result is that investors can now find what they should be seeking – a maximized tail-risk-adjusted return.

## Understanding our Skill Metric (a.k.a. skew-adjusted Sharpe)

This new function modifies the mean-variance relationship by addressing the need to adjust for the skewness of a distribution. In many cases, a return distribution with high negative skewness that results in a high Sharpe Ratio will lead to a Skill Metric that is significantly lower, but in our view, more accurate as to the "skill" involved in achieving the mean-variance. Said differently, we believe, and have shown above, that when skewness is present, the classical Sharpe Ratio inaccurately outputs a mean-variance relationship for which the mean sits on yet unexposed risk. To this end, we summarize some key attributes of our function:

- when the skewness of our distribution is zero, and hence $\eta = 0$, the Skill Metric and the Sharpe Ratio (i.e. $\tau$ and s) coincide. Intuitively, this is desirable since it does not change the results when skewness is not present.

- When the s is zero (or close to zero) the value of $\tau$ is also zero (or close to zero). Thus, the valuation of very low Sharpe Ratio investments are not drastically altered.

- The classical Sharpe Ratio is penalized for negative skewness in a nonlinear manner as given by the following figure (see figure 6)

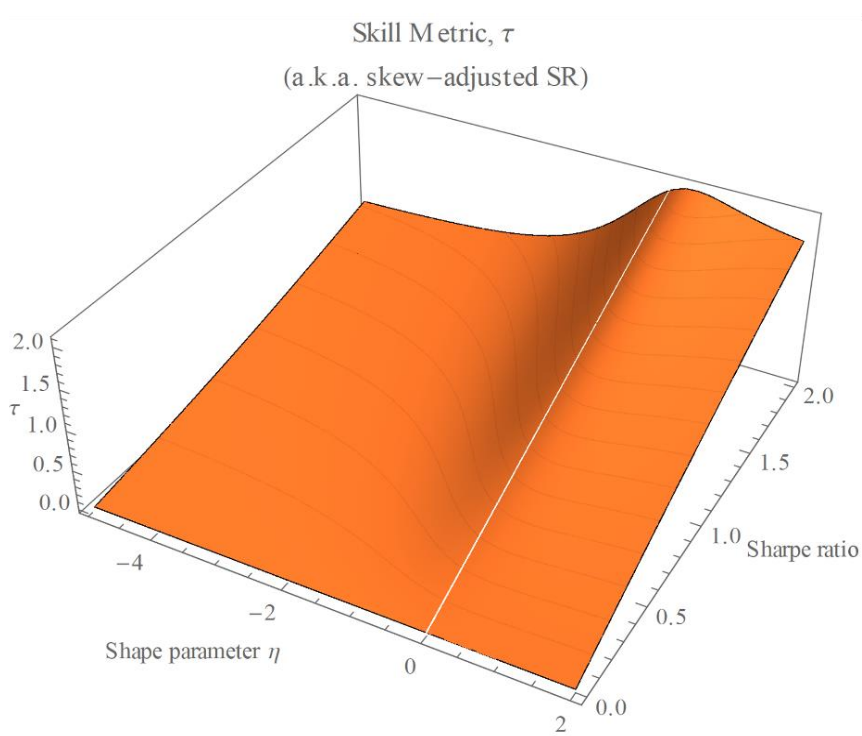



FIGURE 6

# Application to HFRI Indexes: After Skew Adjustment

As a practical application of our work we will apply our Skill Metric, $\tau$, to a familiar set of indices: the HFRI monthly return data from Hedge Fund Research (https://www.hedgefundresearch.com/)[4]. We note their relevant parameters, including the key parameter of skewness that creates the distortion we are most concerned about. Because the skew-normal distribution does not allow for skewness of magnitude greater than or equal to 1, we will restrict analyses to those constituents whose annualized skewness[5] is in this range. For the risk-free rate, we use the 1-month US Treasury rate, obtained from the St. Louis Federal Reserve Economic Data (FRED) website (https://fred.stlouisfed.org/). We summarize our results in Appendix A, where we have computed the value of $\eta$ and our Skill Metric, $\tau$, in terms of annualized monthly return, annualized monthly standard deviation, and annualized monthly skewness. Notably, for the 64 indexes which met our criteria, the rankings in terms of the Skill Metric, a.k.a. the skew-adjusted Sharpe Ratio, differs greatly from the Sharpe Ratio rankings. These differences highlight how profoundly negative skewness can affect performance and risk measures.

The following table lists the five indices whose ranking dropped the most when assessed with our Skill Metric, or equivalently, after adjustment to the Sharpe Ratio to account for negative skewness. Upon further review, or even after a quick glance, we see just how extreme these changes are. As the single highest "rank change", the equivalent skew-adjusted Sharpe Ratio of Relative Value is an incredible 67% lower, going from a Sharpe of 0.81 to a skew-adjusted Sharpe Ratio (Skill Metric) of only 0.27. To state the obvious, any action taken by an allocator after concluding a Sharpe of 0.81 would likely be different to the action taken had the Sharpe been originally assessed to be only 0.27.

Of even greater concern is the 2$^{nd}$ highest percentage "rank change", as it represents not just a single strategy, but in being the "Multi-Strategy", is an index compromised of investment strategies that painstakingly attempt to diversify across underlying individual strategies to reduce portfolio level risk. In fact, there should theoretically be a statistical edge to multi-strategy, wherein the combination of a variety of randomly distributed processes should result in a reduction of skewness. But counterintuitively, their negative skewness is actually higher than most single strategies! While diversification of alpha sources may be their goal, the end result is a further concentration of excess left tail, where accounting for that negative skewness brings their conceptual Sharpe Ratio down from almost 1 (at 0.97) to a lowly 0.34. This is not only contradictory to expectation, it highlights that the "multi" is merely a synonym for "many", where many will often be many of the same types of exposures that, in turn, exacerbate one of the primary undesired exposures. From our perspective, multi-strategy, however diversified by *number* of underlying strategies, has not diversified against, nor mitigated, and potentially even furthered, one of the most lethal exposures – high negative skew.

