CHAPTER 10 — FLEXURE AND AXIAL LOADS

CODE

10.0 — Notation

a = depth of equivalent rectangular stress block as defined in 10.2.7.1

A = effective tension area of concrete surrounding the flexural tension reinforcement and having the same centroid as that reinforcement, divided by the number of bars or wires, in.^{2} When the flexural reinforcement consists of different bar or wire sizes the number of bars or wires shall be computed as the total area of reinforcement divided by the area of the largest bar or wire used

A_c = area of core of spirally reinforced compression member measured to outside diameter of spiral, in.^{2}

A_g = gross area of section, in.^{2}

A_s = area of nonprestressed tension reinforcement, in.^{2}

A_{sk} = area of skin reinforcement per unit height in one side face, in.^{2}/ft. See 10.6.7

$A_{s,min}$ = minimum amount of flexural reinforcement, in.^{2} See 10.5

A_{st} = total area of longitudinal reinforcement, (bars or steel shapes), in.^{2}

A_t = area of structural steel shape, pipe, or tubing in a composite section, in.^{2}

A_1 = loaded area

A_2 = the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal

b = width of compression face of member, in.

b_w = web width, in.

c = distance from extreme compression fiber to neutral axis, in.

C_m = a factor relating actual moment diagram to an equivalent uniform moment diagram

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire located closest thereto, in.

d_t = distance from extreme compression fiber to extreme tension steel, in.

E_c = modulus of elasticity of concrete, psi. See 8.5.1

E_s = modulus of elasticity of reinforcement, psi. See 8.5.2 or 8.5.3

EI = flexural stiffness of compression member. See Eq. (10-12) and Eq. (10-13)

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f_c' = specified compressive strength of concrete, psi

f_s = calculated stress in reinforcement at service loads, ksi

f_y = specified yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section

I_t = moment of inertia of structural steel shape, pipe, or tubing about centroidal axis of composite member cross section

k = effective length factor for compression members

ℓ_c = length of compression member in a frame, measured from center to center of the joints in the frame

ℓ_u = unsupported length of compression member

M_c = factored moment to be used for design of compression member

M_s = moment due to loads causing appreciable sway

M_u = factored moment at section

M_1 = smaller factored end moment on a compression member, positive if member is bent in single curvature, negative if bent in double curvature

M_{1ns} = factored end moment on a compression member at the end at which M_1 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis

M_{1s} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis

M_2 = larger factored end moment on compression member, always positive

$M_{2,min}$ = minimum value of M_2

M_{2ns} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis

M_{2s} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis

P_b = nominal axial load strength at balanced strain conditions. See 10.3.2

P_c = critical load. See Eq. (10-11)

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P_n = nominal axial load strength at given eccentricity

P_o = nominal axial load strength at zero eccentricity

P_u = factored axial load at given eccentricity $\leq \phi P_n$

Q = stability index for a story. See 10.11.4

r = radius of gyration of cross section of a compression member

V_u = factored horizontal shear in a story

z = quantity limiting distribution of flexural reinforcement. See 10.6

β_1 = factor defined in 10.2.7.3

β_d = (a) for non-sway frames, β_d is the ratio of the maximum factored axial dead load to the total factored axial load
(b) for sway frames, except as required in (c), β_d is the ratio of the maximum factored sustained shear within a story to the total factored shear in that story
(c) for stability checks of sway frames carried out in accordance with 10.13.6, β_d is the ratio of the maximum factored sustained axial load to the total factored axial load

δ_{ns} = moment magnification factor for frames braced against sidesway, to reflect effects of member curvature between ends of compression member

δ_s = moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads

Δ_o = relative lateral deflection between the top and bottom of a story due to V_u, computed using a first-order elastic frame analysis and stiffness values satisfying 10.11.1

ε_t = net tensile strain in extreme tension steel at nominal strength

ρ = ratio of nonprestressed tension reinforcement = A_s/bd

ρ_b = reinforcement ratio producing balanced strain conditions. See 10.3.2

ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced compression member

ϕ = strength reduction factor. See 9.3

ϕ_K = stiffness reduction factor. See R10.12.3

10.1 — Scope

Provisions of Chapter 10 shall apply for design of members subject to flexure or axial loads or to combined flexure and axial loads.

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The definition of net tensile strain in 2.1 excludes strains due to effective prestress, creep, shrinkage, and temperature.

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10.2 — Design assumptions

10.2.1 — Strength design of members for flexure and axial loads shall be based on assumptions given in 10.2.2 through 10.2.7, and on satisfaction of applicable conditions of equilibrium and compatibility of strains.

10.2.2 — Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis, except, for deep flexural members with overall depth to clear span ratios greater than $^2/_5$ for continuous spans and $^4/_5$ for simple spans, a nonlinear distribution of strain shall be considered. See 10.7.

10.2.3 — Maximum usable strain at extreme concrete compression fiber shall be assumed equal to 0.003.

10.2.4 — Stress in reinforcement below specified yield strength f_y for grade of reinforcement used shall be taken as E_s times steel strain. For strains greater than that corresponding to f_y, stress in reinforcement shall be considered independent of strain and equal to f_y.

10.2.5 — Tensile strength of concrete shall be neglected in axial and flexural calculations of reinforced concrete, except when meeting requirements of 18.4.

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R10.2 — Design assumptions

R10.2.1 — The strength of a member computed by the strength design method of the code requires that two basic conditions be satisfied: (1) static equilibrium and (2) compatibility of strains. Equilibrium between the compressive and tensile forces acting on the cross section at nominal strength must be satisfied. Compatibility between the stress and strain for the concrete and the reinforcement at nominal strength conditions must also be established within the design assumptions allowed by 10.2.

R10.2.2—Many tests have confirmed that the distribution of strain is essentially linear across a reinforced concrete cross section, even near ultimate strength.

Both the strain in reinforcement and in concrete are assumed to be directly proportional to the distance from the neutral axis. This assumption is of primary importance in design for determining the strain and corresponding stress in the reinforcement.

R10.2.3 — The maximum concrete compressive strain at crushing of the concrete has been observed in tests of various kinds to vary from 0.003 to higher than 0.008 under special conditions. However, the strain at which ultimate moments are developed is usually about 0.003 to 0.004 for members of normal proportions and materials.

R10.2.4 — For deformed reinforcement, it is reasonably accurate to assume that the stress in reinforcement is proportional to strain below the yield strength f_y. The increase in strength due to the effect of strain hardening of the reinforcement is neglected for strength computations. In strength computations, the force developed in tensile or compressive reinforcement is computed as,

when $\varepsilon_s < \varepsilon_y$ (yield strain)

$$A_s f_s = A_s E_s \varepsilon_s$$

when $\varepsilon_s \geq \varepsilon_y$

$$A_s f_s = A_s f_y$$

where ε_s is the value from the strain diagram at the location of the reinforcement. For design, the modulus of elasticity of steel reinforcement E_s may be taken as 29,000,000 psi (see 8.5.2).

R10.2.5 — The tensile strength of concrete in flexure (modulus of rupture) is a more variable property than the compressive strength and is about 10 to 15 percent of the compressive strength. Tensile strength of concrete in flexure is neglected in strength design. For members with normal

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percentages of reinforcement, this assumption is in good agreement with tests. For very small percentages of reinforcement, neglect of the tensile strength at ultimate is usually correct.

The strength of concrete in tension, however, is important in cracking and deflection considerations at service loads.

10.2.6 — Relationship between concrete compressive stress distribution and concrete strain shall be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

R10.2.6 — This assumption recognizes the inelastic stress distribution of concrete at high stress. As maximum stress is approached, the stress-strain relationship for concrete is not a straight line but some form of a curve (stress is not proportional to strain). The general shape of a stress-strain curve is primarily a function of concrete strength and consists of a rising curve from zero to a maximum at a compressive strain between 0.0015 and 0.002 followed by a descending curve to an ultimate strain (crushing of the concrete) from 0.003 to higher than 0.008. As discussed under R10.2.3. the code sets the maximum usable strain at 0.003 for design.

The actual distribution of concrete compressive stress in a practical case is complex and usually not known explicitly. However, research has shown that the important properties of the concrete stress distribution can be approximated closely using any one of several different assumptions as to the form of stress distribution. The code permits any particular stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Many stress distributions have been proposed. The three most common are the parabola, trapezoid, and rectangle.

