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EMAT 101

B. Tech. (Ist SEMESTER) EXAMINATION, 2025-26

Bachelor of Technology

(IT, ME, CSE & ECE)

Engineering Mathematics - I

Time : Three Hours] [Maximum Marks : 75

Note: There are **three** sections (**A**, **B** and **C**) and candidate has to attempt questions from all sections. Marks are indicated against each section.

Section-A

1. Attempt all parts of the following : 5×3=15

(a) Find $\int_0^\infty x^6 e^{-2x} dx$.

(b) Find the value of $\frac{dy}{dx}$ if $e^y = xy$.

(c) Expand $\sin x$ in powers of $\left(x - {}^{\pi}/_{2}\right)$.

(d) Find $\nabla.\left(\frac{1}{r}\vec{r}\right)$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

(e) Construct 3×2 matrix whose elements are given by $aij = \frac{1}{2}|i - 3j|$.

Section-B

Note: Answer **all** questions : 4×5=20

2. (a) If $u = \log\left(\frac{x^2+y^2}{xy}\right)$, find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ and verify that $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$.

Or

(b) Show that $V = xyz + x^4 + y^4 + z^4$ has neither maximum nor minimum at $x = y = z = 0$.

3. (a) Show that $\int_0^\infty \frac{x^c}{c^x} dx = \frac{\lceil(c+1)}{[\log c]^{c+1}}$.

Or

(b) Evaluate $\iint xy(x+y)dxdy$ over the area between $y^2 = x$ and $y = x$.

4. (a) If $\vec{F} = xy^2\,\vec{i} + 2x^2yz\,\vec{j} - 3yz^2\,\vec{k}$, find $\nabla.\vec{F}$ at point $(1, -1, 1)$.

Or

(b) If $\vec{a}$ is a constant vector and $\vec{r}$ is the position vector of any point, prove that –

(i) $\nabla.(\vec{a} \times \vec{r}) = 0$

(ii) $\nabla \times (\vec{a} \times \vec{r}) = 2\vec{a}$

5. (a) Show that –

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

Or

(b) If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, verify that –

(i) $(A')' = A$

(ii) $(A+B)' = A' + B'$

Section-C

Note: Attempt any two questions : 2×20=40

6. Define inverse of matrix and also write its chief properties.

For that matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11 = 0$. Hence, find A^{-1}.

7. State and prove Leibnitz Theorem.

If $y = x^2 e^x$, prove that –

$\frac{d^n y}{dx^n} = \frac{n(n-1)}{2}\frac{d^2 y}{dx^2} - n(n-2)\frac{dy}{dx} + \frac{1}{2}(n-1)(n-2)y.$

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8. State Dirichlet's Theorem.

Find the value of $\iiint x^{l-1}y^{m-1}z^{n-1}dxdydz$, where x, y, z are always positive but $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$.

9. Verify Green's Theorem in the plane for $\int_c [(3x^2 - 8y^2)\,dx + (4y - 6xy)dy]$ where c is the boundary of the region by $x = 0, y = 0, x + y = 1$.