[4] Our data runs from September 2001 until May 2017. We use this period because it both excludes data prior to the most material change in Hedge Fund trading behavior, the Regulation Fair Disclosure (Reg FD) ratified in late 2000, as well as starts during a meaningfully uncertain market environment following the Sep 11, 2001 attacks. That said, our arbitrary starting point satisfied our primary goal of capturing at least 10-15 years of Hedge Fund behavior.

[5] It is commonly known that the sum of $N$ independent identically distributed returns has an average $N$ times as large as an average single return. Similarly, it is common knowledge that both the standard deviation and Sharpe ratio of a sum of $N$ independent identically distributed returns, scales by a factor of $\sqrt{N}$. Less commonly known is that one may use the theory of cumulants to prove that the skewness of a sum of $N$ independent identically distributed random variables scales by a factor of $1/\sqrt{N}$. It is this observation that allows us to annualize sample skewness for our calculations.

TABLE 1: TOP CHANGES TO SHARPE RATIO AFTER NEGATIVE SKEW ADJUSTMENT

| *symbol* | *index* | *Sharpe Ratio* | *skewness* | *Skill Metric* | *Sharpe rank* | *Skew-adjusted Sharpe rank* | *rank change* | *percent change* |
|---|---|---|---|---|---|---|---|---|
| ***HFRIAWRV*** | HFRI Relative Value (Total) Index - Asset Weighted | 0.81 | -0.85 | 0.27 | 14 | 44 | -30 | -66.92% |
| ***HFRIFI*** | HFRI RV: Multi-Strategy Index | 0.97 | -0.81 | 0.34 | 7 | 34 | -27 | -65.26% |
| ***HFRICRED*** | HFRI ED: Credit Arbitrage Index | 0.77 | -0.72 | 0.28 | 17 | 42 | -25 | -64.35% |
| ***HFRIFIHY*** | HFRI RV: Fixed Income-Corporate Index | 0.89 | -0.59 | 0.35 | 10 | 30 | -20 | -60.49% |
| ***HFRICRDT*** | HFRI Credit Index | 0.86 | -0.56 | 0.34 | 13 | 32 | -19 | -59.79% |

Of particular note, because of our equity market neutral (EMN) focus here at Logic$\alpha$, where we take rigorous measures to achieve a zero skew return distribution, we feel particularly pleased to note the reduction in the conceptual Sharpe of the EMN Index by 54% and concurrently compare the apples-to-apples skew-adjusted Sharpe (Skill Metric), of 0.32 with our materially superior one derived via optimization to a symmetric process.

| *symbol* | *index* | *Sharpe Ratio* | *skewness* | *Skill Metric* | *Sharpe rank* | *Skew-adjusted Sharpe rank* | *rank change* | *% change* |
|---|---|---|---|---|---|---|---|---|
| ***HFRIEMNI*** | HFRI EH: Equity Mkt. Neutral Index | 0.71 | -0.39 | 0.32 | 23 | 36 | -13 | -54.55% |

Looking more broadly than the "top changes", if one refers to the Exhibits A1 and A2, denoting the changes in Sharpe Ratio after adjusting for skew, as well as the changes in "rank" of amongst the different hedge fund strategies effected by this adjustment, one must consider the industry wide implications. In fact, one of the most profound points to take away from this paper is the dramatic reduction, in absolute terms, of the range of Sharpe Ratios across all participants. Whereas amongst highly varied strategies, Sharpe Ratios ranged from similar lows to a high of 2.21, after adjusting for negative skewness, the single highest Sharpe Ratio is only 1.12. Likewise, the 2$^{nd}$ highest one went from a Sharpe of 1.49 to an equivalent skew-adjusted Sharpe of 0.97, so instead of being over 70bps lower than the highest, it is only 15 bps lower than the highest. And this trend continues. Overall, Sharpe Ratios contract dramatically when adjusting for negative skewness, highlighting that allocators should broadly revise their expectations, and when coming across a Sharpe Ratio that appears "out of range", should immediately be wary of the high likelihood of negative skewness. Interestingly, this begs the bigger question as to whether there is a theoretical, or more appropriately, a real world limit to how high a skew-adjusted Sharpe Ratio (in our view, the only reliable Sharpe Ratio) can be. Simply put, no human can run a 1 minute mile.

As an even broader reference point, we also chose to assess the performance of the S&P500 Index via the SPY ETF over the same period as the HFR Indexes. The result is equally disheartening, demonstrating that the already low Sharpe Ratio of 0.25 gets chopped almost in half after adjusting for the negative skewness inherent in the returns of the broader market. Thus, it is not only alternative investments to be weary of, but any asset class. Unfortunately, no investment (unless designed with this concern explicitly in mind) is safe from the ever-present danger of a heavy left tail event.

| *symbol* | *Index* | *Sharpe Ratio* | *skewness* | *Skill Metric (skew-adjusted Sharpe Ratio)* | *% change* |
|---|---|---|---|---|---|
| *SPY* | S&P 500 ETF Trust | 0.25 | -0.24 | 0.13 | -48.30% |

# Application to YOUR investments: Using our Skill Metric

As an even more practical application of our work, we have provided for you a means of applying our Skill Metric, $\tau$, (a.k.a. skew-adjusted Sharpe Ratio) to any investment of your choosing. Undoubtedly, a result of reading this paper should lead one to ask – by what degree does the Sharpe Ratio change for the investments I've already made, or are considering making? And to that end, might you want to adjust, or re-allocate, after infusing a method of analysis that captures the one thing you have likely sought to avoid – left tail risk, or simply, exposure to larger magnitude, or more frequent drawdowns than you signed up for.

To make it simple and straightforward for all potential users, we have created a procedure to generate the Skill Metric, or equivalent skew-adjusted Sharpe. The main take away, of course, is that all the attention is on the key parameter of skewness in order to "unwind" the distortion in risk assessment that we are most concerned about.

1) As a starting point, note the skewness (annual) and Sharpe ratio of your investment;
2) Go to our reference chart, Appendix B, which enables you to derive the respective $\eta$ (skewness parameter) of your investment. Accordingly, lookup your skewness, and find your $\eta$;
3) Plug in the $\eta$ you , as well as the Sharpe of your investment, into the formula for Tau (shown above);
4) The resulting $\tau$ is the effective skew adjusted Sharpe ratio of your investment.