10.2.7 — Requirements of 10.2.6 are satisfied by an equivalent rectangular concrete stress distribution defined by the following:

10.2.7.1 — Concrete stress of $\mathbf{0.85}\boldsymbol{f_c'}$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $\boldsymbol{a} = \beta_1 \boldsymbol{c}$ from the fiber of maximum compressive strain.

10.2.7.2 — Distance $\boldsymbol{c}$ from fiber of maximum strain to the neutral axis shall be measured in a direction perpendicular to that axis.

10.2.7.3 — Factor β_1 shall be taken as 0.85 for concrete strengths f_c' up to and including 4000 psi. For strengths above 4000 psi, β_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken less than 0.65.

R10.2.7 — For practical design, the code allows the use of a rectangular compressive stress distribution (stress block) to replace the more exact concrete stress distributions. In the equivalent rectangular stress block, an average stress of $\mathbf{0.85}\boldsymbol{f_c'}$ is used with a rectangle of depth $\boldsymbol{a} = \beta_1 \boldsymbol{c}$. The β_1 of 0.85 for concrete with $f_c' \le 4000$ psi and 0.05 less for each 1000 psi of f_c' in excess of 4000 was determined experimentally.

In the 1976 supplement to ACI 318-71, a lower limit of β_1 equal to 0.65 was adopted for concrete strengths greater than 8000 psi. Research data from tests with high strength concretes[10.1,10.2] supported the equivalent rectangular stress block for concrete strengths exceeding 8000 psi, with a β_1 equal to 0.65. Use of the equivalent rectangular stress distribution specified in ACI 318-71, with no lower limit on β_1, resulted in inconsistent designs for high strength concrete for members subject to combined flexure and axial load.

The rectangular stress distribution does not represent the actual stress distribution in the compression zone at ultimate, but does provide essentially the same results as those obtained in tests.[10.3]

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10.3 — General principles and requirements

10.3.1 — Design of cross section subject to flexure or axial loads or to combined flexure and axial loads shall be based on stress and strain compatibility using assumptions in 10.2.

10.3.2 — Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as concrete in compression reaches its assumed ultimate strain of 0.003.

10.3.3 — For flexural members, and for members subject to combined flexure and compressive axial load when the design axial load strength ϕP_n is less than the smaller of $0.10f_c'A_g$ or ϕP_b, the ratio of reinforcement ρ provided shall not exceed 0.75 of the ratio ρ_b that would produce balanced strain conditions for the section under flexure without axial load. For members with compression reinforcement, the portion of ρ_b equalized by compression reinforcement need not be reduced by the 0.75 factor.

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R10.3 — General principles and requirements

R10.3.1 — Design strength equations for members subject to flexure or combined flexure and axial load are derived in the paper, "Rectangular Concrete Stress Distribution in Ultimate Strength Design."[10.3] Reference 10.3 and previous editions of this commentary also give the derivations of strength equations for cross sections other than rectangular.

R10.3.2 — A balanced strain condition exists at a cross section when the maximum strain at the extreme compression fiber just reaches 0.003 simultaneously with the first yield strain f_y/E_s in the tension reinforcement. The reinforcement ratio ρ_b, which produces balanced conditions under flexure, depends on the shape of the cross section and the location of the reinforcement.

R10.3.3 — The maximum amount of tension reinforcement in flexural members is limited to ensure a level of ductile behavior.

The ultimate flexural strength of a member is reached when the strain in the extreme compression fiber reaches the ultimate (crushing) strain of the concrete. At ultimate strain of the concrete, the strain in the tension reinforcement could just reach the strain at first yield, be less than the yield strain (elastic), or exceed the yield strain (inelastic). Which steel strain condition exists at ultimate concrete strain depends on the relative proportion of steel to concrete and material strengths f_c' and f_y. If $\rho(f_y/f_c')$ is sufficiently low, the strain in the tension steel will greatly exceed the yield strain when the concrete strain reaches its ultimate, with large deflection and ample warning of impending failure (ductile failure condition). With a larger $\rho(f_y/f_c')$, the strain in the tension steel may not reach the yield strain when the concrete strain reaches its ultimate, with consequent small deflection and little warning of impending failure (brittle failure condition). For design it is considered more conservative to restrict the ultimate strength condition so that a ductile failure mode can be expected.

Unless unusual amounts of ductility are required, the **0.75** ρ_b limitation will provide ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in continuous members and frames. Code Section 8.4 permits negative moment redistribution. Since moment redistribution is dependent on adequate ductility in hinge regions, the amount of tension reinforcement in hinging regions is limited to $\mathbf{0.5}\rho_b$.

For ductile behavior of beams with compression reinforcement, only that portion of the total tension steel balanced by compression in the concrete need be limited; that portion of the total tension steel where force is balanced by compression reinforcement need not be limited by the 0.75 factor.

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10.3.4 — Use of compression reinforcement shall be permitted in conjunction with additional tension reinforcement to increase the strength of flexural members.

10.3.5 — Design axial load strength ϕP_n of compression members shall not be taken greater than the following:

10.3.5.1 — For nonprestressed members with spiral reinforcement conforming to 7.10.4 or composite members conforming to 10.16:

$$\phi P_{n(max)} = 0.85\phi\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right] \quad (10\text{-}1)$$

10.3.5.2—For nonprestressed members with tie reinforcement conforming to 7.10.5:

$$\phi P_{n(max)} = 0.80\phi\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right] \quad (10\text{-}2)$$

10.3.5.3 — For prestressed members, design axial load strength ϕP_n shall not be taken greater than 0.85 (for members with spiral reinforcement) or 0.80 (for members with tie reinforcement) of the design axial load strength at zero eccentricity ϕP_o.

10.3.6 — Members subject to compressive axial load shall be designed for the maximum moment that can accompany the axial load. The factored axial load P_u at given eccentricity shall not exceed that given in 10.3.5. The maximum factored moment M_u shall be magnified for slenderness effects in accordance with 10.10.

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R10.3.5 and R10.3.6 — The minimum design eccentricities included in the 1963 and 1971 codes were deleted from the 1977 code except for consideration of slenderness effects in compression members with small or zero computed end moments (see 10.12.3.2). The specified minimum eccentricities were originally intended to serve as a means of reducing the axial load design strength of a section in pure compression to account for accidental eccentricities not considered in the analysis that may exist in a compression member, and to recognize that concrete strength may be less than f_c' under sustained high loads. The primary purpose of the minimum eccentricity requirement was to limit the maximum design axial load strength of a compression member. This is now accomplished directly in 10.3.5 by limiting the design axial load strength of a section in pure compression to 85 or 80 percent of the nominal strength. These percentage values approximate the axial load strengths at e/h ratios of 0.05 and 0.10, specified in the earlier codes for the spirally reinforced and tied members respectively. The same axial load limitation applies to both cast-in-place and precast compression members. Design aids and computer programs based on the minimum eccentricity requirement of the 1963 and 1971 codes are equally applicable for usage.

For prestressed members, the design axial load strength in pure compression is computed by the strength design methods of Chapter 10, including the effect of the prestressing force.

Compression member end moments must be considered in the design of adjacent flexural members. In braced frames, the effects of magnifying the end moments need not be considered in the design of the adjacent beams. In frames which are not braced against sidesway, the magnified end moments must be considered in designing the flexural members, as required in 10.13.7.

Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load. Satisfactory methods are available in the ACI *Design Handbook*[10.4] and the CRSI *Handbook.*[10.5] The reciprocal load method[10.6] and the load contour method[10.7] are the methods used in those two handbooks. Research[10.8,10.9] indicates that using the rectangular stress block provisions of 10.2.7 produces satisfactory strength estimates for doubly symmetric sections. A simple and somewhat conservative estimate of nominal strength P_{ni} can be obtained from the reciprocal load relationship[10.6]

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

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where

P_{ni} = nominal axial load strength at given eccentricity along both axes
P_o = nominal axial load strength at zero eccentricity
P_{nx} = nominal axial load strength at given eccentricity along x-axis
P_{ny} = nominal axial load strength at given eccentricity along y-axis

This relationship is most suitable when values P_{nx} and P_{ny} are greater than the balanced axial force P_b for the particular axis.

10.4 — Distance between lateral supports of flexural members

10.4.1 — Spacing of lateral supports for a beam shall not exceed 50 times the least width b of compression flange or face.

