Institute of Engineering and Technology, Lucknow

Class Test-I | Engg. Mathematics-I(IAS103) | Session: 2023-24

Max Marks:20 | Date: 04.12.2023 | Time: 10.00-11.00 AM

Note: All questions are compulsory. Each question carries equal marks.

Q1. Find inverse of following matrix by using elementary row transformations:

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}.$$

Q2. For what values of a & b, do the equations $x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + az = b$ have (i) no solution (ii) a unique solution (iii) more than one solution.

Q3. Find the eigen values of the following matrix:

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Also find an eigen vector corresponding to any one of the eigen values.

Q4. If $y = \frac{1}{1-5x+6x^2}$, find y_n.

Q5. If $u = \sin^{-1}\left(\frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}\right)$ then evaluate $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y}$.

$\frac{2(-1)^n 2^{n+1} n!}{(\quad)^{n+1}}$

Printed pages: 02

Subject Code: IAS103 | Roll No.

B.TECH.
(SEM I) ODD SEMESTER EXAMINATION 2023-24
ENGINEERING MATHEMATICS-I

[TIME: 3 hrs.] [Max. Marks: 70]

Note: *Attempt All Questions. All Question carry equal marks.*

Marks

Q1. Answer **ALL** parts.

(a) Find the inverse of the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 3 & -3\\ -2 & -4 & -4\end{bmatrix}$ by using elementary row operations. 3.5

(b) Test for consistency the following linear equations in x, y, z, t, and solve them if consistent: $x-2y+3t=2,\ 2x+y+z+t=-4,\ 4x-3y+z+7t=8$. 3.5

(c) Verify that the matrix $A=\begin{bmatrix}\frac{2+i}{3} & \frac{2i}{3}\\ \frac{2i}{3} & \frac{2-i}{3}\end{bmatrix}$ is unitary and find A^{-1}. 3.5

OR

Are the vectors $X_1=(1,1,-1,1),\ X_2=(1,-1,2,-1),\ X_3=(3,1,0,1)$ linearly dependent? If so, find the relation between them.

(d) If $A=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$, use Cayley-Hamilton theorem to express $A^6-4A^5+8A^4-12A^3+14A^2$, as a linear polynomial in A. 3.5

OR

Find the square matrix A whose Eigen values are 1, 2 and 3 and there corresponding Eigen vectors are $\begin{bmatrix}1\\0\\-1\end{bmatrix}$, $\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $\begin{bmatrix}1\\0\\1\end{bmatrix}$ respectively.

Q2. Answer **ALL** parts.

(a) If $y=e^{a\sin^{-1}x}$, where a is a constant, prove that 7

$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$. Deduce that $\lim_{x\to 0}\frac{y_{n+2}}{y_n}=n^2+a^2$.

Hence find y_n at $x=0$. Here, y_n stands for n^{th} derivative of y.

(b) If $f(x,y)=\frac{1}{x^2}+\frac{1}{xy}+\frac{\log_e x-\log_e y}{x^2+y^2}$, prove that (i) $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+2f(x,y)=0$, 7

(ii) $x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial x\partial y}+y^2\frac{\partial^2 f}{\partial y^2}-6f(x,y)=0$.

OR

Transform the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ into polar coordinates.

Q3. Answer **ALL** parts.

(a) Expand $f(x,y)=x^y$ in powers of $(x-1)$ and $(y-1)$ up to the third degree terms and hence evaluate $(1.1)^{1.02}$. 7

(b) Find the dimensions of a rectangular box, which is open on the top, of maximum capacity whose surface area is given, by using Lagrange's method of multipliers. 7

OR

If $u=\dfrac{x^2-y^2}{x^2+y^2}, v=\dfrac{2xy}{x^2+y^2}$, show that u,v are functionally related and find the relationship.

Q4. Answer **ALL** parts.

(a) Let D be the region in the first quadrant bounded by the curves $xy=16, x=y$, $y=0$ and $x=8$. Sketch the region of integration of the following integral $\iint_D x^2\,dx\,dy$ and evaluate it over D. 7

(b) Evaluate the integral $\iiint x^{l-1}y^{m-1}z^{n-1}dxdydz$, where x,y,z are all positive but limited by the condition, $\left(\dfrac{x}{a}\right)^p+\left(\dfrac{y}{b}\right)^q+\left(\dfrac{z}{c}\right)^r\le 1$. Here l, m, n, a, b, c, p, q and r, are all positive constants. 7

OR

Show that $\iiint \dfrac{dxdydz}{(x+y+z+1)^3}=\dfrac{1}{2}\log 2-\dfrac{5}{16}$, integral being taken throughout the volume bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$.