# Summary:

In conclusion, if there is only one single point that we have successfully illustrated, and ideally burned into your mind's eye, it is this – **negative skew is the enemy of investing**. And the "profits" derived from it are actually not profits at all; like borrowing money from the bank to fill up your wallet, you may feel like your cash position has improved, but your looming debt is not only pending, its growing! The quintessential problem with negative skewness is that the investment is not increasing its return symmetric to its risk, it is merely borrowing more future risk to provide additional increments of current return. Even worse, the accruing debt that must be paid off is often done so in a more violent or extreme manner – the infamous "tail event" which inevitably comes around to equalize the accretion of risk induced smaller wins. Accordingly, negative skewness MUST BE adjusted for, per our Tables above, and per the Appendices below. More so, and as touched upon in the preceding paragraphs, all readily accepted measures of risk vs. reward must be revised if one is to accurately assess an investment, or more appropriately, a manager's skill. Like any enemy you are trying to stay clear of, their tendency to blend in as a means of infiltrating your environment is analogous to the rampant distortion of industry standard measures and metrics for risk assessment and return evaluation and their resulting dependence, more so reliance, on inflated alpha and hidden risk.

# References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# Appendix A: effect of skew-adjusted Sharpe Ratio

Note that all the average return, volatility, Sharpe ratio (Sharpe) and skewness are annualized numbers.

## Table A1

HFRI indexes sorted by (decreasing) annualized Sharpe ratio:

