HEISENBERG'S UNCERTAINTY PRINCIPLE

In 1927 the German physicist Werner Heisenberg provided an interesting addition to the meaning of the wave particle concept. He stated that-

"It is impossible to determine the exact position and velocity of the particle simultaneously."

If Δx is error in determining the position of a particle and ΔP is error in determining its momentum at the same instant. The error Δx and ΔP is related by

$$\Delta x.\Delta P \geq \frac{h}{2\pi}$$

In addition to the uncertainty relation between coordinates and momentum of a moving particle, there is uncertainty relation between energy and time and angle and angular momentum.

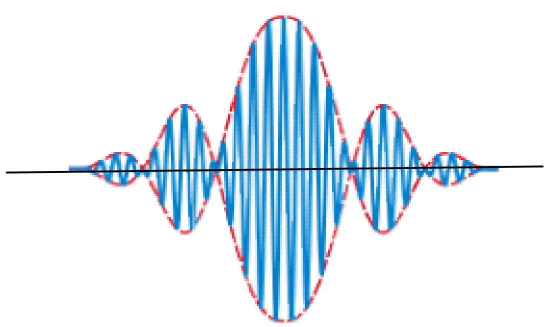
$$\Delta E.\Delta t \geq \frac{h}{2\pi}$$

And

$$\Delta\theta.\Delta\omega \geq \frac{h}{2\pi}$$

Proof of Heisenberg's Uncertainty Principle

Position and momentum uncertainty relation: - let us assume a particle in motion can be taken as a group of wave, the group velocity being equal to the particle velocity. These is shown in figure below-



Let us consider two waves of angular frequency of ω_1 and ω_2 and propagation constant k_1 and k_2 travelling along a single direction.

$\psi_1 = A\sin(\omega_1 t - k_1 x)$ ………………(1)

$\psi_2 = A\sin(\omega_2 t - k_2 x)$ ………………(2)

According to the principle of superposition, the resultant wave is-

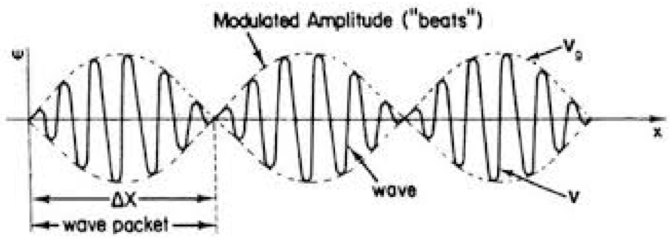
$$\psi = \psi_1 + \psi_2$$

$$= A\sin(\omega_1 t - k_1 x) + A\sin(\omega_2 t - k_2 x)$$

$$= 2A\sin(\omega t - kx).\cos\left(\frac{\partial\omega}{2}t - \frac{\partial k}{2}x\right) \quad \text{………………(3)}$$

where $= \frac{\omega_1+\omega_2}{2}$, $k = \frac{k_1+k_2}{2}$, $\partial\omega = \omega_1 - \omega_2$, $\partial k = k_1 - k_2$

the resultant is poltted in figure where the loop formed will travel with group velocity v_g.



Now this group velocity is equal to particle velocity and hence the loop formed is equivalent to the position of the particle. Hence the position of the particle cannot be given with certainty. It is somewhere between the one node and the next node. The error in the measurement of the position of the particle is therefore equal to the distance between two nodes.

Now a node is formed when $\cos\left(\frac{\partial\omega}{2}t - \frac{\partial k}{2}x\right) = 0$and it is possible when

$\cos\left(\frac{\partial\omega}{2}t - \frac{\partial k}{2}x\right) = \pi/2, 3\pi/2, 5\pi/2,$…….

Thus if x_1 & x_2 represent the position of two successive nodes, then at any instant t, we get-

$$\cos\left(\frac{\partial\omega}{2}t - \frac{\partial k}{2}x_1\right) = (2n+1).\frac{\pi}{2} \quad \text{……….} \quad (4)$$

$$\cos\left(\frac{\partial\omega}{2}t - \frac{\partial k}{2}x_2\right) = (2n+3).\frac{\pi}{2} \quad \text{……………(5)}$$