10.4.2 — Effects of lateral eccentricity of load shall be taken into account in determining spacing of lateral supports.

R10.4 — Distance between lateral supports of flexural members

Tests have shown that laterally unbraced reinforced concrete beams of any reasonable dimensions, even when very deep and narrow, will not fail prematurely by lateral buckling provided the beams are loaded without lateral eccentricity that could cause torsion[10.10,10.11]

Laterally unbraced beams are frequently loaded off center (lateral eccentricity) or with slight inclination. Stresses and deformations set up by such loading become detrimental for narrow, deep beams, the more so as the unsupported length increases. Lateral supports spaced closer than **50*b*** may be required by actual loading conditions.

10.5 — Minimum reinforcement of flexural members

10.5.1 — At every section of a flexural member where tensile reinforcement is required by analysis, except as provided in 10.5.2, 10.5.3, and 10.5.4, the area A_s provided shall not be less than that given by

$$A_{s,min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \qquad \text{(10-3)}$$

f'c in psi

and not less than **200** $b_w d/f_y$.

10.5.2 — For a statically determinate T-section with flange in tension, the area $A_{s,min}$ shall be equal to or greater than the smaller value given either by

$$A_{s,min} = \frac{6\sqrt{f_c'}}{f_y} b_w d \qquad \text{(10-4)}$$

or Eq. (10-3) with b_w set equal to the width of the flange.

R10.5 — Minimum reinforcement of flexural members

The provision for a minimum amount of reinforcement applies to flexural members, which for architectural or other reasons, are larger in cross section than required for strength. With a very small amount of tensile reinforcement, the computed moment strength as a reinforced concrete section using cracked section analysis becomes less than that of the corresponding unreinforced concrete section computed from its modulus of rupture. Failure in such a case can be sudden.

To prevent such a failure, a minimum amount of tensile reinforcement is required by 10.5.1. This is required in both positive and negative moment regions. The **200/f_y** value formerly used was originally derived to provide the same 0.5 percent minimum (for mild grade steel) as required in earlier editions of the ACI Building Code. When concrete strength higher than about 5000 psi is used, the **200/f_y** value previously used may not be sufficient. The value given by Eq. (10-3) gives the same amount as **200/f_y** when f_c' equals 4440 psi. When the flange of a T-section is in tension, the amount of tensile reinforcement needed to make the

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strength of a reinforced concrete section equal that of an unreinforced section is about twice that for a rectangular section or that of a T-section with the flange in compression. It was concluded that this higher amount is necessary, particularly for cantilevers and other statically determinate situations where the flange is in tension.

10.5.3 — The requirements of 10.5.1 and 10.5.2 need not be applied if at every section the area of tensile reinforcement provided is at least one-third greater than that required by analysis.

R10.5.3 — The minimum reinforcement required by Eq. (10-3) or (10-4) must be provided wherever reinforcement is needed, except where such reinforcement is at least one-third greater than that required by analysis. This exception provides sufficient additional reinforcement in large members where the amount required by 10.5.1 or 10.5.2 would be excessive.

10.5.4 — For structural slabs and footings of uniform thickness the minimum area of tensile reinforcement in the direction of the span shall be the same as that required by 7.12. Maximum spacing of this reinforcement shall not exceed the lesser of three times the thickness and 18 in.

R10.5.4 — The minimum reinforcement required for slabs should be equal to the same amount as that required by 7.12 for shrinkage and temperature reinforcement.

Soil-supported slabs such as slabs on grade are not considered to be structural slabs in the context of this section, unless they transmit vertical loads from other parts of the structure to the soil. Reinforcement, if any, in soil-supported slabs should be proportioned with due consideration of all design forces. Mat foundations and other slabs which help support the structure vertically should meet the requirements of this section.

In reevaluating the overall treatment of 10.5, the maximum spacing for reinforcement in structural slabs (including footings) was reduced from the $5h$ for temperature and shrinkage reinforcement to the compromise value of $3h$, which is somewhat larger than the $2h$ limit of 13.3.2 for two-way slab systems.

10.6 — Distribution of flexural reinforcement in beams and one-way slabs

10.6.1 — This section prescribes rules for distribution of flexural reinforcement to control flexural cracking in beams and in one-way slabs (slabs reinforced to resist flexural stresses in only one direction).

R10.6 — Distribution of flexural reinforcement in beams and one-way slabs

R10.6.1 — Many structures designed by working stress methods and with low steel stress served their intended functions with very limited flexural cracking. When high strength reinforcing steels are used at high service load stresses, however, visible cracks must be expected, and steps must be taken in detailing of the reinforcement to control cracking. To assure protection of reinforcement against corrosion, and for aesthetic reasons, many fine hairline cracks are preferable to a few wide cracks.

Control of cracking is particularly important when reinforcement with a yield strength in excess of 40,000 psi is used. Current good detailing practices will usually lead to adequate crack control even when reinforcement of 60,000 psi yield is used.

Extensive laboratory work[10.12-10.14] involving modern deformed bars has confirmed that crack width at service

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loads is proportional to steel stress. However, the significant variables reflecting steel detailing were found to be thickness of concrete cover and the area of concrete in the zone of maximum tension surrounding each individual reinforcing bar.

Crack width is inherently subject to wide scatter even in careful laboratory work and is influenced by shrinkage and other time-dependent effects. The best crack control is obtained when the steel reinforcement is well distributed over the zone of maximum concrete tension.

10.6.2 — Distribution of flexural reinforcement in two-way slabs shall be as required by 13.3.

10.6.3 — Flexural tension reinforcement shall be well distributed within maximum flexural tension zones of a member cross section as required by 10.6.4.

R10.6.3 — Several bars at moderate spacing are much more effective in controlling cracking than one or two larger bars of equivalent area.

10.6.4 — When design yield strength f_y for tension reinforcement exceeds 40,000 psi, cross sections of maximum positive and negative moment shall be so proportioned that the quantity z given by

$$z = f_s \sqrt[3]{d_c A} \qquad (10\text{-}5)$$

does not exceed 175 kips/in. for interior exposure and 145 kips/in. for exterior exposure. Calculated stress in reinforcement at service load f_s (kips/in.^{2}) shall be computed as the moment divided by the product of steel area and internal moment arm. Alternatively, it shall be permitted to take f_s as 60 percent of specified yield strength f_y.

R10.6.4 — Eq. (10-5) will provide a distribution that will reasonably control flexural cracking. The equation is written in a form emphasizing reinforcement details rather than crack width w, per se. It is based on the Gergely-Lutz expression:

$$w = 0.076\beta f_s \sqrt[3]{d_c A}$$

in which w is in units of 0.001 in. To simplify practical design, an approximate value of 1.2 is used for β (ratio of distances to the neutral axis from the extreme tension fiber and from the centroid of the main reinforcement). Laboratory tests[10.15] have shown that the Gergely-Lutz expression

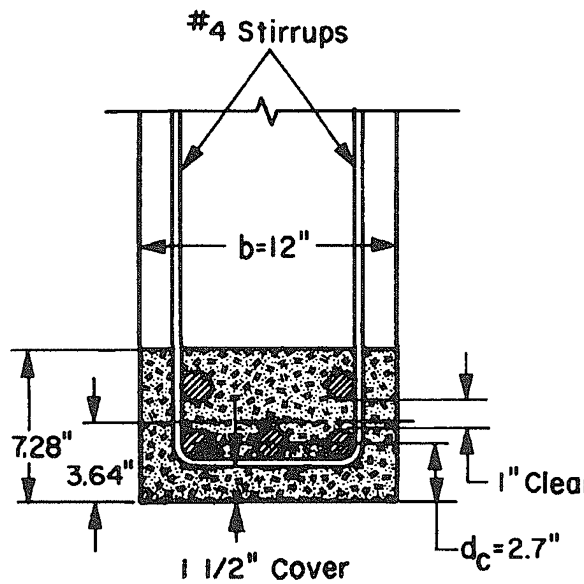

Fig. R10.6.4—Effective tension area of concrete (beam with five No. 11 bars)

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applies reasonably to one-way slabs. The average ratio β is about 1.35 for floor slabs, rather than the value 1.2 used for beams. Accordingly it would be consistent to reduce the maximum values for z by the factor 1.2/1.35.