| *symbol* | index | return | volatility | Sharpe ratio | skewness | kappa | Skew-adjusted Sharpe | Sharpe rank | Skew-adjusted Sharpe rank | rank change | percent change |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***HFRIFIMB*** | HFRI RV: Fixed Income-Asset Backed Index | 7.31% | 3.24% | 2.21 | -0.45 | -1.98 | 1.12 | 1 | 1 | 0 | -49.34% |
| ***HFRIFISV*** | HFRI RV: Fixed Income-Sovereign Index | 4.85% | 3.26% | 1.49 | -0.13 | -0.97 | 0.97 | 2 | 2 | 0 | -35.00% |
| ***HFRIRVA*** | HFRI Relative Value (Total) Index | 4.93% | 4.16% | 1.19 | -0.79 | -4.02 | 0.43 | 3 | 16 | -13 | -63.58% |
| ***HFRIMAI*** | HFRI ED: Merger Arbitrage Index | 3.18% | 3.00% | 1.07 | -0.22 | -1.26 | 0.6 | 4 | 7 | -3 | -44.07% |
| ***HFRIDSI*** | HFRI ED: Distressed/Restructuring Index | 6.39% | 6.18% | 1.04 | -0.34 | -1.62 | 0.51 | 5 | 12 | -7 | -50.85% |
| ***HFRIHLTH*** | HFRI EH: Sector - Healthcare Index | 9.13% | 9.35% | 0.97 | -0.27 | -1.4 | 0.51 | 6 | 11 | -5 | -47.59% |
| ***HFRIFI*** | HFRI RV: Multi-Strategy Index | 4.18% | 4.30% | 0.97 | -0.81 | -4.27 | 0.34 | 7 | 34 | -27 | -65.26% |
| ***HFRIFWIG*** | HFRI Fund Weighted Composite Index - GBP | 5.14% | 5.77% | 0.9 | -0.31 | -1.52 | 0.45 | 8 | 14 | -6 | -50.09% |
| ***HFRIVOL*** | HFRI RV: Volatility Index | 4.14% | 4.68% | 0.89 | -0.58 | -2.51 | 0.35 | 9 | 28 | -19 | -60.21% |
| ***HFRIFIHY*** | HFRI RV: Fixed Income-Corporate Index | 4.87% | 5.47% | 0.89 | -0.59 | -2.55 | 0.35 | 10 | 30 | -20 | -60.49% |
| ***HFRIEDI*** | HFRI Event-Driven (Total) Index | 5.44% | 6.27% | 0.87 | -0.33 | -1.59 | 0.42 | 11 | 18 | -7 | -51.20% |
| ***HFRIMI*** | HFRI Macro (Total) Index | 4.14% | 4.84% | 0.86 | 0.11 | 0.91 | 0.9 | 12 | 3 | 9 | 4.29% |
| ***HFRICRDT*** | HFRI Credit Index | 4.88% | 5.68% | 0.86 | -0.56 | -2.41 | 0.34 | 13 | 32 | -19 | -59.79% |
| ***HFRIAWRV*** | HFRI Relative Value (Total) Index - Asset Weighted | 4.46% | 5.50% | 0.81 | -0.85 | -4.9 | 0.27 | 14 | 44 | -30 | -66.92% |
| ***HFRIDVRS*** | HFRI Diversity Index | 4.64% | 5.79% | 0.8 | -0.19 | -1.17 | 0.46 | 15 | 13 | 2 | -43.11% |
| ***HFRIFWI*** | HFRI Fund Weighted Composite Index | 4.57% | 5.73% | 0.8 | -0.26 | -1.38 | 0.42 | 16 | 19 | -3 | -47.80% |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***HFRICRED*** | HFRI ED: Credit Arbitrage Index | 4.77% | 6.15% | 0.77 | -0.72 | -3.35 | 0.28 | 17 | 42 | -25 | -64.35% |
| ***HFRIEMG*** | HFRI Emerging Markets: Global Index | 6.41% | 8.64% | 0.75 | -0.26 | -1.38 | 0.39 | 18 | 22 | -4 | -48.02% |
| ***HFRIEMA*** | HFRI Emerging Markets: Asia ex-Japan Index | 8.85% | 12.10% | 0.73 | -0.17 | -1.1 | 0.43 | 19 | 17 | 2 | -41.77% |
| ***HFRIEM*** | HFRI Emerging Markets (Total) Index | 7.98% | 10.97% | 0.73 | -0.26 | -1.38 | 0.38 | 20 | 23 | -3 | -48.09% |
| ***HFRIFWIE*** | HFRI Fund Weighted Composite Index - EUR | 4.18% | 5.76% | 0.73 | -0.32 | -1.56 | 0.35 | 21 | 27 | -6 | -51.34% |
| ***HFRIWEU*** | HFRI Western/Pan Europe Index | 3.68% | 5.04% | 0.73 | -0.26 | -1.38 | 0.38 | 22 | 24 | -2 | -48.10% |
| ***HFRIEMNI*** | HFRI EH: Equity Market Neutral Index | 1.71% | 2.43% | 0.71 | -0.39 | -1.77 | 0.32 | 23 | 36 | -13 | -54.55% |
| ***HFRISTI*** | HFRI EH: Sector - Technology/Healthcare (Total) Index | 6.33% | 9.03% | 0.7 | -0.17 | -1.1 | 0.41 | 24 | 21 | 3 | -41.89% |
| ***HFRISRE*** | HFRI RV: Yield Alternatives Index | 5.64% | 8.08% | 0.7 | -0.3 | -1.49 | 0.34 | 25 | 31 | -6 | -50.45% |
| ***HFRIAWM*** | HFRI Macro (Total) Index - Asset Weighted | 3.32% | 4.86% | 0.68 | -0.03 | -0.54 | 0.53 | 26 | 10 | 16 | -21.98% |
| ***HFRIWOMN*** | HFRI Women Index | 4.48% | 6.73% | 0.67 | -0.24 | -1.32 | 0.35 | 27 | 29 | -2 | -47.12% |
| ***HFRIMACT*** | HFRI Macro: Active Trading Index | 2.47% | 3.72% | 0.66 | 0 | 0 | 0.66 | 28 | 5 | 23 | 0.00% |
| ***HFRITECH*** | HFRI EH: Sector - Technology Index | 5.24% | 8.07% | 0.65 | -0.2 | -1.2 | 0.36 | 29 | 26 | 3 | -44.45% |
| ***HFRIMTI*** | HFRI Macro: Systematic Diversified Index | 4.77% | 7.46% | 0.64 | 0.08 | 0.79 | 0.68 | 30 | 4 | 26 | 5.92% |
| ***HFRIEDSS*** | HFRI ED: Special Situations Index | 4.79% | 7.50% | 0.64 | -0.25 | -1.35 | 0.33 | 31 | 35 | -4 | -47.85% |
