BSM-110

B.Tech.

Year: I Semester: I

Minor Test (Examination): 2024-2025

Subject Name: Engineering Mathematics-I

Time: 2 hrs. **Max. Marks: 30**

Note: Attempt all questions.

			Marks	CO	BL	PO	PI
Q.1		Attempt any three parts of the following.					
	(a)	If u, v, w are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ, then find $\frac{\partial(u,\ v,\ w)}{\partial(x,\ y,\ z)}$.	4	1,2	2,3	1	1.1.1
	(b)	If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, then prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$.	4	1,2	2,3	1	1.1.1
	(c)	Find the inverse of the matrix $A = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}$	4	6	2,3	1	1.1.1
	(d)	Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Find matrix P such that $P^{-1}AP$ is a diagonal matrix.	4	6	2,3	1	1.1.1
Q.2		Attempt any three parts of the following.					
	(a)	The Temperature T at any point (x, y, z) in space is $T(x, y, z) = kxyz^2$, k is positive constant. Find the highest temperature on the surface of sphere $x^2 + y^2 + z^2 = a^2$.	3	2	2,3	1	1.1.1
	(b)	If u be the homogeneous function of degree n in x and y, then show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$	3	1,2	2,3	1	1.1.1
	(c)	Expand $f(x, y) = tan^{-1}(xy)$ in powers of $(x-1)$ and $(y-1)$ up to the second-degree term. Hence compute $f(0.9,\ 1.1)$ approximately.	3	2	2,3	1	1.1.1
	(d)	If $\log y = \tan^{-1} x$, show that $(1+x^2)y_{n+2} + \{2(n+1)x - 1\}y_{n+1} + n(n+1)y_n = 0$.	3	1,2	2,3	1	1.1.1
Q. 3		Attempt any three parts of the following.					
	(a)	Find the rank of the matrix A by changing it into normal form. $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$	3	6	2,3	1	1.1.1
	(b)	Find the value of a and b for which the system of equations $3x - 2y + z = b, 5x - 8y + 9z = 3, \quad 2x + y + az = -1$ has (i) unique solution (ii) no solution (iii) infinitely many solutions.	3	3,6	2,3	1	1.1.1
	(c)	State the Cayley-Hamilton theorem. Verify this theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$. Hence express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as linear polynomial in A.	3	6	1,2	1	1.1.1
	(d)	Find the value of k such that the system of equations $x + ky + 3z = 0, 4x + 3y + kz = 0, 2x + y + 2z = 0$ has nontrivial solution.	3	3,6	2,3	1	1.1.1

BSM-110

B.Tech.

Year: I Semester: I

Major Examination-2024-2025

Subject Name: Engineering Mathematics-I

Time: 3 hrs. Max. Marks: 50

Note: Attempt all questions. All questions carry equal marks.

Q.1	Attempt any five parts of the following.	Marks	CO	BL	PO	PI CODE
a)	If $u = \sin nx + \cos nx$, then show that $u_r = n^r[1 + (-1)^r \sin 2nx]^{1/2}$, where u_r is the r^{th} differential coefficient of u w.r.t. x.	2	6	2	1	1.1.1
b)	Find the n^{th} derivative of $e^{ax}\sin(bx + c)$.	2	1	2	1	1.1.1
c)	Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.	2	2	2	1	1.1.1
d)	If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, then prove that $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$	2	4	2	1	1.1.1
e)	The eigen values of a matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ are -2 and 3, then find the value of a and b.	2	5	2	1	1.1.1
f)	Check the consistency of the system of equations $x + 2y - z = 6$, $3x - y - 2z = 3$ and $4x + 3y + z = 9$.	2	4	2	1	1.1.1
g)	Find inverse of the following matrix using Gauss Jorden Method. $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$	2	4	2	1	1.1.1
Q.2	Attempt any Two parts of the following.					
a)	Show that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)\ldots\Gamma\left(\frac{n-1}{n}\right) = \frac{(2\pi)^{\left(\frac{n-1}{2}\right)}}{n^{1/2}}$.	5	6	3	1	1.1.1
b)	Show that $\Gamma m \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.	5	2	3	1	1.1.1
c)	Evaluate (i) $\int_0^\infty \frac{x^c}{c^x}dx$, (ii) $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$.	5	1	3	1	1.1.1
Q.3	Attempt any Two parts of the following.					
a)	Evaluate $\iint_R (x - y)^4 e^{x+y}\,dxdy$, where R is the square with vertices at (1,0), (2,1), (1,2) and (0,1).	5	3	3	1	1.1.1
b)	Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate it.	5	3	3	1	1.1.1

c)	(i) Using double integration, find the area enclosed by curves $y = 2x^2$ and $y^2 = 4x$. (ii) Find the volume of the solid region contained in the first octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	5	6	3	1	1.1.1
Q.4	**Attempt any Two parts of the following.**					
a)	Evaluate $$\iint_S F.\hat{n}\, dS.$$ Here $\vec{F} = 4y\hat{\imath} + 18z\hat{\jmath} - x\hat{k}$ and S is the surface of the plane $3x + 2y + 6z = 6$, contained in the first octant.	5	5	3	1	1.1.1
b)	State Stoke's theorem. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$, over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy –plane.	5	5	3	1	1.1.1
c)	Verify Green's theorem in a plane for the integral $$\int_C (x - 2y)dx + xdy$$ taken around the circle $x^2 + y^2 = 4$.	5	5	3	1	1.1.1
Q.5	**Attempt any Two parts of the following.**					
a)	(i) Show that $r^n\vec{r}$ is solenoidal for $n = -3$ and irrotational for all values of n. (ii) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in the direction of the normal to the surface $3xy^2 + y + z$ at $(0,1,1)$.	5	4	3	1	1.1.1
b)	Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped bounded by $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.	5	4	3	1	1.1.1
c)	(i) Show that $div\left\{grad\left(\frac{x}{r^3}\right)\right\} = 0$, where r is the magnitude of position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. (ii) Find the work done in moving a particle by the force field $\vec{F} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} - z\hat{k}$, from $t = 0$ to $t = 1$ along the curve $x = 2t^2, y = t, z = 4t^3$.	5	4	3	1	1.1.1