The numerical limitations of z = **175 and 145** kips/in. for interior and exterior exposure, respectively, correspond to limiting crack widths of 0.016 and 0.013 in.

The effective tension area of concrete surrounding the principal reinforcement is defined as having the same centroid as the reinforcement. Moreover, this area is to be bounded by the surfaces of the cross section and a straight line parallel to the neutral axis. Computation of the effective area per bar, A (see notation definition), is illustrated by the example shown in Fig. R10.6.4 in which the centroid of the main reinforcement is located 3.64 in. from the bottom of the beam. The effective tension area is then taken as twice 3.64 in. times the beam width b. Divided by the number of bars, this gives 17.6 in.^{2} per bar.

10.6.5 — Provisions of 10.6.4 are not sufficient for structures subject to very aggressive exposure or designed to be watertight. For such structures, special investigations and precautions are required.

R10.6.5 — Although a number of studies have been conducted, clear experimental evidence is not available regarding the crack width beyond which a corrosion danger exists. Exposure tests indicate that concrete quality, adequate compaction, and ample concrete cover may be of greater importance for corrosion protection than crack width at the concrete surface. The limiting values for z were, therefore, chosen primarily to give reasonable reinforcement details in terms of practical experiences with existing structures.

10.6.6 — Where flanges of T-beam construction are in tension, part of the flexural tension reinforcement shall be distributed over an effective flange width as defined in 8.10, or a width equal to $^1/_{10}$ the span, whichever is smaller. If the effective flange width exceeds $^1/_{10}$ the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

R10.6.6 — In major T-beams, distribution of the negative reinforcement for control of cracking must take into account two considerations: (1) wide spacing of the reinforcement across the full effective width of flange may cause some wide cracks to form in the slab near the web and, (2) close spacing near the web leaves the outer regions of the flange unprotected. The $^1/_{10}$ limitation is to guard against too wide a spacing, with some additional reinforcement required to protect the outer portions of the flange.

10.6.7 — If the effective depth d of a beam or joist exceeds 36 in., longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member for a distance $d/2$ nearest the flexural tension reinforcement. The area of skin reinforcement A_{sk} per foot of height on each side face shall be $\geq 0.012(d-30)$. The maximum spacing of the skin reinforcement shall not exceed the lesser of $d/6$ and 12 in. It shall be permitted to include such reinforcement in strength computations if a strain compatibility analysis is made to determine stress in the individual bars or wires. The total area of longitudinal skin reinforcement in both faces need not exceed one-half of the required flexural tensile reinforcement.

R10.6.7 — For relatively deep flexural members, some reinforcement should be placed near the vertical faces in the tension zone to control cracking in the web. Without such auxiliary steel, the width of the cracks in the web may greatly exceed the crack widths at the level of the flexural tension reinforcement.

The requirements for skin reinforcement were modified in the 1989 edition of the code, as the previous requirements were found to be inadequate in some cases. See Reference 10.16. For lightly reinforced members, these requirements may be reduced to one-half of the main flexural reinforcement. Where the provisions for deep beams, walls, or precast panels require more steel, those provisions (along with their spacing requirements) will govern.

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10.7 — Deep flexural members

10.7.1 — Flexural members with overall depth to clear span ratios greater than $^2/_5$ for continuous spans, or $^4/_5$ for simple spans, shall be designed as deep flexural members taking into account nonlinear distribution of strain and lateral buckling. (See also 12.10.6.)

10.7.2 — Shear strength of deep flexural members shall be in accordance with 11.8.

10.7.3 — Minimum flexural tension reinforcement shall conform to 10.5.

10.7.4 — Minimum horizontal and vertical reinforcement in the side faces of deep flexural members shall be the greater of the requirements of 11.8.8, 11.8.9, and 11.8.10 or 14.3.2 and 14.3.3.

10.8 — Design dimensions for compression members

10.8.1 — Isolated compression member with multiple spirals

Outer limits of the effective cross section of a compression member with two or more interlocking spirals shall be taken at a distance outside the extreme limits of the spirals equal to the minimum concrete cover required by 7.7.

10.8.2 — Compression member built monolithically with wall

Outer limits of the effective cross section of a spirally reinforced or tied reinforced compression member built monolithically with a concrete wall or pier shall be taken not greater than $1^1/_2$ in. outside the spiral or tie reinforcement.

10.8.3 — Equivalent circular compression member

As an alternative to using the full gross area for design of a compression member with a square, octagonal, or other shaped cross section, it shall be permitted to use a circular section with a diameter equal to the least lateral dimension of the actual shape. Gross area considered, required percentage of reinforcement, and design strength shall be based on that circular section.

10.8.4 — Limits of section

For a compression member with a cross section larger than required by considerations of loading, it shall be permitted to base the minimum reinforcement and

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R10.7 — Deep flexural members

The code does not contain detailed requirements for designing deep beams for flexure except that nonlinearity of strain distribution and lateral buckling must be considered.

Suggestions for the design of deep beams for flexure are given in References 10.17, 10.18, and 10.19.

R10.8 — Design dimensions for compression members

With the 1971 edition of the ACI Building Code, minimum sizes for compression members were eliminated to allow wider utilization of reinforced concrete compression members in smaller size and lightly loaded structures, such as low rise residential and light office buildings. The engineer should recognize the need for careful workmanship, as well as the increased significance of shrinkage stresses with small sections.

R10.8.2, R10.8.3, R10.8.4 — For column design,[10.20] the code provisions for quantity of reinforcement, both vertical and spiral, are based on the gross column area and core area, and the design strength of the column is based on the gross area of the column section. In some cases, however, the gross area is larger than necessary to carry the factored load. The basis of 10.8.2, 10.8.3, and 10.8.4 is that it is satisfactory to design a column of sufficient size to carry the factored load and then simply add concrete around the designed section without increasing the reinforcement to meet the minimum percentages required by 10.9.1. The additional concrete must not be considered as carrying load; however, the effects of the additional concrete on member stiffness must be included in the structural analysis. The effects of the additional concrete also must be considered in design of the other parts of the structure that interact with the oversize member.

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strength on a reduced effective area A_g not less than one-half the total area. This provision shall not apply in regions of high seismic risk.

10.9 — Limits for reinforcement of compression members

10.9.1 — Area of longitudinal reinforcement for noncomposite compression members shall be not less than 0.01 nor more than 0.08 times gross area A_g of section.

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R10.9 — Limits for reinforcement of compression members

R10.9.1 — This section prescribes the limits on the amount of longitudinal reinforcement for noncomposite compression members. If the use of high reinforcement ratios would involve practical difficulties in the placing of concrete, a lower percentage and hence a larger column, or higher strength concrete or reinforcement (see R9.4) should be considered. The percentage of reinforcement in columns should usually not exceed 4 percent if the column bars are required to be lap spliced.

Minimum reinforcement. Since the design methods for columns incorporate separate terms for the load carried by concrete and by reinforcement, it is necessary to specify some minimum amount of reinforcement to ensure that only reinforced concrete columns are designed by these procedures. Reinforcement is necessary to provide resistance to bending, which may exist whether or not computations show that bending exists, and to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses. Tests have shown that creep and shrinkage tend to transfer load from the concrete to the reinforcement, with a consequent increase in stress in the reinforcement, and that this increase is greater as the ratio of reinforcement decreases. Unless a lower limit is placed on this ratio, the stress in the reinforcement may increase to the yield level under sustained service loads. This phenomenon was emphasized in the report of ACI Committee 105[10.21] and minimum reinforcement ratios of 0.01 and 0.005 were recommended for spiral and tied columns, respectively. However, in all editions of the code since 1936, the minimum ratio has been 0.01 for both types of laterally reinforced columns.

Maximum reinforcement. Extensive tests of the ACI column investigation[10.21] included reinforcement ratios no greater than 0.06. Although other tests with as much as 17 percent reinforcement in the form of bars produced results similar to those obtained previously, it is necessary to note that the loads in these tests were applied through bearing plates on the ends of the columns and the problem of transferring a proportional amount of the load to the bars was thus minimized or avoided. Maximum ratios of 0.08 and 0.03 were recommended by ACI Committee 105[10.21] for spiral and tied columns, respectively. In the 1936 ACI Building Code, these limits were made 0.08 and 0.04, respectively. In the 1956 code, the limit for tied columns with bending was raised to 0.08. Since the 1963 code, it has been required that bending be considered in the design of all

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columns, and the maximum ratio of 0.08 has been applied to both types of columns. This limit can be considered a practical maximum for reinforcement in terms of economy and requirements for placing.