| ***HFRICIS*** | HFRI Emerging Markets: Russia/Eastern Europe Index | 10.46% | 17.32% | 0.61 | -0.31 | -1.52 | 0.3 | 32 | 37 | -5 | -51.32% |
| ***HFRIAWC*** | HFRI Asset Weighted Composite Index | 3.20% | 5.28% | 0.6 | -0.42 | -1.87 | 0.26 | 33 | 46 | -13 | -56.22% |
| ***HFRINA*** | HFRI North America Index | 4.06% | 6.73% | 0.6 | -0.33 | -1.59 | 0.29 | 34 | 38 | -4 | -52.33% |
| ***HFRIAWED*** | HFRI Event-Driven (Total) Index - Asset Weighted | 4.39% | 7.31% | 0.6 | -0.34 | -1.62 | 0.28 | 35 | 39 | -4 | -52.82% |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *HFRIENHI* | HFRI EH: Quantitative Directional Index | 5.59% | 9.37% | 0.6 | -0.18 | -1.13 | 0.34 | 36 | 33 | 3 | -43.07% |
| *HFRIWRLD* | HFRI World Index | 2.62% | 4.41% | 0.59 | -0.07 | -0.75 | 0.41 | 37 | 20 | 17 | -30.76% |
| *HFRICAI* | HFRI RV: Fixed Income-Convertible Arbitrage Index | 4.37% | 7.45% | 0.58 | -0.74 | -3.52 | 0.2 | 38 | 55 | -17 | -65.67% |
| *HFRIFOFM* | HFRI FOF: Market Defensive Index | 2.78% | 4.85% | 0.58 | 0.04 | 0.6 | 0.62 | 39 | 6 | 33 | 7.11% |
| *HFRIFOFC* | HFRI FOF: Conservative Index | 2.03% | 3.69% | 0.56 | -0.7 | -3.2 | 0.2 | 40 | 57 | -17 | -64.86% |
| *HFRIEHI* | HFRI Equity Hedge (Total) Index | 4.31% | 7.81% | 0.55 | -0.27 | -1.4 | 0.28 | 41 | 40 | 1 | -49.34% |
| *HFRIMMS* | HFRI Macro: Multi-Strategy Index | 2.42% | 4.38% | 0.55 | 0.03 | 0.54 | 0.59 | 42 | 8 | 34 | 7.31% |
| *HFRIFWIC* | HFRI Fund Weighted Composite Index - CHF | 3.12% | 5.73% | 0.55 | -0.31 | -1.52 | 0.26 | 43 | 45 | -2 | -51.54% |
| *HFRIFWIJ* | HFRI Fund Weighted Composite Index - JPY | 3.02% | 5.67% | 0.53 | -0.26 | -1.38 | 0.27 | 44 | 43 | 1 | -48.82% |
| *HFRIFOFD* | HFRI FOF: Diversified Index | 2.41% | 4.58% | 0.53 | -0.42 | -1.87 | 0.23 | 45 | 49 | -4 | -56.49% |
| *HFRIJPN* | HFRI Japan Index | 4.29% | 8.15% | 0.52 | 0.03 | 0.54 | 0.56 | 46 | 9 | 37 | 7.36% |
| *HFRIFOF* | HFRI Fund of Funds Composite Index | 2.46% | 4.77% | 0.52 | -0.43 | -1.91 | 0.22 | 47 | 50 | -3 | -56.91% |
| *HFRIMCUR* | HFRI Macro: Currency Index | 1.57% | 3.09% | 0.51 | -0.01 | -0.36 | 0.44 | 48 | 15 | 33 | -14.09% |
| *HFRIFOFS* | HFRI FOF: Strategic Index | 2.90% | 6.20% | 0.47 | -0.36 | -1.68 | 0.22 | 49 | 52 | -3 | -54.18% |
| *HFRIEDMS* | HFRI ED: Multi-Strategy Index | 3.82% | 8.21% | 0.46 | -0.36 | -1.68 | 0.21 | 50 | 54 | -4 | -54.20% |
| *HFRIEHFV* | HFRI EH: Fundamental Value Index | 4.10% | 9.31% | 0.44 | -0.22 | -1.26 | 0.24 | 51 | 48 | 3 | -46.54% |
| *HFRIAWJ* | HFRI Asia with Japan Index | 2.71% | 6.64% | 0.4 | -0.07 | -0.75 | 0.28 | 52 | 41 | 11 | -31.19% |
| *HFRIEMLA* | HFRI Emerging Markets: Latin America Index | 5.37% | 13.34% | 0.4 | -0.15 | -1.04 | 0.24 | 53 | 47 | 6 | -41.02% |
| *HFRIEHMS* | HFRI EH: Multi-Strategy Index | 3.15% | 7.81% | 0.4 | -0.19 | -1.17 | 0.22 | 54 | 51 | 3 | -44.44% |
| *HFRISEN* | HFRI EH: Sector - Energy/Basic Materials Index | 5.07% | 13.65% | 0.37 | -0.22 | -1.26 | 0.2 | 55 | 56 | -1 | -46.71% |
| *HFRIMCOM* | HFRI Macro: Commodity Index | 2.01% | 5.64% | 0.36 | 0.32 | 1.56 | 0.36 | 56 | 25 | 31 | 0.88% |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *HFRICHN* | HFRI Emerging Markets: China Index | 4.70% | 14.01% | 0.33 | -0.1 | -0.87 | 0.22 | 57 | 53 | 4 | -35.69% |
| *HFRIAWEH* | HFRI Eq. Hedge (Total) Index - Asset Weighted | 2.70% | 8.14% | 0.33 | -0.29 | -1.46 | 0.16 | 58 | 59 | -1 | -51.10% |
| *HFRIACT* | HFRI ED: Activist Index | 4.02% | 13.36% | 0.3 | -0.21 | -1.23 | 0.16 | 59 | 58 | 1 | -46.16% |
| *HFRIMENA* | HFRI Emerging Markets: MENA Index | 2.39% | 9.76% | 0.24 | -0.16 | -1.07 | 0.14 | 60 | 61 | -1 | -42.25% |
| *HFRIEHFG* | HFRI EH: Fundamental Growth Index | 2.05% | 11.64% | 0.18 | -0.21 | -1.23 | 0.09 | 61 | 63 | -2 | -46.36% |
| *HFRIIND* | HFRI Emerging Markets: India Index | 3.00% | 22.25% | 0.13 | 0.06 | 0.71 | 0.14 | 62 | 60 | 2 | 7.39% |
| *HFRIMDT* | HFRI Macro: Discretionary Thematic Index | 0.51% | 4.72% | 0.11 | 0.01 | 0.36 | 0.11 | 63 | 62 | 1 | 7.17% |
| *HFRISHSE* | HFRI EH: Short Bias Index | -4.80% | 11.13% | -0.43 | 0.07 | 0.75 | -0.46 | 64 | 64 | 0 | 6.73% |

