

Institute Of Engineering and Technology, Lucknow
Department of Applied Science and Humanities

Subject code-IAS-102

(Session 2024-25)

Subject Name – Mathematics IV

CT-I (06/03/2025)

Max. Marks: 20 (5×4)

B. TECH. (4th Semester) Branches: CS (AI), EC, EE & ME

Time: 1 hour

Note: Attempt All Questions. Each question carries equal marks.

Q1. Find the partial differential equation of $(x - a)^2 + (y - b)^2 = z^2 \cot^2 \alpha$ by elimination of arbitrary constant.

Q2. Solve $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$. Symbols p and q have their usual meanings.

Q3. Find the complete integral of given equation $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$.
OR

$$\text{Solve } x^2 D^2 - xy DD' - 2y^2 D'^2 + xD - 2yD' = x(e^y - 1).$$

Q4. Solve the equation by the method of separation of variables

$$\frac{\partial^2 u}{\partial x \partial t} = e^{-x} \cos x \text{ given that } u = 0 \text{ when } t = 0 \text{ and } \frac{\partial u}{\partial t} = 0 \text{ when } x = 0$$

Q5. Solve the wave equation under the conditions $u(0, t) = u(l, t) = u(x, 0) = 0, u_t(x, 0) = x(l - x)$.

B.Tech.
(SEM IV) EVEN SEMESTER EXAMINATION 2024-25
MATHS IV(PDF , Probability and Statistics)

[TIME: 3 hrs.]

Note: Attempt All Questions. All Question carry equal marks.

[Max. Marks: 70]

- Q 1 Answer ALL parts
- (a) Find the partial differential equation by the elimination of a and b from $z = (x^3 + a)(y^3 - b)$. Marks
3.5
- (b) Solve $(x - y)p + (y - z)q = (z - x)$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ by Lagrange's method. 3.5
- (c) Solve $p \cot x + q \cot y = \cot z$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 3.5

OR

- Solve $\frac{\partial^2 z}{\partial x^2} - 8 \frac{\partial^2 z}{\partial x \partial y} + 16 \frac{\partial^2 z}{\partial y^2} = 0$.
- (d) Find the Particular integral of PDE $4 \frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{2x+5y}$ 3.5

OR

Find the solution of $x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y} + 4y^2 \frac{\partial^2 z}{\partial y^2} = 0$

- Q 2 Answer ALL parts
- (a) Solve the following equation by the method of separation of variables: 7
- $$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$
- (b) Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, under the conditions $u = 0$ when $x = 0$ and $x = \pi$. 7

OR

Find by the method of separation of variables, a particular solution of the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ that tends to zero as x tends to infinity and is equal to $2 \cos y$ when $x = 0$.

- Q 3 Answer ALL parts
- (a) Fit a parabola $y = a + bx + cx^2$, least square method to the following data: 7

X	1	2	3	4	5
Y	1	3	8	10	17

(b) Find the regression line of y on x and x on y from the following data 7

X	1	2	3	4	4
Y	1	3	7	10	16

OR

The first four moments of a distribution about the value '5' of the distribution are - 2.5, 17, -35 and 58 respectively. Find the first four moments about mean. Also discuss Skewness and Kurtosis of the distribution.

- Q 4 Answer ALL parts
- (a) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals : 7
 (i) exactly 2 (ii) more than 2 individuals (iii) none (iv) more than one individual will suffer a bad reaction.
- (b) (i) If the probability of a defective bolts is 0.1, find (i) the mean (ii) the 7
 standard deviation for the distribution bolts in a total of 400.
 (ii) Find the recurrence relation Binomial distribution.

OR

If the variance of the Poisson distribution is 4, find the probabilities for $r = 1, 2, 3, 4$ from the recurrence relation of the Poisson distribution. Also find $P(X \geq 4)$.

- Q 5 Answer ALL parts
- (a) Below are given the gain in weight (in lbs) of lions on two diets X and Y: 7
 Gain in weight:
 Diet X : 22 28 25 28 24 17 34 32 40
 Diet Y : 21 31 25 33 40 33 38 32 35
 32

Test at 5% level of significance whether the two diets differ significantly in increasing weight. (Given that the value of 't' at 5% level of significance and 15 degrees of freedom is 2.13).

- (b) The following table gives for a sample of married women, the level of education and the marriage adjustment score: 7

Level of Education	Marriage adjustment				Total
	Very low	low	high	Very high	
College	24	97	62	58	241
High School	22	28	30	41	121
Middle School	32	10	11	20	73
Total	78	135	103	119	435

Can you conclude from the above data that the higher the level of education, the greater is the degree of adjustment in marriage (Given that at 5% level of significance $\chi^2_{0.05} = 12.59$ at 6 degree of freedom?)

OR

A machine is set to deliver packets of a given weight. Ten samples of size 5 each were recorded. Data being given below:

Sample No.	1	2	3	4	5	6	7	8	9	10
\bar{X}	22	28	26	29	26	27	28	31	31	25
R	13	14	10	13	15	13	16	17	15	11

Calculate the value for the central line and control limits of mean chart and the range. Construct \bar{X} and R charts and examine whether the process is under control and state your recommendations. Assume for $n = 5$, $A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.115$.