10.9.2 — Minimum number of longitudinal bars in compression members shall be 4 for bars within rectangular or circular ties, 3 for bars within triangular ties, and 6 for bars enclosed by spirals conforming to 10.9.3.

R10.9.2 — For compression members, a minimum of four longitudinal bars are required when bars are enclosed by rectangular or circular ties. For other shapes, one bar should be provided at each apex or corner and proper lateral reinforcement provided. For example, tied triangular columns require three longitudinal bars, one at each apex of the triangular ties. For bars enclosed by spirals, six bars are required.

When the number of bars in a circular arrangement is less than eight, the orientation of the bars will affect the moment strength of eccentrically loaded columns and must be considered in design.

10.9.3 — Ratio of spiral reinforcement ρ_s shall be not less than the value given by

$$\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_y} \qquad (10\text{-}6)$$

where f_y is the specified yield strength of spiral reinforcement but not more than 60,000 psi.

R10.9.3 — The effect of spiral reinforcement in increasing the load-carrying strength of the concrete within the core is not realized until the column has been subjected to a load and deformation sufficient to cause the concrete shell outside the core to spall off. The amount of spiral reinforcement required by Eq. (10-6) is intended to provide additional load-carrying strength for concentrically loaded columns equal to or slightly greater than the strength lost when the shell spalls off. This principle was recommended by ACI Committee 105[10.21] and has been a part of the code since 1963. The derivation of Eq. (10-6) is given in the ACI Committee 105 report. Tests and experience show that columns containing the amount of spiral reinforcement required by this section exhibit considerable toughness and ductility.

10.10 — Slenderness effects in compression members

*COMPLETE REWRITE

R10.10 — Slenderness effects in compression members

Provisions for slenderness effects in compression members and frames were revised in the 1995 code to better recognize the use of second-order analyses and to improve the arrangement of the provisions dealing with braced and sway frames.[10.22] The use of a refined nonlinear second-order analysis is permitted in 10.10.1. Sections 10.11, 10.12, and 10.13 present an approximate design method based on the traditional moment magnifier method. For sway frames, the magnified sway moment $\delta_s M_s$ may be calculated using a second-order elastic analysis, by an approximation to such an analysis, or by the traditional sway moment magnifier.

10.10.1 — Except as allowed in 10.10.2, the design of compression members, restraining beams, and other supporting members shall be based on the factored forces and moments from a second-order analysis considering material nonlinearity and cracking, as well as the effects of member curvature and lateral drift,

R10.10.1 — Two limits are placed on the use of the refined second-order analysis. First, the structure which is analyzed must have members similar to those in the final structure. If the members in the final structure have cross-sectional dimensions more than 10 percent different from those assumed in the analysis, new member properties should be

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duration of the loads, shrinkage and creep, and interaction with the supporting foundation. The dimensions of each member cross section used in the analysis shall be within 10 percent of the dimensions of the members shown on the design drawings or the analysis shall be repeated. The analysis procedure shall have been shown to result in prediction of strength in substantial agreement with the results of comprehensive tests of columns in statically indeterminate reinforced concrete structures.

Analysis Procedure Predicts test Results Within 15%

10.10.2 — As an alternate to the procedure prescribed in 10.10.1, it shall be permitted to base the design of compression members, restraining beams, and other supporting members on axial forces and moments from the analyses described in 10.11.

10.11 — Magnified moments — General

10.11.1 — The factored axial forces P_u, the factored moments M_1 and M_2 at the ends of the column, and, where required, the relative lateral story deflections Δ_o shall be computed using an elastic first-order frame analysis with the section properties determined taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and effects of duration of the loads. Alternatively, it shall be permitted to use the following properties for the members in the structure:

(a) Modulus of elasticity .... E_c from 8.5.1

(b) Moments of inertia
- Beams .... $0.35\ I_g$
- Columns .... $0.70\ I_g$
- Walls—Uncracked .... $0.70\ I_g$
- —Cracked .... $0.35\ I_g$
- Flat plates and flat slabs .... $0.25\ I_g$

(c) Area .... $1.0\ A_g$

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computed and the analysis repeated. Second, the refined second-order analysis procedure should have been shown to predict ultimate loads within 15 percent of those reported in tests of indeterminate reinforced concrete structures. At the very least, the comparison should include tests of columns in planar braced frames, sway frames, and frames with varying column stiffnesses. To allow for variability in the actual member properties and in the analysis, the member properties used in analysis should be multiplied by a stiffness reduction factor ϕ_K less than one. For consistency with the second-order analysis in 10.13.4.1, the stiffness reduction factor ϕ_K can be taken as 0.80. The concept of a stiffness reduction factor ϕ_K is discussed in R10.12.3.

R10.10.2 — As an alternate to the refined second-order analysis of 10.10.1, design may be based on elastic analyses and the moment magnifier approach.[10.23,10.24] For sway frames the magnified sway moments may be calculated using a second-order elastic analysis based on realistic stiffness values. See R10.13.4.1.

R10.11 — Magnified moments — General

This section describes an approximate design procedure which uses the moment magnifier concept to account for slenderness effects. Moments computed using an ordinary first-order frame analysis are multiplied by a "moment magnifier" which is a function of the factored axial load P_u and the critical buckling load P_c for the column. Nonsway and sway frames are treated separately in 10.12 and 10.13. Provisions applicable to both non-sway and sway columns are given in 10.11. A first-order frame analysis is an elastic analysis which does not include the internal force effects resulting from deflections.

R10.11.1 — The stiffnesses EI used in an elastic analysis used for strength design should represent the stiffnesses of the members immediately prior to failure. This is particularly true for a second-order analysis which should predict the lateral deflections at loads approaching ultimate. The EI values should not be based totally on the moment-curvature relationship for the most highly loaded section along the length of each member. Instead, they should correspond to the moment-end rotation relationship for a complete member.

The alternative values of E, I, and A given in 10.11.1 have been chosen from the results of frame tests and analyses and include an allowance for the variability of the computed deflections. The modulus of elasticity E is based on the specified concrete strength while the sway deflections are a function of the average concrete strength which is higher. The moments of inertia were taken as $^7/_8$ of those in Reference 10.25. These two effects result in an overestimation of the second-order deflections in the order of 20 to 25 percent, corresponding to an implicit stiffness reduction factor ϕ_K of

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The moments of inertia shall be divided by $(1 + \beta_d)$

(a) When sustained lateral loads act, or

(b) For stability checks made in accordance with 10.13.6.

10.11.2 — It shall be permitted to take the radius of gyration r equal to 0.30 times the overall dimension in the direction stability is being considered for rectangular compression members and 0.25 times the diameter for circular compression members. For other shapes, it shall be permitted to compute the radius of gyration for the gross concrete section.

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0.80 to 0.85 on the stability calculation. The concept of a stiffness reduction factor ϕ_K is discussed in R10.12.3

The moment of inertia of T-beams should be based on the effective flange width defined in 8.10. It is generally sufficiently accurate to take I_g of a T-beam as two times the I_g for the web, $2(b_w h^3/12)$.

If the factored moments and shears from an analysis based on the moment of inertia of a wall taken equal to $0.70I_g$ indicate that the wall will crack in flexure, based on the modulus of rupture, the analysis should be repeated with $I = 0.35I_g$ in those stories where cracking is predicted at factored loads.

The alternative values of the moments of inertia given in 10.11.1 were derived for nonprestressed members. For prestressed members, the moments of inertia may differ from the values in 10.11.1 depending on the amount, location, and type of the reinforcement and the degree of cracking prior to ultimate. The stiffness values for prestressed concrete members should include an allowance for the variability of the stiffnesses.

Sections 10.11 through 10.13 provide requirements for strength and assume frame analyses will be carried out using factored loads. Analyses of deflections, vibrations, and building periods are needed at various service (unfactored) load levels[10.26,10.27] to determine the serviceability of the structure and to estimate the wind forces in wind tunnel laboratories. The seismic base shear is also based on the service load periods of vibration. The magnified service loads and deflections by a second-order analysis should also be computed using service loads. The moments of inertia of the structural members in the service load analyses should, therefore, be representative of the degree of cracking at the various service load levels investigated. Unless a more accurate estimate of the degree of cracking at design service load level is available, it is satisfactory to use **1/0.70 = 1.43** times the moments of inertia given in 10.11.1 for service load analyses.