# Table A2

HFRI indexes sorted by (increasing) rank change:

| ***symbol*** | **index** | **return** | **volatility** | **Sharpe ratio** | **skewness** | **kappa** | **Skew-adjusted Sharpe** | **Sharpe rank** | **Skew-adjusted Sharpe rank** | **rank change** | **percent change** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***HFRIAWRV*** | HFRI Relative Value (Total) Index - Asset Weighted | 4.46% | 5.50% | 0.81 | -0.85 | -4.9 | 0.27 | 14 | 44 | -30 | -66.92% |
| ***HFRIFI*** | HFRI RV: Multi-Strategy Index | 4.18% | 4.30% | 0.97 | -0.81 | -4.27 | 0.34 | 7 | 34 | -27 | -65.26% |
| ***HFRICRED*** | HFRI ED: Credit Arbitrage Index | 4.77% | 6.15% | 0.77 | -0.72 | -3.35 | 0.28 | 17 | 42 | -25 | -64.35% |
| ***HFRIFIHY*** | HFRI RV: Fixed Income-Corporate Index | 4.87% | 5.47% | 0.89 | -0.59 | -2.55 | 0.35 | 10 | 30 | -20 | -60.49% |
| ***HFRIVOL*** | HFRI RV: Volatility Index | 4.14% | 4.68% | 0.89 | -0.58 | -2.51 | 0.35 | 9 | 28 | -19 | -60.21% |
| ***HFRICRDT*** | HFRI Credit Index | 4.88% | 5.68% | 0.86 | -0.56 | -2.41 | 0.34 | 13 | 32 | -19 | -59.79% |
| ***HFRICAI*** | HFRI RV: Fixed Income-Convertible Arbitrage Index | 4.37% | 7.45% | 0.58 | -0.74 | -3.52 | 0.2 | 38 | 55 | -17 | -65.67% |
| ***HFRIFOFC*** | HFRI FOF: Conservative Index | 2.03% | 3.69% | 0.56 | -0.7 | -3.2 | 0.2 | 40 | 57 | -17 | -64.86% |
| ***HFRIRVA*** | HFRI Relative Value (Total) Index | 4.93% | 4.16% | 1.19 | -0.79 | -4.02 | 0.43 | 3 | 16 | -13 | -63.58% |
| ***HFRIEMNI*** | HFRI EH: Equity Market Neutral Index | 1.71% | 2.43% | 0.71 | -0.39 | -1.77 | 0.32 | 23 | 36 | -13 | -54.55% |
| ***HFRIAWC*** | HFRI Asset Weighted Composite Index | 3.20% | 5.28% | 0.6 | -0.42 | -1.87 | 0.26 | 33 | 46 | -13 | -56.22% |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***HFRIDSI*** | HFRI ED: Distressed/Restructuring Index | 6.39% | 6.18% | 1.04 | -0.34 | -1.62 | 0.51 | 5 | 12 | -7 | -50.85% |
| ***HFRIEDI*** | HFRI Event-Driven (Total) Index | 5.44% | 6.27% | 0.87 | -0.33 | -1.59 | 0.42 | 11 | 18 | -7 | -51.20% |
| ***HFRIFWIG*** | HFRI Fund Weighted Composite Index - GBP | 5.14% | 5.77% | 0.9 | -0.31 | -1.52 | 0.45 | 8 | 14 | -6 | -50.09% |
| ***HFRIFWIE*** | HFRI Fund Weighted Composite Index - EUR | 4.18% | 5.76% | 0.73 | -0.32 | -1.56 | 0.35 | 21 | 27 | -6 | -51.34% |
| ***HFRISRE*** | HFRI RV: Yield Alternatives Index | 5.64% | 8.08% | 0.7 | -0.3 | -1.49 | 0.34 | 25 | 31 | -6 | -50.45% |
| ***HFRIHLTH*** | HFRI EH: Sector - Healthcare Index | 9.13% | 9.35% | 0.97 | -0.27 | -1.4 | 0.51 | 6 | 11 | -5 | -47.59% |
| ***HFRICIS*** | HFRI Emerging Markets: Russia/Eastern Europe Index | 10.46% | 17.32% | 0.61 | -0.31 | -1.52 | 0.3 | 32 | 37 | -5 | -51.32% |
| ***HFRIEMG*** | HFRI Emerging Markets: Global Index | 6.41% | 8.64% | 0.75 | -0.26 | -1.38 | 0.39 | 18 | 22 | -4 | -48.02% |
| ***HFRIEDSS*** | HFRI ED: Special Situations Index | 4.79% | 7.50% | 0.64 | -0.25 | -1.35 | 0.33 | 31 | 35 | -4 | -47.85% |
| ***HFRINA*** | HFRI North America Index | 4.06% | 6.73% | 0.6 | -0.33 | -1.59 | 0.29 | 34 | 38 | -4 | -52.33% |
| ***HFRIAWED*** | HFRI Event-Driven (Total) Index - Asset Weighted | 4.39% | 7.31% | 0.6 | -0.34 | -1.62 | 0.28 | 35 | 39 | -4 | -52.82% |
| ***HFRIFOFD*** | HFRI FOF: Diversified Index | 2.41% | 4.58% | 0.53 | -0.42 | -1.87 | 0.23 | 45 | 49 | -4 | -56.49% |
| ***HFRIEDMS*** | HFRI ED: Multi-Strategy Index | 3.82% | 8.21% | 0.46 | -0.36 | -1.68 | 0.21 | 50 | 54 | -4 | -54.20% |
| ***HFRIMAI*** | HFRI ED: Merger Arbitrage Index | 3.18% | 3.00% | 1.07 | -0.22 | -1.26 | 0.6 | 4 | 7 | -3 | -44.07% |
| ***HFRIFWI*** | HFRI Fund Weighted Composite Index | 4.57% | 5.73% | 0.8 | -0.26 | -1.38 | 0.42 | 16 | 19 | -3 | -47.80% |
| ***HFRIEM*** | HFRI Emerging Markets (Total) Index | 7.98% | 10.97% | 0.73 | -0.26 | -1.38 | 0.38 | 20 | 23 | -3 | -48.09% |
| ***HFRIFOF*** | HFRI Fund of Funds Composite Index | 2.46% | 4.77% | 0.52 | -0.43 | -1.91 | 0.22 | 47 | 50 | -3 | -56.91% |
| ***HFRIFOFS*** | HFRI FOF: Strategic Index | 2.90% | 6.20% | 0.47 | -0.36 | -1.68 | 0.22 | 49 | 52 | -3 | -54.18% |
| ***HFRIWEU*** | HFRI Western/Pan Europe Index | 3.68% | 5.04% | 0.73 | -0.26 | -1.38 | 0.38 | 22 | 24 | -2 | -48.10% |
| ***HFRIWOMN*** | HFRI Women Index | 4.48% | 6.73% | 0.67 | -0.24 | -1.32 | 0.35 | 27 | 29 | -2 | -47.12% |
