Department of Computer Science and Engineering

B. Tech. III[rd] Semester (CSE & CSE_SF) Session – 2024 – 25

Discrete Structures and Theory of Logic (ICS 303)

Time: One hour

Class Test II

M.M.:20

Q1 Define a binary operation * on the set R of real numbers by $x * y = xy + x + y$, where the symbols on the right-hand side of the equation have their usual meanings. 5

a) Show that with respect to *, R satisfies three of the axioms for a group, but that not every element of R has an inverse

b) Now remove one element z from R so that R-$\{z\}$ forms a group under operation * . Is this group abelian?

Q2 Let C_m be the cyclic group of order m. List the elements of $C_2 \times C_3$ and $C_2 \times C_4$. Show that $C_2 \times C_3$ is isomorphic to C_6, but $C_2 \times C_4$ is not isomorphic to C_8. 5

Q3 Define the subgroup and normal subgroup. Let H be a subgroup of a finite group G. Justify the statement "the order of H is a divisor of the order of G". 5

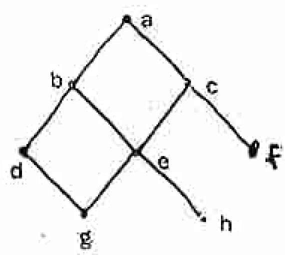
Q4 (a) Draw the Hasse diagram for a poset that has exactly five members, two of which are maximal and one of which is the poset's minimum. Label the maximal and minimum members on your diagram. 5

(b) Consider the poset P with Hasse diagram as shown below:

(i) List the maximal members and minimal members of P.

(ii) Is there a largest member of P? If so, what is it? Is there a smallest member of P? If so, what is it?

(iii) Determine: LUB{b, c}, GLB{b, c}, LUB{b, f}, GLB{b, f}, LUB{g, c}, and GLB{g, c}.



Department of Computer Science and Engineering

B. Tech. IIIrd Semester (CSE) Session – 2022 – 23

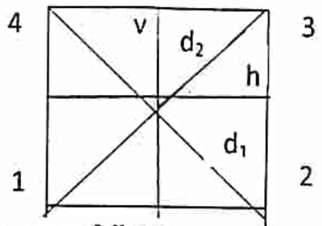
Discrete Structures and Theory of Logic (KCS 303)

Time: One hour — Class Test II — M.M.:30

Q1 Prove the following results for a group *G*. **[6]**
(i) The identity element is unique.(ii) Each a in G has unique inverse a^{-1}(iii) $(ab)^{-1} = b^{-1}a^{-1}$

Q2 Show that a group (G, *) is abelian if and only if for a, b in G, $(a * b)^2 = a^2 * b^2$ **[4]**

Q3 Consider the square shown below **[4]**



The symmetries of the square are as follows:
Rotations f_1, f_2, f_3, and f_4 through 0°, 90°, 180°, and 270°, respectively
f_5 and f_6, reflections about the lines v and h respectively
f_7 and f_8, reflections about the diagonals d_1 and d_2 respectively
Write the multiplication table of D_4, the group of symmetries of the square.

Q4 Let R be the set of all real numbers and define a relation R on R × R as follows: For all (a, b) and (c, d) in R × R **[4]**
(a, b) R (c,d) ⇔ either $a < c$ or both $a = c$ and $b \le d$.
Is R a partial order relation? Prove or give a counterexample

Q5 Consider the "divides" relation defined on the set **[4]**
$A=\{1, 2, 2^2, 2^3, \ldots, 2^n\}$, where n is a nonnegative integer.
a. Prove that this relation is a total order relation on *A*.
b. Draw the Hasse diagram for this relation for $n = 4$

Q6 Use the laws of Boolean algebra: commutative, associative, distributive, De Morgan's, etc., and the shorthand p → q for ¬p ∨ q, to prove the following equivalences. **[4]**
(a) $p \to (q \to r) \equiv (p \wedge q) \to r$
(b) $(p \wedge q) \to r \equiv (p \to r) \vee (q \to r)$

Q7 Define the distributive lattice. Show that in bounded distributive lattice if a complement exists it is unique. **[4]**

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B.TECH.
(SEM III) ODD SEMESTER EXAMINATION 2024-25
DISCRETE STRUCTURES AND THEORY OF LOGIC