The last sentence in 10.11.1 refers to the unusual case of sustained lateral loads. Such a case might exist, for example, if there were permanent lateral loads resulting from unequal earth pressures on two sides of a building.

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10.11.3 — Unsupported length of compression members

10.11.3.1 — The unsupported length ℓ_u of a compression member shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered.

10.11.3.2 — Where column capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

10.11.4 — Columns and stories in structures shall be designated as non-sway or sway columns or stories. The design of columns in non-sway frames or stories shall be based on 10.12. The design of columns in sway frames or stories shall be based on 10.13.

10.11.4.1 — It shall be permitted to assume a column in a structure is non-sway if the increase in column end moments due to second-order effects does not exceed 5 percent of the first-order end moments.

10.11.4.2 — It also shall be permitted to assume a story within a structure is non-sway if:

$$Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} \qquad (10\text{-}7)$$

is less than or equal to 0.05, where ΣP_u and V_u are the total vertical load and the story shear, respectively, in the story in question and Δ_o is the first-order relative deflection between the top and bottom of that story due to V_u.

10.11.5 — Where an individual compression member in the frame has a slenderness $k\ell_u/r$ of more than 100, 10.10.1 shall be used to compute the forces and moments in the frame.

10.11.6 — For compression members subject to bending about both principal axes, the moment about each axis shall be magnified separately based on the conditions of restraint corresponding to that axis.

COMMENTARY

Non Sway | Sway

Col $\frac{M_{2nd} - M_{1st}}{M_{1st}} \le 0.05$ > 0.05

Story $Q = \frac{\Sigma P_u \Delta_o}{V_c \ell_c} \le 0.05$ > 0.05

R10.11.4 — The moment magnifier design method requires the designer to distinguish between non-sway frames which are designed according to 10.12 and sway frames which are designed according to 10.13. Frequently this can be done by inspection by comparing the total lateral stiffness of the columns in a story to that of the bracing elements. A compression member may be assumed braced by inspection if it is located in a story in which the bracing elements (shearwalls, shear trusses, or other types of lateral bracing) have such substantial lateral stiffness to resist the lateral deflections of the story that any resulting lateral deflection is not large enough to affect the column strength substantially. If not readily apparent by inspection, 10.11.4.1 and 10.11.4.2 give two possible ways of doing this. In 10.11.4.1, a story in a frame is said to be non-sway if the increase in the lateral load moments resulting from $P\Delta$ effects does not exceed 5 percent of the first-order moments.[10.25] Section 10.11.4.2 gives an alternative method of determining this based on the stability index for a story Q. In computing Q, ΣP_u should correspond to the lateral loading case for which ΣP_u is greatest. It should be noted that a frame may contain both non-sway and sway stories. This test would not be suitable if V_u were zero.

If the lateral load deflections of the frame have been computed using service loads and the service load moments of inertia given in 10.11.1, it is permissible to compute Q in Eq. (10-7) using 1.2 times the sum of the service gravity loads, the service load story shear, and 1.43 times the first-order service load story deflections.

R10.11.5 — An upper limit is imposed on the slenderness ratio of columns designed by the moment magnifier method of 10.11 to 10.13. No similar limit is imposed if design is carried out according to 10.10.1. The limit of $k\ell_u/r = 100$ represents the upper range of actual tests of slender compression members in frames.

R10.11.6 — When biaxial bending occurs in a compression member, the computed moments about each of the principal axes must be magnified. The magnification factors δ are computed considering the buckling load P_c about each axis

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separately based on the appropriate effective length $k\ell_u$ and the stiffness EI. If the buckling capacities are different about the two axes different magnification factors will result.

10.12 — Magnified moments — Non-sway frames

10.12.1 — For compression members in non-sway frames, the effective length factor k shall be taken as 1.0, unless analysis shows that a lower value is justified. The calculation of k shall be based on the E and I values used in 10.11.1.

R10.12 — Magnified moments — Non-sway frames

R10.12.1 — The moment magnifier equations were derived for hinged end columns and must be modified to account for the effect of end restraints. This is done by using an "effective length" $k\ell_u$ in the computation of P_c.

The primary design aid to estimate the effective length factor k is the Jackson and Moreland Alignment Charts (Fig. R10.12.1) which allow a graphical determination of k for a column of constant cross section in a multibay frame.[10.28,10.29]

The effective length is a function of the relative stiffness at each end of the compression member. Studies have indicated that the effects of varying beam and column reinforcement percentages and beam cracking should be considered in determining the relative end stiffnesses. In determining ψ for use in evaluating the effective length factor k, the rigidity of the flexural members may be calculated on the basis of $0.35I_g$ for flexural members to account for the effect of cracking and reinforcement on relative stiffness, and $0.70I_g$ for compression members.

The following simplified equations for computing the effective length factors for braced and unbraced members may be used. Eq. (A), (B), and (E) are taken from the 1972 British Standard Code of Practice.[10.30,10.31] Eq. (C) and (D) for unbraced members were developed in Reference 10.29.

For braced compression members, an upper bound to the effective length factor may be taken as the smaller of the following two expressions:

$$k = 0.7 + 0.05\,(\psi_A + \psi_B) \le 1.0 \qquad \text{(A)}$$

$$k = 0.85 + 0.05\psi_{min} \le 1.0 \qquad \text{(B)}$$

where ψ_A and ψ_B are the values of ψ at the two ends of the column and ψ_{min} is the smaller of the two values.

For unbraced compression members restrained at both ends, the effective length factor may be taken as:

For $\psi_m < 2$

$$k = \frac{20 - \psi_m}{20}\sqrt{1 + \psi_m} \qquad \text{(C)}$$

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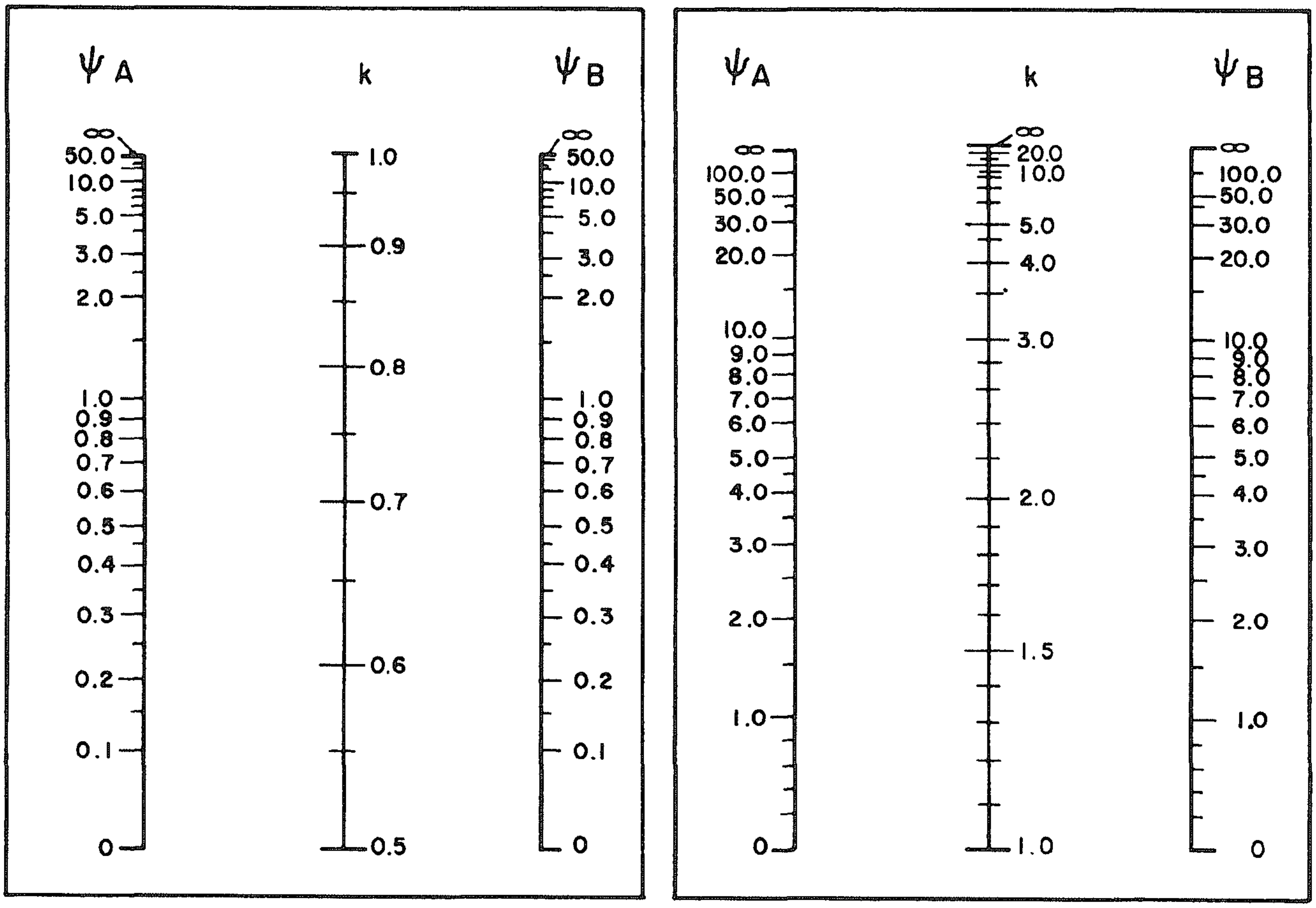