| ***HFRIFWIC*** | HFRI Fund Weighted Composite Index - CHF | 3.12% | 5.73% | 0.55 | -0.31 | -1.52 | 0.26 | 43 | 45 | -2 | -51.54% |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ***HFRIEHFG*** | HFRI EH: Fundamental Growth Index | 2.05% | 11.64% | 0.18 | -0.21 | -1.23 | 0.09 | 61 | 63 | -2 | -46.36% |
| ***HFRISEN*** | HFRI EH: Sector - Energy/Basic Materials Index | 5.07% | 13.65% | 0.37 | -0.22 | -1.26 | 0.2 | 55 | 56 | -1 | -46.71% |
| ***HFRIAWEH*** | HFRI Equity Hedge (Total) Index - Asset Weighted | 2.70% | 8.14% | 0.33 | -0.29 | -1.46 | 0.16 | 58 | 59 | -1 | -51.10% |
| ***HFRIMENA*** | HFRI Emerging Markets: MENA Index | 2.39% | 9.76% | 0.24 | -0.16 | -1.07 | 0.14 | 60 | 61 | -1 | -42.25% |
| ***HFRIFIMB*** | HFRI RV: Fixed Income-Asset Backed Index | 7.31% | 3.24% | 2.21 | -0.45 | -1.98 | 1.12 | 1 | 1 | 0 | -49.34% |
| ***HFRIFISV*** | HFRI RV: Fixed Income-Sovereign Index | 4.85% | 3.26% | 1.49 | -0.13 | -0.97 | 0.97 | 2 | 2 | 0 | -35.00% |
| ***HFRISHSE*** | HFRI EH: Short Bias Index | -4.80% | 11.13% | -0.43 | 0.07 | 0.75 | -0.46 | 64 | 64 | 0 | 6.73% |
| ***HFRIEHI*** | HFRI Equity Hedge (Total) Index | 4.31% | 7.81% | 0.55 | -0.27 | -1.4 | 0.28 | 41 | 40 | 1 | -49.34% |
| ***HFRIFWIJ*** | HFRI Fund Weighted Composite Index - JPY | 3.02% | 5.67% | 0.53 | -0.26 | -1.38 | 0.27 | 44 | 43 | 1 | -48.82% |
| ***HFRIACT*** | HFRI ED: Activist Index | 4.02% | 13.36% | 0.3 | -0.21 | -1.23 | 0.16 | 59 | 58 | 1 | -46.16% |
| ***HFRIMDT*** | HFRI Macro: Discretionary Thematic Index | 0.51% | 4.72% | 0.11 | 0.01 | 0.36 | 0.11 | 63 | 62 | 1 | 7.17% |
| ***HFRIDVRS*** | HFRI Diversity Index | 4.64% | 5.79% | 0.8 | -0.19 | -1.17 | 0.46 | 15 | 13 | 2 | -43.11% |
| ***HFRIEMA*** | HFRI Emerging Markets: Asia ex-Japan Index | 8.85% | 12.10% | 0.73 | -0.17 | -1.1 | 0.43 | 19 | 17 | 2 | -41.77% |
| ***HFRIIND*** | HFRI Emerging Markets: India Index | 3.00% | 22.25% | 0.13 | 0.06 | 0.71 | 0.14 | 62 | 60 | 2 | 7.39% |
| ***HFRISTI*** | HFRI EH: Sector - Technology/Healthcare (Total) Index | 6.33% | 9.03% | 0.7 | -0.17 | -1.1 | 0.41 | 24 | 21 | 3 | -41.89% |
| ***HFRITECH*** | HFRI EH: Sector - Technology Index | 5.24% | 8.07% | 0.65 | -0.2 | -1.2 | 0.36 | 29 | 26 | 3 | -44.45% |
| ***HFRIENHI*** | HFRI EH: Quantitative Directional Index | 5.59% | 9.37% | 0.6 | -0.18 | -1.13 | 0.34 | 36 | 33 | 3 | -43.07% |
| ***HFRIEHFV*** | HFRI EH: Fundamental Value Index | 4.10% | 9.31% | 0.44 | -0.22 | -1.26 | 0.24 | 51 | 48 | 3 | -46.54% |
| ***HFRIEHMS*** | HFRI EH: Multi-Strategy Index | 3.15% | 7.81% | 0.4 | -0.19 | -1.17 | 0.22 | 54 | 51 | 3 | -44.44% |
| ***HFRICHN*** | HFRI Emerging Markets: China Index | 4.70% | 14.01% | 0.33 | -0.1 | -0.87 | 0.22 | 57 | 53 | 4 | -35.69% |
| ***HFRIEMLA*** | HFRI Emerging Markets: Latin America Index | 5.37% | 13.34% | 0.4 | -0.15 | -1.04 | 0.24 | 53 | 47 | 6 | -41.02% |
| ***HFRIMI*** | HFRI Macro (Total) Index | 4.14% | 4.84% | 0.86 | 0.11 | 0.91 | 0.9 | 12 | 3 | 9 | 4.29% |
| ***HFRIAWJ*** | HFRI Asia with Japan Index | 2.71% | 6.64% | 0.4 | -0.07 | -0.75 | 0.28 | 52 | 41 | 11 | -31.19% |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***HFRIAWM*** | HFRI Macro (Total) Index - Asset Weighted | 3.32% | 4.86% | 0.68 | -0.03 | -0.54 | 0.53 | 26 | 10 | 16 | -21.98% |
| ***HFRIWRLD*** | HFRI World Index | 2.62% | 4.41% | 0.59 | -0.07 | -0.75 | 0.41 | 37 | 20 | 17 | -30.76% |
| ***HFRIMACT*** | HFRI Macro: Active Trading Index | 2.47% | 3.72% | 0.66 | 0 | 0 | 0.66 | 28 | 5 | 23 | 0.00% |
| ***HFRIMTI*** | HFRI Macro: Systematic Diversified Index | 4.77% | 7.46% | 0.64 | 0.08 | 0.79 | 0.68 | 30 | 4 | 26 | 5.92% |
| ***HFRIMCOM*** | HFRI Macro: Commodity Index | 2.01% | 5.64% | 0.36 | 0.32 | 1.56 | 0.36 | 56 | 25 | 31 | 0.88% |
| ***HFRIFOFM*** | HFRI FOF: Market Defensive Index | 2.78% | 4.85% | 0.58 | 0.04 | 0.6 | 0.62 | 39 | 6 | 33 | 7.11% |
| ***HFRIMCUR*** | HFRI Macro: Currency Index | 1.57% | 3.09% | 0.51 | -0.01 | -0.36 | 0.44 | 48 | 15 | 33 | -14.09% |
| ***HFRIMMS*** | HFRI Macro: Multi-Strategy Index | 2.42% | 4.38% | 0.55 | 0.03 | 0.54 | 0.59 | 42 | 8 | 34 | 7.31% |
| ***HFRIJPN*** | HFRI Japan Index | 4.29% | 8.15% | 0.52 | 0.03 | 0.54 | 0.56 | 46 | 9 | 37 | 7.36% |