[TIME: 3 hrs.] [Max. Marks: 70]

Note: *Attempt All Questions. All Question carry equal marks. All notations are their usual meanings. Make suitable assumptions wherever necessary.*

Q1. Answer ALL parts. — Marks

(a) The following statements about sets are false. Give a counterexample to each statement. — 3.5

(i) $A \cap B = A \cap C$ *implies* B = C.
(ii) $A \cup B = A \cup C$ *implies* B = C.
(iii) $A \subseteq B \cup C$ *implies* $A \subseteq B$ *or* $A \subseteq C$

(b) Let $S = \{1, 2, 3, 4, 5, 6, 7\}$ and define m R n if $m^2 \equiv n^2 \pmod 5$. Show that R be an equivalence relation on set S. — 3.5

(c) Given the relations, R, that are defined on the sets S, determine if R is reflexive, symmetric, transitive, and/or antisymmetric. Explain your reasoning. — 3.5

(i) Let S denote the set of all nonempty subsets of $\{a; b; c; d; e\}$ and define $A\ R\ B$ to mean that $A \cap B = \emptyset$, for $A, B \subseteq S$.
(ii) Let S be the set of ordered pairs of real numbers with $(x_1; x_2)\ R\ (y_1; y_2)$ if and only if $x_1 = y_1$ and $x_2 \leq y_2$.

OR

Let X be the set of prime numbers less than 100, and define x R y to means that x and y have the same final digit; x,y∈X. What are the equivalence classes of X with respect to R

(d) Which of the following functions from N to N are surjections, which of them are injections, and which of them are bijections? — 3.5

$$f(x) = x^3,\ g(x) = x + 3,\ h(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x-1 & \text{if } x \text{ is even} \end{cases}$$

OR

Prove by mathematical induction that for all $n \in N$, $n^3 + 5n$ is a multiple of 6.

Q2. Answer ALL parts.

(a) Each of the following sentences expresses an implication. Rewrite each in the form "If p, then q." Also, write the contrapositive of each. — 7

(i) Touch those cookies if you want a spanking.
(ii) Touch those cookies and you will be sorry.
(iii) You leave or I will set the dog on you.
(iv) I will if you will.
(v) I will go unless you stop that.

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OR

Answer the followings(i to iii):

(i) Determine the truth value of the statement if the universe of each variable consists of all real numbers

$$\forall x \exists y (x + y = 2 \wedge 2x - y = 2)$$

(ii) Rewrite the following statement so that negations appear only within predicates (i.e.no negation is outside quantifier or expression involving logical connectives)

$$\neg \forall x ((\exists y \forall z\, P(x, y, z)) \leftrightarrow (\exists z \exists y\, R(x, y, z))).$$

(iii) Are the following statements logically equivalent?

$$\exists x P(x) \wedge Q(x) \text{ and } (\exists x P(x)) \wedge (\exists y Q(y))$$

(b) Convert each of the following arguments into logical notation using the suggested variables (c, b, r, p). Then provide a formal proof. — 7

"If my computations are correct and I pay the electric bill, then I will run out of money. If I don't pay the electric bill, the power will be turned off. Therefore. I don't run out of money and the power is still on, then my computations are incorrect."

Q3. Answer ALL parts.

(a) (i) Define a group. Suppose the table given below is a group table. Fill in the blank entries. — 7

	e	a	b	c	d
e	e				
a		b			e
b		c	d	e	
c		d		a	b
d					

(ii) If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i for all a, b ∈ G, show that G is abelian.

(b) Define a subgroup. Show that if H be a subgroup of finite group G, the order of H divides the order of G. Let A, B are subgroups of a group G. Prove or disprove that A∩B is also a subgroup of G. — 7

OR

Define the mathematical structure ring. Show that if $S = \{a + b\sqrt{2}, a, b \in Z\}$, together with ordinary addition and multiplication, is a commutative ring with identity.

Q4. Answer ALL parts.

