

B.Tech.

Year: I Semester: I

Minor Test (Examination): 2024-2025

Subject Name: Engineering Mathematics-I

Time: 2 hrs.

Max. Marks: 30

Note: Attempt all questions.

Q.1	Attempt any three parts of the following.	Marks	CO	BL	PO	PI
(a)	If $u, v, w$ are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in $\lambda$ , then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ .	4	1,2	2,3	1	1.1.1
(b)	If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$ , then prove that $(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 + (\frac{\partial u}{\partial z})^2 = 2(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z})$ .	4	1,2	2,3	1	1.1.1
(c)	Find the inverse of the matrix $A = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}$	6	2,3	1	1.1.1	
(d)	Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ . Find matrix $P$ such that $P^{-1}AP$ is a diagonal matrix.	4	6	2,3	1	1.1.1
Q.2	Attempt any three parts of the following.					
(a)	The Temperature $T$ at any point $(x, y, z)$ in space is $T(x, y, z) = kxyz^2$ , $k$ is positive constant. Find the highest temperature on the surface of sphere $x^2 + y^2 + z^2 = a^2$ .	3	2	2,3	1	1.1.1
(b)	If $u$ be the homogeneous function of degree $n$ in $x$ and $y$ , then show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$	3	1,2	2,3	1	1.1.1
(c)	Expand $f(x, y) = \tan^{-1}(xy)$ in powers of $(x-1)$ and $(y-1)$ up to the second-degree term. Hence compute $f(0.9, 1.1)$ approximately.	3	2	2,3	1	1.1.1
(d)	If $\log y = \tan^{-1} x$ , show that $(1+x^2)y_{n+2} + \{2(n+1)x-1\}y_{n+1} + n(n+1)y_n = 0$ .	3	1,2	2,3	1	1.1.1
Q.3	Attempt any three parts of the following.					
(a)	Find the rank of the matrix $A$ by changing it into normal form. $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$	3	6	2,3	1	1.1.1
(b)	Find the value of $a$ and $b$ for which the system of equations $3x - 2y + z = b, 5x - 8y + 9z = 3, 2x + y + az = -1$ has (i) unique solution (ii) no solution (iii) infinitely many solutions.	3	3,6	2,3	1	1.1.1
(c)	State the Cayley-Hamilton theorem. Verify this theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ . Hence express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as linear polynomial in $A$ .	3	6	1,2	1	1.1.1
(d)	Find the value of $k$ such that the system of equations $x + ky + 3z = 0, 4x + 3y + kz = 0, 2x + y + 2z = 0$ has nontrivial solution.	3	3,6	2,3	1	1.1.1

B.Tech.

Year: I Semester: I

Major Examination-2024-2025

Subject Name: Engineering Mathematics-I

Max. Marks: 50

Time: 3 hrs.

Note: Attempt all questions. All questions carry equal marks.

Q.1	Attempt any five parts of the following.	Marks	C	BL	PO	PI
			O			CODE
a)	If $u = \sin nx + \cos nx$ , then show that $u_r = n^r [1 + (-1)^r \sin 2nx]^{1/2}$ , where $u_r$ is the $r^{\text{th}}$ differential coefficient of $u$ w.r.t. $x$ .	2	6	2	1	1.1.1
b)	Find the $n^{\text{th}}$ derivative of $e^{ax} \sin(bx + c)$ .	2	1	2	1	1.1.1
c)	Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ .	2	2	2	1	1.1.1
d)	If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$ , then prove that $\frac{1}{x} \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial u}{\partial y} + \frac{1}{z} \frac{\partial u}{\partial z} = 0$	2	4	2	1	1.1.1
e)	The eigen values of a matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ are $-2$ and $3$ , then find the value of $a$ and $b$ .	2	5	2	1	1.1.1
f)	Check the consistency of the system of equations $x + 2y - z = 6$ , $3x - y - 2z = 3$ and $4x + 3y + z = 9$ .	2	4	2	1	1.1.1
g)	Find inverse of the following matrix using Gauss Jordan Method. $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$	2	4	2	1	1.1.1
Q.2	Attempt any Two parts of the following.					
a)	Show that $\Gamma\left(\frac{1}{n}\right) \Gamma\left(\frac{2}{n}\right) \Gamma\left(\frac{3}{n}\right) \dots \Gamma\left(\frac{n-1}{n}\right) = \frac{(2\pi)^{\frac{n-1}{2}}}{n^{1/2}}$	5	6	3	1	1.1.1
b)	Show that $\Gamma m \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$ .	5	2	3	1	1.1.1
c)	Evaluate (i) $\int_0^{\infty} \frac{x^c}{c^x} dx$ , (ii) $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ .	5	1	3	1	1.1.1
Q.3	Attempt any Two parts of the following.					
a)	Evaluate $\iint_R (x-y)^4 e^{x+y} dx dy$ , where $R$ is the square with vertices at $(1,0)$ , $(2,1)$ , $(1,2)$ and $(0,1)$ .	5	3	3	1	1.1.1
b)	Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy dy dx$ and hence evaluate it.	5	3	3	1	1.1.1

c)	(i) Using double integration, find the area enclosed by curves $y = 2x^2$ and $y^2 = 4x$ . (ii) Find the volume of the solid region contained in the first octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .	5	6	3	1	1.1.1
Q.4	Attempt any Two parts of the following.					
a)	Evaluate $\iint_S F \cdot \hat{n} dS.$ Here $\vec{F} = 4y\hat{i} + 18z\hat{j} - x\hat{k}$ and $S$ is the surface of the plane $3x + 2y + 6z = 6$ , contained in the first octant.	5	5	3	1	1.1.1
b)	State Stoke's theorem. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ , over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ , bounded by its projection on the $xy$ -plane.	5	5	3	1	1.1.1
c)	Verify Green's theorem in a plane for the integral $\int_C (x - 2y)dx + xdy$ taken around the circle $x^2 + y^2 = 4$ .	5	5	3	1	1.1.1
Q.5	Attempt any Two parts of the following.					
a)	(i) Show that $r^n \vec{r}$ is solenoidal for $n = -3$ and irrotational for all values of $n$ . (ii) Find the directional derivative of $(xyz^2 + xz)$ at $(1, 1, 1)$ in the direction of the normal to the surface $3xy^2 + y + z$ at $(0, 1, 1)$ .	5	4	3	1	1.1.1
b)	Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped bounded by $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$ .	5	4	3	1	1.1.1
c)	(i) Show that $\text{div} \left\{ \text{grad} \left( \frac{x}{r^3} \right) \right\} = 0$ , where $r$ is the magnitude of position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ . (ii) Find the work done in moving a particle by the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} - z\hat{k}$ , from $t = 0$ to $t = 1$ along the curve $x = 2t^2, y = t, z = 4t^3$ .	5	4	3	1	1.1.1