(a)

Braced Frames

(b)

Unbraced Frames

ψ = ratio of $\Sigma(EI/\ell_c)$ of compression members to $\Sigma(EI/\ell)$ of flexural members in a plane at one end of a compression member

ℓ = span length of flexural member measured center-to-center of joints

Fig. R10.12.1—Effective length factors, k

For $\psi_m \geq 2$

$$k = 0.9\sqrt{1+\psi_m} \tag{D}$$

where ψ_m is the average of the ψ-values at the two ends of the compression member.

For unbraced compression members hinged at one end, the effective length factor may be taken as:

$$k = 2.0 + 0.3\psi \tag{E}$$

where ψ is the value at the restrained end.

The use of the charts in Fig. R10.12.1, or the equations in this section, may be considered as satisfying the requirements of the code to justify $\boldsymbol{k}$ less than 1.0.

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10.12.2 — In non-sway frames it shall be permitted to ignore slenderness effects for compression members which satisfy:

$$\frac{k\ell_u}{r} \leq 34 - 12\,(M_1/M_2) \qquad (10\text{-}8)$$

where M_1/M_2 is not taken less than −0.5. The term M_1/M_2 is positive if the column is bent in single curvature.

10.12.3 — Compression members shall be designed for the factored axial load P_u and the moment amplified for the effects of member curvature M_c as follows:

$$M_c = \delta_{ns} M_2 \qquad (10\text{-}9)$$

where

δns - non sway Magnifier

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0 \qquad (10\text{-}10)$$

Euler's Buckling Eq.

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad (10\text{-}11)$$

EI shall be taken as

$$EI = \frac{(0.2E_cI_g + E_sI_{se})}{1 + \beta_d} \qquad (10\text{-}12)$$

or

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \qquad (10\text{-}13)$$

Optional 10-12 OR 10-13

$$\beta d = \frac{\text{Max Fact' Axial DL}}{\text{Total Fact Axial LOAD}}$$

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R10.12.2 — Eq. (10-8) is derived from Eq. (10-10) assuming that a 5 percent increase in moments due to slenderness is acceptable.[10.23] The derivation did not include ϕ in the calculation of the moment magnifier. As a first approximation, k may be taken equal to 1.0 in Eq. (10-8).

R10.12.3 — The ϕ-factors used in the design of slender columns represent two different sources of variability. First, the stiffness reduction ϕ-factors in the magnifier equations in the 1989 and earlier codes were intended to account for the variability in the stiffness EI and the moment magnification analysis. Second, the variability of the strength of the cross section is accounted for by strength reduction ϕ-factors of 0.70 for tied columns and 0.75 for spiral columns. Studies reported in Reference 10.32 indicate that the stiffness reduction factor ϕ_K, and the cross-sectional strength reduction ϕ-factors do not have the same values, contrary to the assumption in the 1989 and earlier codes. These studies suggest the stiffness reduction factor ϕ_K for an isolated column should be 0.75 for both tied and spiral columns. The 0.75 factors in Eq. (10-10) and (10-19) are stiffness reduction factors ϕ_K and replace the ϕ-factors in these equations in the 1989 and earlier codes. This has been done to avoid confusion between a stiffness reduction factor ϕ_K in Eq. (10-10) and (10-19), and the cross-sectional strength reduction ϕ-factors.

In defining the critical load, the main problem is the choice of a stiffness EI which reasonably approximates the variations in stiffness due to cracking, creep, and the nonlinearity of the concrete stress-strain curve. Eq. (10-12) was derived for small eccentricity ratios and high levels of axial load where the slenderness effects are most pronounced.

Creep due to sustained load will increase the lateral deflections of a column and hence the moment magnification. This is approximated for design by reducing the stiffness EI used to compute P_c and hence δ_{ns} by dividing EI by $(1 + \beta_d)$. Both the concrete and steel terms in Eq. (10-12) are divided by $(1 + \beta_d)$. This reflects the premature yielding of steel in columns subjected to sustained load.

Either Eq. (10-12) or (10-13) may be used to compute EI. Eq. (10-13) is a simplified approximation to Eq. (10-12). It is less accurate than Eq. (10-12).[10.33] Eq. (10-13) may be simplified further by assuming $\beta_d = 0.6$. When this is done Eq. (10-13) becomes

$$EI = 0.25E_cI_g \qquad \text{(F)}$$

The term β_d is defined differently for non-sway and sway

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frames. See 10.0. For non-sway frames, β_d is the ratio of the maximum factored axial dead load to the total factored axial load.

10.12.3.1 — For members without transverse loads between supports, C_m shall be taken as

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \quad (10\text{-}14)$$

where M_1/M_2 is positive if the column is bent in single curvature. For members with transverse loads between supports, C_m shall be taken as 1.0.

R10.12.3.1 — The factor C_m is an equivalent moment correction factor. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum moment occurs at one end of the column, design must be based on an "equivalent uniform moment" C_mM_2 which would lead to the same maximum moment when magnified.[10.23]

In the case of compression members that are subjected to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of M_2 in Eq. (10-9). In accordance with the last sentence of 10.12.3.1, C_m must be taken as 1.0 for this case.

10.12.3.2 — The factored moment M_2 in Eq. (10-9) shall not be taken less than

$$M_{2,min} = P_u\,(0.6 + 0.03h) \quad (10\text{-}15)$$

h is not height

about each axis separately, where 0.6 and h are in inches. For members for which $M_{2,min}$ exceeds M_2, the value of C_m in Eq. (10-14) shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moments M_1 and M_2.

R10.12.3.2 — In this code, slenderness is accounted for by magnifying the column end moments. If the factored column moments are very small or zero, the design of slender columns must be based on the minimum eccentricity given in this section. It is not intended that the minimum eccentricity be applied about both axes simultaneously.

The factored column end moments from the structural analysis are used in Eq. (10-14) in determining the ratio M_1/M_2 for the column when the design must be based on minimum eccentricity. This eliminates what would otherwise be a discontinuity between columns with computed eccentricities less than the minimum eccentricity and columns with computed eccentricities equal to or greater than the minimum eccentricity.

10.13 — Magnified moments — Sway frames

R10.13 — Magnified moments — Sway frames

The design of sway frames for slenderness has been revised in the 1995 ACI Building Code. The revised procedure consists of three steps:

(1) The magnified sway moments δ_sM_s are computed. This should be done in one of three ways. First, a second-order elastic frame analysis may be used (10.13.4.1). Second, an approximation to such analysis (10.13.4.2) may be used. The third option is to use the sway magnifier δ_s from previous editions of the ACI Building Code (10.13.4.3).

(2) The magnified sway moments δ_sM_s are added to the unmagnified non-sway moment M_{ns} at each end of each column (10.13.3). The non-sway moments may be computed using a first-order elastic analysis.

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(3) If the column is slender and the loads on it are high, it is checked to see whether the moments at points between the ends of the column exceed those at the ends of the column. As specified in 10.13.5 this is done using the non-sway frame magnifier δ_{ns} with P_c computed assuming k = **1.0** or less.