(a) Let $A = \{1, 2, 3, 5, 6, 7, 10, 20, 30, 60, 70\}$. Consider the partial order divisibility on A, i.e. if a and b belongs to A, $a \leq b$ if and only if a|b. — 7

(i) Draw the Hasse diagram of poset (A, |).
(ii) Find the maximal and minimal element.
(iii) Find the greatest element and least element or explain why there is no greatest element or least element.
(iv) Find all upper bounds and least upper bound of {2, 5}.
(v) Find all lower bounds and greatest lower bound of {6, 10}

(b) (i) Let L be a lattice. Prove that for every a, b, and c in L, if a ≤ b and c ≤ d, then 7

a ∨ c ≤ b ∨ d and a ∧ c ≤ b ∧ d.

(ii) Show that if (L_1, ≤) and (L_2, ≤) are lattices, then then (L, ≤) is a lattice, where L = L_1 x L_2, and the partial order ≤ on L is the product partial order.

OR

Draw the Karnaugh map of the following Boolean expressions in x, y, z, and w, and minimize the expression.

(i) The Boolean function E has minterms

x'y'z'w + x'y'zw + xy'zw' + xyz'w + x'yz'w' + xy'z'w'

(ii) The Boolean function E has value 1 if and only if at least two of x, y, z, and w have the value 1.

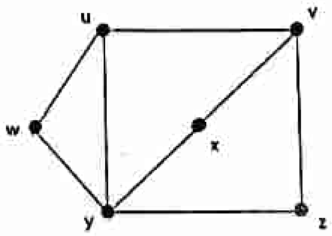
Q5. Answer ALL parts.

(a) Discuss how to solve a second order homogeneous linear recurrence relation. Solve the recurrence 7

$$a_n = 3a_{n-1} + 10a_{n-2} + 7(5)^n, n \geq 2$$

given $a_0 = 4, a_1 = 3$

(b) 7



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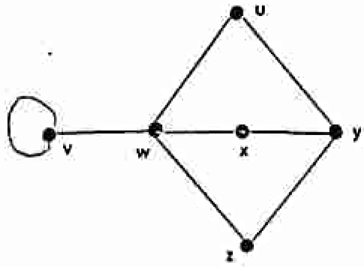

H

For the graphs G and H as shown in above answer the following:

(i) Does the graph G have an Euler circuit?

(ii) Is every simple closed path in G a cycle? Explain.

(iii) Does G have a Hamiltonian path? Explain.

(iv) Give the vertex sequence of an Euler path for the graph H.

(v) Does H have a Hamiltonian path? Explain.

OR

Write short notes on:

(i) Planar graphs

(ii) Graph coloring

(iii) Pigeonhole principle

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Subject Code: KCS-303 Roll No.:

B. Tech.

(SEM III) ODD SEMESTER EXAMINATION 2022-23

DISCRETE STRUCTURES AND THEORY OF LOGIC

[TIME: 3 hrs.] [Max. Marks: 100]

Note: *Attempt All Questions. All Question carry equal marks. All notations are their usual meanings. Make suitable assumptions wherever necessary.*

Q1. Answer ALL parts. Marks

(a) Let $S = \{a, b, c\}$ and for each integer $i = 0, 1, 2, 3$, let S_i be the set of all subsets of S that have i elements. List the elements in S_0, S_1, S_2, and S_3. Is $\{S_0, S_1, S_2, S_3\}$ a partition of $P(S)$? 5

OR

To prove that a composition of onto functions is onto, a student wrote,

"Suppose $f : X \to Y$ and $g : Y \to Z$ are both onto. Then
$\forall y \in Y, \exists x \in X$ such that $f(x) = y$
and
$\forall z \in Z, \exists y \in Y$ such that $f(y) = z$.
So
$(g \circ f)(x) = g(f(x)) = g(y) = z$,
and thus $g \circ f$ is onto."

Explain the mistakes in this "proof."

(b) Let S be the set of all strings of 0's and 1's, and define $l: S \to Z^+ \cup \{0\}$ by 5

$l(s)$ = the length of s, for all strings s in S.

(i) Is l one-to-one? Prove or give a counterexample.

(ii) Is l onto? Prove or give a counterexample.

OR

Let $A = R \times R$. A relation F is defined on A as follows:

For all (x_1, y_1) and (x_2, y_2) in A,

$(x_1, y_1)\ F\ (x_2, y_2) \Leftrightarrow x_1 = x_2$.

Determine whether the given relation is reflexive, symmetric, transitive, or none of these. Justify your answers.

(c) A is the set of all strings of length 4 in a's and b's. R is defined on A as follows: For all strings s and t in A, 5

$s\ R\ t \Leftrightarrow s$ has the same first two characters as t.

