

Institute of Engineering and Technology, Lucknow

Class-Test-I

Branches: CE, CS-SF, CS-R, CHE

Time: 1H

All questions are compulsory and each question carries 4 marks.

1. Solve $2y(z-3)p + (2x-z)q = y(2x-3)$.
2. Write the Charpit's auxiliary equations of the PDE $p^2x^2 + q^2y^2 = z^2$, and hence solve it.
3. Solve the PDE $r + s - 6t = y \cos(x)$.
4. Solve the PDE $r + t + 2s - 2p - 2q = \sin(x + 2y)$, where r, s, t, p and q have their usual meaning.
5. If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\frac{\partial u}{\partial y} = kx(l-x)$ at $y = 0$, find the displacement $u(x, y)$.

B.Tech.

(SEM III) ODD SEMESTER EXAMINATION 2024-25
MATHEMATICS-IV (PDE, PROBABILITY & STATISTICS)

[TIME: 3 hrs.]

[Max. Marks: 70]

Note: Attempt All Questions. All Question carry equal marks.

- Q1.** Answer ALL parts.
- (a) Form the partial differential equation by eliminating the arbitrary constant from the following: $2z = (ax + y)^2 + b$. Marks 3.5
- (b) Solve the partial differential equation $x(y - z)p + y(z - x)q = z(x - y)$, where symbols have their usual meanings. 3.5
- (c) Solve the partial differential equation by Charpit's method : $px + qy = pq$. 3.5

OR

- Solve: $r - 2s + t = 2(y - x) + \sin(x - y)$ where symbols have their usual meanings.
- (d) Solve: $(D^2 - DD' + D' - 1)z = e^{(y)} + \cos(x + 2y)$, where $D = \frac{\partial}{\partial x}$ & $D' = \frac{\partial}{\partial y}$. 3.5

OR

Solve : $x^2r - y^2t = xy$, where symbols have their usual meanings.**Q2.** Answer ALL parts.

- (a) Use the method of separation of variables to solve the equation 7
- $$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0.$$
- (b) A rectangular plate with insulated surface is 8 cm wide and so long as compared to its width that it may considered infinite in length. If the temperature along one short edge $y = 0$ is given by $u(x, 0) = 100 \sin \frac{\pi x}{8}$, $0 < x < 8$ while the two long edges $x = 0$ and $x = 8$ as well as the other short edges are kept at 0°C . Find the steady state temperature. 7

OR

Find the Fourier transform of the function $F(x) = \begin{cases} 1 - x^2; & |x| < 1 \\ 0; & |x| > 1 \end{cases}$ and hence evaluate the integral $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$.

Q3. Answer ALL parts.

- (a) The pressure and volume of a gas are related by the equation $pv^\gamma = k$, γ and k being constants. Fit this equation to the following set of observations: 7

$p(\text{kg/cm}^2)$	0.5	1.0	1.5	2.0	2.5	8.0
v (litres)	1.62	1.00	0.75	0.62	0.52	0.46

OR

Obtain the line of regression of y on x for the data given below.

x	1.53	1.78	2.60	2.95	3.42
y	33.50	36.30	40.00	45.80	53.50

(b) The first moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also calculate β_1 and β_2 and comment upon the skewness and kurtosis of the distribution. Symbols have their usual meanings. 7

Q4. Answer ALL parts.

(a) A bag contains 5 distinct white and 10 distinct black balls. Random sample of three balls are taken out without replacement. Find the probability that the sample contains (i) Exactly one white ball (ii) No white ball. 7

(b) In a normal distribution 31% of items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. 7

OR

What is the probability that at most 5 defective diodes will be found in a pack of 600 diodes if previous data shows that 3% of such diodes are defective?

Q5. Answer ALL parts.

(a) In a hospital 475 female and 525 male babies were born in a week. Do these figures confirm the hypothesis that males and females are born in equal number? 7

(b) Samples of sizes 10 and 14 were taken from two normal populations with S.D. 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. test whether the means of the two populations are the same at 5% level. 7

OR

The demand for a particular spare part in a factory was found to vary from day-to-day. In a sample study, the following information was obtained.

Days: Mon. Tue. Wed. Thurs. Fri. Sat.
 No. of parts demanded: 1124 1125 1110 1120 1126 1115
 Use χ^2 - test to test the hypothesis that the number of parts demanded does not depend on the day of the week. Given $\chi^2_{0.05} = 11.07$ for 5 degrees of freedom.

CO-BL Mapping

Odd Semester Examination : 2024-25			
Subject Code: IAS-302			
Subject Name: (PDE, PROBABILITY & STATISTICS)			
Q.No.	Marks	CO	BL
1(a)	3.5	1	2
1(b)	3.5	1	4
1(c)	3.5	1	3
1(d)	3.5	1	3
2(a)	7	2	2
2(b)	7	2	4
3(a)	7	3	3
3(b)	7	3	2
4(a)	7	4	5
4(b)	7	4	3
5(a)	7	5	2
5(b)	7	5	3