# Appendix B: table for numerically determining $\eta$ in terms of skewness

## Table B1

The following table expresses the numerical relationship between the shape parameter, $\eta$, and the skewness of the skew-normal distribution.

| skewness | $\eta$ |
|---|---|
| **-0.99** | -18.1753 |
| **-0.98** | -14.5016 |
| **-0.97** | -11.65765 |
| **-0.96** | -10.01645 |
| **-0.95** | -8.9066 |
| **-0.94** | -8.0889 |
| **-0.93** | -7.45265 |
| **-0.92** | -6.9385 |
| **-0.91** | -6.5112 |
| **-0.9** | -6.1483 |
| **-0.89** | -5.8348 |
| **-0.88** | -5.56015 |
| **-0.87** | -5.31675 |
| **-0.86** | -5.09885 |
| **-0.85** | -4.90215 |
| **-0.84** | -4.72335 |
| **-0.83** | -4.55975 |
| **-0.82** | -4.40925 |
| **-0.81** | -4.27005 |
| **-0.8** | -4.1407 |
| **-0.79** | -4.0201 |
| **-0.78** | -3.9072 |
| **-0.77** | -3.80115 |
| **-0.76** | -3.7012 |
| **-0.75** | -3.6068 |
| **-0.74** | -3.5174 |
| **-0.73** | -3.43245 |
| **-0.72** | -3.35165 |
| **-0.71** | -3.2746 |
| **-0.7** | -3.20095 |
| **-0.69** | -3.1305 |
| **-0.68** | -3.06295 |
| **-0.67** | -2.99805 |
| **-0.66** | -2.9356 |
| **-0.65** | -2.87545 |
| **-0.64** | -2.81745 |
| **-0.63** | -2.7614 |
| **-0.62** | -2.7072 |
| **-0.61** | -2.6547 |
| **-0.6** | -2.6038 |
| **-0.59** | -2.55445 |
| **-0.58** | -2.5065 |
| **-0.57** | -2.45985 |
| **-0.56** | -2.41445 |
| **-0.55** | -2.3702 |
| **-0.54** | -2.32705 |
| **-0.53** | -2.2849 |
| **-0.52** | -2.2437 |
| **-0.51** | -2.20345 |
| **-0.5** | -2.1641 |
| **-0.49** | -2.12555 |
| **-0.48** | -2.0877 |
| **-0.47** | -2.05055 |
| **-0.46** | -2.0141 |
| **-0.45** | -1.9783 |
| **-0.44** | -1.9431 |
| **-0.43** | -1.90845 |
| **-0.42** | -1.87435 |
| **-0.41** | -1.84075 |
| **-0.4** | -1.80755 |
| **-0.39** | -1.7748 |
| **-0.38** | -1.7425 |
| **-0.37** | -1.71055 |
| **-0.36** | -1.6789 |
| **-0.35** | -1.64755 |
| **-0.34** | -1.6165 |
| **-0.33** | -1.58575 |
| **-0.32** | -1.5552 |
| **-0.31** | -1.52485 |
| **-0.3** | -1.4947 |
| **-0.29** | -1.46465 |
| **-0.28** | -1.43475 |
| **-0.27** | -1.40495 |
| **-0.26** | -1.37515 |
| **-0.25** | -1.3454 |
| **-0.24** | -1.31565 |
| **-0.23** | -1.28585 |
| **-0.22** | -1.256 |
| **-0.21** | -1.226 |
| **-0.2** | -1.1958 |
| **-0.19** | -1.16535 |
| **-0.18** | -1.13465 |
| **-0.17** | -1.10365 |
| **-0.16** | -1.0722 |
| **-0.15** | -1.04025 |
| **-0.14** | -1.0077 |
| **-0.13** | -0.97445 |
| **-0.12** | -0.9404 |
| **-0.11** | -0.90535 |
| **-0.1** | -0.86905 |
| **-0.09** | -0.8313 |
| **-0.08** | -0.79175 |
| **-0.07** | -0.74995 |
| **-0.06** | -0.70525 |
| **-0.05** | -0.65665 |
| **-0.04** | -0.6027 |
| **-0.03** | -0.54065 |
| **-0.02** | -0.46465 |
| **-0.01** | -0.355 |
| **0** | 0 |
| **0.01** | 0.355 |
| **0.02** | 0.46465 |
| **0.03** | 0.54065 |
| **0.04** | 0.6027 |
| **0.05** | 0.65665 |
| **0.06** | 0.70525 |
| **0.07** | 0.74995 |
| **0.08** | 0.79175 |
| **0.09** | 0.8313 |
| **0.1** | 0.86905 |
| **0.11** | 0.90535 |
| **0.12** | 0.9404 |
| **0.13** | 0.97445 |
| **0.14** | 1.0077 |
| **0.15** | 1.04025 |
| **0.16** | 1.0722 |
| **0.17** | 1.10365 |
| **0.18** | 1.13465 |
| **0.19** | 1.16535 |
| **0.2** | 1.1958 |
| **0.21** | 1.226 |
| **0.22** | 1.256 |

| | |
|---|---|
| **0.23** | 1.28585 |
| **0.24** | 1.31565 |
| **0.25** | 1.3454 |
| **0.26** | 1.37515 |
| **0.27** | 1.40495 |
| **0.28** | 1.43475 |
| **0.29** | 1.46465 |
| **0.3** | 1.4947 |
| **0.31** | 1.52485 |
| **0.32** | 1.5552 |
| **0.33** | 1.58575 |
| **0.34** | 1.6165 |
| **0.35** | 1.64755 |
| **0.36** | 1.6789 |
| **0.37** | 1.71055 |
| **0.38** | 1.7425 |
| **0.39** | 1.7748 |
| **0.4** | 1.80755 |
| **0.41** | 1.84075 |
| **0.42** | 1.87435 |
| **0.43** | 1.90845 |
| **0.44** | 1.9431 |
| **0.45** | 1.9783 |
| **0.46** | 2.0141 |
| **0.47** | 2.05055 |
| **0.48** | 2.0877 |

| | |
|---|---|
| **0.49** | 2.12555 |
| **0.5** | 2.1641 |
| **0.51** | 2.20345 |
| **0.52** | 2.2437 |
| **0.53** | 2.2849 |
| **0.54** | 2.32705 |
| **0.55** | 2.3702 |
| **0.56** | 2.41445 |
| **0.57** | 2.45985 |
| **0.58** | 2.5065 |
| **0.59** | 2.55445 |
| **0.6** | 2.6038 |
| **0.61** | 2.6547 |
| **0.62** | 2.7072 |
| **0.63** | 2.7614 |
| **0.64** | 2.81745 |
| **0.65** | 2.87545 |
| **0.66** | 2.9356 |
| **0.67** | 2.99805 |
| **0.68** | 3.06295 |
| **0.69** | 3.1305 |
| **0.7** | 3.20095 |
| **0.71** | 3.2746 |
| **0.72** | 3.35165 |
| **0.73** | 3.43245 |
| **0.74** | 3.5174 |

| | |
|---|---|
| **0.75** | 3.6068 |
| **0.76** | 3.7012 |
| **0.77** | 3.80115 |
| **0.78** | 3.9072 |
| **0.79** | 4.0201 |
| **0.8** | 4.1407 |
| **0.81** | 4.27005 |
| **0.82** | 4.40925 |
| **0.83** | 4.55975 |
| **0.84** | 4.72335 |
| **0.85** | 4.90215 |
| **0.86** | 5.09885 |
| **0.87** | 5.31675 |
| **0.88** | 5.56015 |
| **0.89** | 5.8348 |
| **0.9** | 6.1483 |
| **0.91** | 6.5112 |
| **0.92** | 6.9385 |
| **0.93** | 7.45265 |
| **0.94** | 8.0889 |
| **0.95** | 8.9066 |
| **0.96** | 10.01645 |
| **0.97** | 11.65765 |
| **0.98** | 14.5016 |
| **0.99** | 18.17535 |