Show that R is an equivalence relation on the set A. Find the distinct equivalence classes of R.

(d) Use mathematical induction to prove that for all integers $n \geq 3$, 5

$2n + 1 < 2^n$.

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Q2. Answer ALL parts.

(a) (i) Let G be a group. Show that each element a in G has only one inverse in G. 10

(ii) Let G be a group and let a and b be elements of G then show that $(a\,b)^{-1} = b^{-1}\,a^{-1}$

(b) Consider the groups $(G_1, *)$ and $(G_2, \oplus)$ with identity elements e_1 and e_2 respectively. If $f : G_1 \to G_2$ is a group homomorphism, then prove that 10

(i) $f(e_1) = e_2$

(ii) $f(a^{-1}) = [f(a)]^{-1}$

(iii) If H1 is a sub group of G_1 and $H2 = f(H_1)$, then H_2 is a sub group of G_2.

(iv) If f is an isomorphism from G_1 onto G_2, then f^{-1} is an isomorphism from G_2 onto G_1

OR

(i) Let G be a finite group with identity e, and let a be an arbitrary element of G. Prove that there exists a nonnegative integer n such that $a^n = e$.

(ii) Define the algebraic structures Rings and Fields. Show that set of integers along with addition and multiplication operations forms a ring and set of real numbers with these operations form a field.

Q3. Answer ALL parts.

(a) Consider the "divides" relation on each of the following sets A. Draw the Hasse diagram for the following:. 10

(i) $A = \{1, 2, 4, 5, 10, 15, 20\}$

(ii) $A = \{2, 3, 4, 6, 8, 9, 12, 18\}$

(b) Define a lattice. Give an example of a poset with five elements that is a lattice and an example of a poset with five elements that is not a lattice. Show that every lattice with finite number of elements has a greatest element and a least element. 10

OR

Explain how K-maps can be used to simplify the sum-of-products expression in four Boolean variables. Use K-map to simplify the sum-of-products expression

wxyz + wxyz' + wxy'z + wxy'z' + wx'yz + wx'y'z + w'xyz + w'x'yz + w'x'yz'

Q4. Answer ALL parts.

(a) (i) Write each of the two statements in symbolic form and determine whether they are logically equivalent. 10

If you paid full price, you did not buy it at Crown Books.

You did not buy it at Crown Books or you paid full price.

(ii) Use logical equivalence to rewrite the given statement forms without using the symbol → or ↔, and using only ∧, ∨ and ~.

$p \vee \sim q \rightarrow r \vee q$ $\quad\quad$ $(p \rightarrow r) \leftrightarrow (q \rightarrow r)$

(b) You are about to leave for school in the morning and discover that you do not have your glasses. You know the following statements are true: 10

(i) If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table

(ii) If my glasses are on the kitchen table, then I saw them at breakfast.

(iii) I did not see my glasses at breakfast.

(iv) I was reading the newspaper in the living room, or I was reading the newspaper in the kitchen.

(v) If I was reading the newspaper in the living room then my glasses are on the coffee table.

Where are the glasses?

OR

(i) Let P(x), Q(x) be predicates. Then the proposition $(\exists x)[P(x)] \rightarrow (\exists y)[Q(y)]$ is logically equivalent to $(\exists x)(\exists y)[P(x) \rightarrow Q(y)]$.

(ii) Let P(x), Q(x) be predicates involving an integer-valued variable x.

Prove or disprove: $\forall x\ [P(x) \Rightarrow Q(x)]$ is logically equivalent to $\forall x\ [P(x)] \Rightarrow \forall x\ [Q(x)]$.

Q5. Answer ALL parts.

(a) Find an explicit formula for Fibonacci sequence. 10

(b) Determine if the relation R = {(t, u), (u, w), (u, x), (u, v), (v, z), (v, y)} is a tree on the set A = {t, u, v, w, x, y, z}. If it is tree, what is the root and height? If it is not a tree, make the least number of changes necessary to make it a tree and give the root and height. Also, list the vertices of the tree when they are visited in a preorder traversal and in a post order traversal. 10

OR

Write short note on the following:

(i) Euler graphs

(ii) Homomorphism and isomorphism of graphs

(iii) Pigeonhole principle