Subtracting equation 4 from 5, we get

$\Delta k/2\ (x_2 - x_1) = \pi$

$x_2 - x_1 = 2\pi/\Delta k$

,$k = 2\pi/\lambda$

$\Delta x = 2\pi/\Delta\ (2\pi/\lambda)$

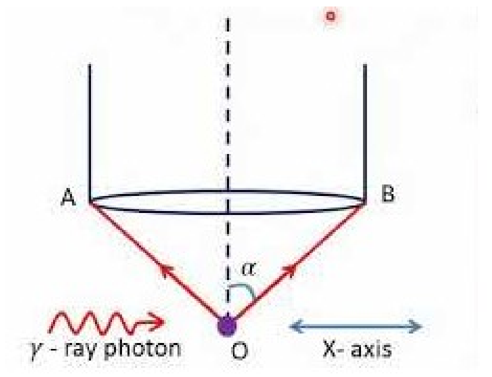
$= h/\Delta p$

$\Delta x \,.\, \Delta p \sim h$

Or $\Delta x \,.\, \Delta p \geq h \; (= h/2\pi)$

Experimental Verification of Heisenberg's Uncertainty Principle

γ – RAY MICROSCOPE



Let us imagine that we are using a γ – ray microscope of angular aperture α. From physical optics it is well known that the exactness of position determination improves with a decrease in wavelength of light. If λ be the wavelength of light employed, the uncertainty in the position is of the order

$$\Delta x \sim \frac{\lambda}{sin\alpha} \quad \rightarrow 1$$

The photon that makes it possible for us to observe the electron also imparts it Compton recoil of the order of magnitude $\frac{h\vartheta}{c}$. the uncertainty in momentum becomes-

$$\Delta p \sim \frac{h\vartheta}{c} \sin \alpha \quad \rightarrow 2$$

$$\Delta p \,.\, \Delta x \sim \frac{\lambda}{sin\alpha} \frac{h\vartheta}{c} \sin \alpha$$

$\Delta p \,.\, \Delta x \sim \frac{\lambda\vartheta}{c} .h$ since $c = \lambda\vartheta$

Thus

$$\Delta p \,.\, \Delta x \sim h \quad \rightarrow 3$$

2. Diffraction of electron beam by a slit: -

Let us consider that an e- has wave character a new beam of e- of momentum P passing through a narrow slit of width d .The precision of the position of e- limited by the size of the slit i.e. d is the measure of uncertainly in the position of electron

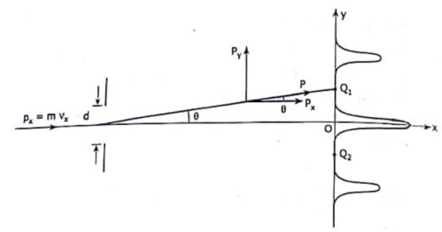


Figure 2.5 : Single slit electron diffraction

$$d \sin \alpha = \lambda \quad \Rightarrow \quad d = \frac{\lambda}{sin\alpha}$$

If the slit is narrow enough , it causes a change in motion of the electron as evidenced from diffraction pattern observed on the screen.

The uncertainly in the e- momentum parallel to y axis depends on the diffraction angle α. When e- deviate from their original path to from a pattern on the screen and hence a component of momentum P$sin\alpha$ in the Y- direction. Now as the e- may be any where within the pattern from angle $-\alpha$ to $+\alpha$ uncertainly in momentum of e- is

$$\Delta P = 2P\ sin\alpha$$

$$= \frac{2h}{\lambda} sin\alpha$$

$$\Delta x . \Delta P = \frac{\lambda}{\sin\alpha} . \frac{2h}{\lambda} . \sin\alpha$$

$$\Delta x . \Delta P = 2h$$

Which is in a agreement with uncertainly principle.

Application of Heisenberg's Uncertainty Principle

1. To prove that electron cannot exist inside the nucleus

Let us consider that electron can exist inside the nucleus .

According to uncertainty principle,

$$\Delta x . \Delta P \geq \frac{h}{2\pi}$$

The radius of the nucleus is of the order of 10^{-14} m . hence the uncertainty in the position of the electron is

$$\Delta x = 2 * 10^{-14}$$

$$\Delta x . \Delta P \geq \frac{h}{2\pi}$$

$$\Delta P \geq \frac{h}{2\pi . \Delta x}$$

$$\Delta P \geq \frac{6.63 * 10^{-34}}{2 * 3.14 * 2 * 10^{-14}}$$

$$\Delta P \geq 5.27 * 10^{-21}\ kg\ m/s$$

If this is p the uncertainty in the momentum of electron ,then the momentum of electron should be at least of this order of $5.27 * 10^{-21}\ kg\ m/s$,

An electron having this much high momentum must have a velocity comparable to the velocity of light. Thus, its energy should be calculated by the following relativistic formula

$$E = \sqrt{(P^2C^2 + m_0^2\,C^4)}$$

$on\ neglecting\ m_0^2\,C^4$

E= PC

E= $\frac{5.27*10^{-21}*3*10^{8}}{1.6*10^{-1}}$ eV

E= 9.88 MeV

Therefore, if the electron exists in the nucleus, it should have an energy of the order of 9.88 MeV. However, it is observed that beta-particles (electrons) ejected from the nucleus during b –decay have energies of approximately 3 Me V, which is quite different from the calculated value of aprrox 10 MeV.

Second reason that electron cannot exist inside the nucleus is that experimental results show that no electron or particle in the atom possess energy greater than 4 MeV.

Therefore, it is confirmed that electrons do not exist inside the nucleus.