BSM-211 Roll No. 20240

B. Tech. ODD SEMESTER
MINOR TEST 2025-2026
Complex Variables and Numerical Techniques

Time:2 Hrs. **Max. Marks:30**

Note: Answer all questions

1. Attempt any Three parts of the following.

(a) Show that the function $v(r,\theta) = 3r^2 \sin 2\theta - 2\,r \sin\theta$ is harmonic. Find the corresponding conjugate harmonic function $u(r,\theta)$ and construct the analytic function. 4

(b) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u + v = \frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$ then find u, v and the analytic function $f(z)$. 4

(c) Find Taylors Series expansion of $f(z) = \frac{1}{z^2+(1+2i)z+2i}$ about z=0. Also find the radius of convergence 4

(d) State and prove Laurent's theorem. 4

2. Attempt any Three parts of the following.

(a) Show that the function $f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$

Satisfies the Cauchy-Riemann equations but not differentiable at z=0. 3

(b) State and prove Cauchy's integral theorem. 3

(c) Evaluate the integral $\oint_C \frac{(z-2)dz}{(z-1)^3(z+2)^3}$; where C is $|z| = 3$, by using Cauchy integral formula. 3

(d) Evaluate $\int_c (12z^2 - 4iz)dz$ along the curve C joining the points (1,1) and (2,3). 3

3. Attempt any Three parts of the following.

(a) Define singularity of a function. Find the singularity(ies) of the functions 3

(i) $f(z) = \sin\frac{1}{z}$ (ii) $g(z) = \frac{e^{\frac{1}{z}}}{z^2}$

(b) Find out zeros and discuss the nature of the singularities of $f(z) = \frac{(z-2)}{z^2}\sin\left(\frac{1}{z-1}\right)$ 3

(c) Find the order of each pole and residue at it of $\frac{1-2z}{z(z-1)(z-2)}$. 3

(d) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{(5+4\cos\theta)^2}$, by method of residue. 3

BSM-211

Roll No. 2 0 2 4 0

B. Tech.
Year: 2nd Semester: IIIrd
Major Examination: 2025
Complex Variables and Numerical Techniques

Time: 3 Hrs. Max Marks: 50

Note: Attempt ALL questions. ALL questions carry equal marks.

Q1.		Attempt any five parts of the following. (Unit I and II)	Marks	CO	BL	PO	PI Code
	a)	Show that the function $f(z) = \frac{z}{z+1}$ is analytic at $z = \infty$.	2	1.2,3	2	1,8	1.3.1
	b)	Classify the singular point z=0 of the function $\frac{e^z}{z-\sin z}$.	2	1.2,3	4	1,8	1.4.1
	c)	Find the residue at all singular points of $f(z) = \frac{\sin z}{(z^2+1)}$.	2	1.2,3	3	1	1.3.1
	d)	Evaluate the integral $\oint_C z.e^{1/z}dz, C: \|z\| = 1$.	2	1.2,3	1	1	1.3.1
	e)	Classify the singular points of the following functions in the finite complex plane $\frac{\sin z}{e^z-1}$.	2	4,5,6	3	1	1.3.1
	f)	Find the residue at $z = \infty$ for the function $f(z) = \frac{1}{z^3-z^5}$.	2	4,5,6	4	1	1.2.1
	g)	Evaluate the integral $\oint_C \frac{dz}{(z^2+4)^2}$; where C is $\|z-i\| = 2$, by using Cauchy integral formula.	2	4,5,6	2	1	1.3.1
Q2.		Attempt any two parts of the following. (Unit III only)					
	a)	Find an interval of unit length which contains the smallest positive root of equation $x^3 - 5x - 1 = 0$. Hence, determine the number of iteration required by the bisection method so that $\|error\| < 10^{-5}$. Also find the approximate root with $\|error\| < 10^{-5}$.	5	1.2,3	3	1	1.3.1
	b)	Find an interval of unit length which contains the smallest negative root in magnitude of the equation $2x^3 + 3x^2 + 2x + 5 = 0$. Using the end points of this interval as initial approximation, perform five iteration of the Regula Falsi method.	5	1.2,3	4	1	1.3.1
	c)	Find first positive root of equation $10\int_0^x e^{-x^2}dt - 1 = 0$ by Newton-Raphson`s method correct upto six decimal places.	5	1.2,3	3	1	1.4.1
Q3.		Attempt any two parts of the following. (Unit III only)					
	a)	From the table below, find the number of student who secured marks between 40-45 by suitable interpolation formula.	5	1.2,3	4	1	1.3.1

Marks	30-40	40-50	50-60	60-70	70-80
No of Students	31	42	51	35	31

		Question	Marks	CO	BL	PO	PI Code
	b)	Using the following table, find the f(x) as a polynomial in power of (x-6) using Newton Divided difference formula, Also find $f'(6)$, x: -1, 0, 2, 3, 7, 10 f(x): -11, 1, 1, 1, 141, 561	5	1.2,3	4	1	1.4.1
	c)	Find y(1.5), Using Lagrange's interpolation formula from the following table x: 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 y: 0.2420, 0.1942, 0.1497, 0.1109, 0.0790, 0.0540	5	1.2,3	4	1	1.4.1
Q4.		**Attempt any two parts of the following. (Unit IV only)**					
	a)	Solve, by Crouts method, the following system of linear equation: $2x + 3y + z = -1;\ 5x + y + z = 9; 3x + 2y + 4z = 11.$	5	4,5,6	4	1	1.3.1
	b)	Solve, by Gauss Seidel method, the following system of linear equation: $28x + 4y - z = 32;\ x + 3y + 10z = 24; 2x + 17y + 4z = 35.$	5	4,5,6	2	1	1.3.1
	c)	The velocity v of a particle at distance S from a point on its path is given by the table below. S in meter: 0, 10, 20, 30, 40, 50, 60 V m/sec: 47, 58, 64, 65, 61, 52, 38 Estimate the time taken to travel 60 meters by using Simpson's one third rule.	5	4,5,6	2	1	1.3.1
Q5.		**Attempt any two parts of the following. (Unit IV only)**					
	a)	Using fourth order Runge-Kutta method find $y\ (0.2)$, given that $\frac{dy}{dx} = x + y, y\ (0) = 1$ and $h = 0.1$.	5	4,5,6	3	1	1.3.1
	b)	Using Modified Euler's method find $y(0.2)$, given that $dy/dx = log\ (x + y), y(0) = 1.0$ and $h = 0.1$	5	4,5,6	2	1	1.2.1
	c)	Using Taylors series method solve $dy/dx = y + x^3$ for $x = 1.1, 1.2, \&1.3$ given that $y\ (1) = 1$.	5	4,5,6	3	1,8	1.3.1

Tables within questions:

Q3 b)

x	-1	0	2	3	7	10
f(x)	-11	1	1	1	141	561

Q3 c)

x	1.0	1.2	1.4	1.6	1.8	2.0
y	0.2420	0.1942	0.1497	0.1109	0.0790	0.0540

Q4 c)

S in meter	0	10	20	30	40	50	60
V m/sec	47	58	64	65	61	52	38

BL – Bloom's Taxonomy Levels (1- Remembering, 2- Understanding, 3 – Applying, 4 – Analysing, 5 – Evaluating, 6 - Creating)

CO – Course Outcomes

PO – Program Outcomes

PI Code – Performance Indicator Code