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Subject Code: IAS-203 Roll No.:

B. TECH.

(SEM II) EVEN SEMESTER EXAMINATION 2023-24

ENGINEERING MATHEMATICS-II

[TIME: 3 hrs.] **[Max. Marks: 70]**

Note: *Attempt All Questions. All Question carry equal marks.*

Marks

Q1. Answer **ALL** parts. 3.5

(a) Solve: $(D^3 + D^2 - D - 1)y = \cos 2x, D = \frac{d}{dx}$. 3.5

(b) Solve the simultaneous differential equations: $\frac{dx}{dt} = 2y + x, \frac{dy}{dt} = y$. 3.5

(c) Using method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = sec\, x$.

OR

By changing independent variable, solve $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3y = x^5$. 3.5

(d) Solve the differential equation: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$.

OR

A resistance R of 5 Ω and an inductance L of 0.1 H are connected in series with a battery of 12V. Find the current i in the circuit as a function of time using following differential equation $Ri + L\frac{di}{dt} = E$.

Q2. Answer **ALL** parts.

(a) Find the Laplace transform of the function $f(x) = x^2 \sin x$. Hence, find $\int_0^\infty e^{-x}x^2 \sin x dx$. 7

(b) State convolution theorem of the Laplace transforms. Hence, find inverse Laplace transform of $F(s) = \frac{1}{s^3(s^2+1)}$. 7

OR

Using Laplace transform, solve the following differential equation:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 6e^{-x}, y(0) = -2 \,\&\, y'(0) = 8.$$

Q3. Answer **ALL** parts.

(a) Test the convergence of following series: 7

$1 + \frac{x}{2} + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^3 + ..., x > 0$ and x is a real number.

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OR

Find the half range cosine series for the function $f(x)=x(\pi-x)$ in the interval $(0,\pi)$. Hence, prove that

$$\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\frac{1}{8^2}+\ldots=\frac{\pi^2}{24}.$$

7

(b) Find a Fourier series to represent $f(x)=x+x^2, -\pi\le x\le\pi$. Hence, show that

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}.$$

Q4. Answer **ALL** parts.

(a) Show that the function *f(z)* defined by $f(z)=\frac{2xy(x+iy)}{x^2+y^2}, z\neq 0, f(0)=0$ is not analytic at the origin even though it satisfies Cauchy-Riemann equations at the origin. 7

(b) Determine an analytic function $f(z)=u+iv$ in terms of z whose real part $u(x,y)$ is $e^x(x\cos y-y\sin y)$ and $f(1)=e$. 7

OR

Find the image of the region bounded by (0,0), (1,0), (1,2),(0,2) by the transformation $w=(1+i)z+2-i$. Sketch the image.

Q5. Answer **ALL** parts.

(a) Using Taylor's and Laurent series, expand $f(z)=\frac{z^2-1}{z^2+5z+6}$ in the following regions: 7

$(i)\ |z|<2 \quad (ii)\ 2<|z|<3 \quad (iii)\ |z|>3.$

(b) Using contour integration, evaluate the real integral $\int_0^{2\pi}\frac{d\theta}{a+b\sin\theta}, a>|b|$. 7

OR

Using Cauchy's residue theorem, evaluate the following integral:

$$\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}dz,$$ Where C is the circle $|z|=3$.