Q5. Answer **ALL** parts.

(a) A fluid motion is given by $\vec{v}=(y\sin z-\sin x)\hat{i}+(x\sin z+2yz)\hat{j}+(xy\cos z+y^2)\hat{k}$. Is the motion irrotational? If so, find the velocity potential. 7

(b) Apply Gauss divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where $\vec{F}=2x^2y\hat{i}-y^2\hat{j}+4xz^2\hat{k}$ and S is the surface bounding the region $y^2+z^2=9$, and $x=2$ in the first octant. 7

OR

Evaluate $\int_C \vec{F}.\vec{dr}$ by Stoke's theorem, where $\vec{F}=y^2\hat{i}+x^2\hat{j}-(x+z)\hat{k}$ and C is the boundary of the triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0).

B.Tech.

(SEM I) ODD SEMESTER EXAMINATION 2022-23

ENGINEERING MATHEMATICS-I

[TIME: 3 hrs.] **[Max. Marks: 70]**

Note: ***Attempt All Questions. All Question carry equal marks.***

Q1. Answer **ALL** parts. Marks

(a) Find the inverse of the matrix $A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$ by using elementary row operations. 3.5

(b) Find the rank of the matrix $A=\begin{bmatrix}6 & 1 & 3 & 8\\ 4 & 2 & 6 & -1\\ 10 & 3 & 9 & 7\\ 16 & 4 & 12 & 15\end{bmatrix}$ by reducing it to canonical form. 3.5

(c) Find the value of k so that the equations $x+y+z=1,\ x+2y+4z=k,\ x+4y+10z=k^2$ have a solution. 3.5

OR

If the vectors (0, 1, a), (1, a, 1) and (a, 1, 0) are linearly dependent, then find the value of a.

(d) Find the characteristic equation of the matrix $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and hence find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$. 3.5

OR

Find the eigen values and corresponding eigen vectors of the matrix

$$A=\begin{bmatrix}2 & -3 & 1\\ 3 & 1 & 3\\ -5 & 2 & -4\end{bmatrix}.$$

Q2. Answer **ALL** parts.

(a) If $y=\sqrt{1-x^2}\sin^{-1}x$, prove that $(1-x^2)y_{n+3}-(2n+3)xy_{n+2}-n(n+2)y_{n+1}=0$ and hence find y_n at $x=0$. 7

(b) If $z=x^4y^2\sin^{-1}\left(\frac{x}{y}\right)+\log_e x-\log_e y$, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=6x^4y^2\sin^{-1}\left(\frac{x}{y}\right)$. 7

OR

If $u=u\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0$.

Q3. Answer **ALL** parts.

(a) Expand $f(x,y)=e^x \tan^{-1} y$ in powers of $(x-1)$ and $(y-1)$ up to terms of degree 2. **7**

(b) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ by using Lagrange's method of multipliers. **7**

OR

If $u=x\sqrt{1-y^2}+y\sqrt{1-x^2}, v=\sin^{-1}x+\sin^{-1}y$, show that u,v are functionally related and find the relationship.

Q4. Answer **ALL** parts.

(a) Evaluate the double integral $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{(y^4-a^2x^2)}}dxdy$ by changing the order of integration. **7**

(b) The plane $\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$ meets the axes in A, B, and C. Apply Dirichlet's integral to find the volume of the tetrahedron OABC. Also, find it's mass if the density at any point is $4xyz$. **7**

OR

Evaluate $\iiint \sqrt{\frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}}\,dxdydz$, integral being taken over all positive values of x,y,z such that $x^2+y^2+z^2 \le 1$.

Q5. Answer **ALL** parts.

(a) Find the directional derivative of $\vec{\nabla}.(\vec{\nabla}\phi)$ at the point $(1,-2,1)$ in the direction of the normal to the surface $xy^2z=3x+z^2$, where $\phi=2x^3y^2z^4$. **7**

(b) Apply Green's theorem to evaluate $\int_C [(y-\sin x)dx+\cos x dy]$, where C is the plane triangle enclosed by the lines $y=0, x=\frac{\pi}{2}$ and $y=\frac{2}{\pi}x$. **7**

OR

Apply Gauss divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and S is the surface bounding the region $x^2+y^2=4, z=0$ and $z=3$.