10.13.1 — For compression members not braced against sidesway, the effective length factor k shall be determined using E and I values in accordance with 10.11.1 and shall be greater than 1.0.

R10.13.1 — See R10.12.1.

10.13.2 — For compression members not braced against sidesway, effects of slenderness may be neglected when $k\ell_u/r$ is less than 22.

10.13.3 — The moments M_1 and M_2 at the ends of an individual compression member shall be taken as

$$M_1 = M_{1ns} + \delta_s M_{1s} \quad (10\text{-}16)$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \quad (10\text{-}17)$$

where $\delta_s M_{1s}$ and $\delta_s M_{2s}$ shall be computed according to 10.13.4.

R10.13.3 — The analysis described in this section deals only with plane frames subjected to loads causing deflections in that plane. If torsional displacements are significant, a three-dimensional second-order analysis should be used.

10.13.4 — Calculation of $\delta_s M_s$

10.13.4.1 — The magnified sway moments $\delta_s M_s$ shall be taken as the column end moments calculated using a second-order elastic analysis based on the member stiffnesses given in 10.11.1.

10.13.4 — Calculation of $\delta_s M_s$

R10.13.4.1 — A second-order analysis is a frame analysis which includes the internal force effects resulting from deflections. When a second-order elastic analysis is used to compute $\delta_s M_s$ the deflections must be representative of the stage immediately prior to the ultimate load. For this reason the reduced EI values given in 10.11.1 must be used in the second-order analysis.

The term β_d is defined differently for non-sway and sway frames. See 10.0. Sway deflections due to short-term loads such as wind or earthquake are a function of the short-term stiffness of the columns following a period of sustained gravity load. For this case the definition of β_d in 10.0 gives β_d = **0**. In the unusual case of a sway frame where the lateral loads are sustained, β_d will not be zero. This might occur if a building on a sloping site is subjected to earth pressure on one side but not on the other.

In a second-order analysis the axial loads in all columns which are not part of the lateral load resisting elements and depend on these elements for stability must be included.

In the 1989 and earlier codes, the moment magnifier equations for δ_b and δ_s included a stiffness reduction factor ϕ_K to cover the variability in the stability calculation. The second-order analysis method is based on the values of E and I

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from 10.11.1. These lead to a 20 to 25 percent overestimation of the lateral deflections which corresponds to a stiffness reduction factor ϕ_K between 0.80 and 0.85 on the $P\Delta$ moments. No additional ϕ-factor is needed in the stability calculation. Once the moments are established, selection of the cross sections of the columns involves the strength reduction factors ϕ from 9.3.2.2.

10.13.4.2 — Alternatively it shall be permitted to calculate $\delta_s M_s$ as

$$\delta_s M_s = \frac{M_s}{1-Q} \geq M_s \qquad (10\text{-}18)$$

Q = Stability Index

If δ_s calculated in this way exceeds 1.5, $\delta_s M_s$ shall be calculated using 10.13.4.1 or 10.13.4.3.

R10.13.4.2 — The iterative $P\Delta$ analysis for second-order moments can be represented by an infinite series. The solution of this series is given by Eq. (10-18).[10.25] Reference 10.34 shows that Eq. (10-18) closely predicts the second-order moments in an unbraced frame until δ_s exceeds 1.5.

The $P\Delta$ moment diagrams for deflected columns are curved, with Δ related to the deflected shape of the columns. Eq. (10-18) and most commercially available second-order frame analyses have been derived assuming that the $P\Delta$ moments result from equal and opposite forces of $P\Delta/\ell_c$ applied at the bottom and top of the story. These forces give a straight line $P\Delta$ moment diagram. The curved $P\Delta$ moment diagrams lead to lateral displacements in the order of 15 percent larger than those from the straight line $P\Delta$ moment diagrams. This effect can be included in Eq. (10-18) by writing the denominator as **(1–1.15*Q*)** rather than **(1–*Q*)**. The 1.15 factor has been left out of Eq. (10-18) to maintain consistency with commercially available computer programs.

If deflections have been calculated using service loads, Q in Eq. (10-18) should be calculated in the manner explained in R10.11.4.

In the 1989 and earlier codes, the moment magnifier equations for δ_b and δ_s included a stiffness reduction factor ϕ_K to cover the variability in the stability calculation. The Q factor analysis is based on deflections calculated using the values of E and I from 10.11.1 which include the equivalent of a stiffness reduction factor ϕ_K as explained in R10.13.4.1. As a result, no additional ϕ-factor is needed in the stability calculation. Once the moments are established using Eq. (10-18), selection of the cross sections of the columns involves the strength reduction factors ϕ from 9.3.2.2.

10.13.4.3 — Alternatively it shall be permitted to calculate the magnified sway moment $\delta_s M_s$ as

$$\delta_s M_s = \frac{M_s}{1 - \dfrac{\Sigma P_u}{0.75\Sigma P_c}} \geq M_s \qquad (10\text{-}19)$$

where ΣP_u is the summation for all the vertical loads in a story and ΣP_c is the summation for all sway resisting columns in a story. P_c is calculated using Eq. (10-11) using k from 10.13.1 and EI from Eq. (10-12) or Eq. (10-13).

R10.13.4.3 — To check the effects of story stability, δ_s is computed as an averaged value for the entire story based on use of $\Sigma P_u / \Sigma P_c$. This reflects the interaction of all sway resisting columns in the story in the $P\Delta$ effects since the lateral deflection of all columns in the story must be equal in the absence of torsional displacements about a vertical axis. In addition, it is possible that a particularly slender individual column in an unbraced frame could have substantial midheight deflections even if adequately braced against lateral end deflections by other columns in the story. Such a column will have ℓ_u/r greater than the value given in Eq. (10-20) and would have to be checked using 10.13.5.

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If the lateral load deflections involve a significant torsional displacement, the moment magnification in the columns farthest from the center of twist may be underestimated by the moment magnifier procedure. In such cases a three-dimensional second-order analysis should be considered.

The 0.75 in the denominator of Eq. (10-19) is a stiffness reduction factor ϕ_K as explained in R10.12.3.

In the calculation of EI, β_d will normally be zero for an unbraced frame because the lateral loads are generally of short duration. (See R10.13.4.1).

10.13.5 — If an individual compression member has

$$\frac{\ell_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f_c' A_g}}} \qquad (10\text{-}20)$$

it shall be designed for the factored axial load P_u and the moment M_c calculated using 10.12.3 in which M_1 and M_2 are computed in accordance with 10.13.3, β_d as defined for the load combination under consideration, and k as defined in 10.12.1.

R10.13.5 — The unmagnified non-sway moments at the ends of the columns are added to the magnified sway moments at the same points. Generally one of the resulting end moments is the maximum moment in the column. However, for slender columns with high axial loads the point of maximum moment may be between the ends of the column so that the end moments are no longer the maximum moments. If ℓ_u/r is less than the value given by Eq. (10-20) the maximum moment at any point along the height of such a column will be less than 1.05 times the maximum end moment. When ℓ_u/r exceeds the value given by Eq. (10-20), the maximum moment will occur at a point between the ends of the column and will exceed the maximum end moment by more than 5 percent.[10.22] In such a case the maximum moment is calculated by magnifying the end moments using Eq. (10-9).

10.13.6 — In addition to load cases involving lateral loads, the strength and stability of the structure as a whole under factored gravity loads shall be considered.

(a) When $\delta_s M_s$ is computed from 10.13.4.1, the ratio of second-order lateral deflections to first-order lateral deflections for 1.4 dead load and 1.7 live load plus lateral load applied to the structure shall not exceed 2.5.

(b) When $\delta_s M_s$ is computed according to 10.13.4.2, the value of Q computed using ΣP_u for 1.4 dead load plus 1.7 live load shall not exceed 0.60.

(c) When $\delta_s M_s$ is computed from 10.13.4.3, δ_s computed using ΣP_u and ΣP_c corresponding to the factored dead and live loads shall be positive and shall not exceed 2.5.

In cases (a), (b), and (c) above, β_d shall be taken as the ratio of the maximum factored sustained axial load to the total factored axial load.

R10.13.6 — The possibility of sidesway instability under gravity loads alone must be investigated. When using second-order analyses to compile $\delta_s M_s$ (10.13.4.1), the frame should be analyzed twice for the case of factored gravity loads plus a lateral load applied to the frame. This load may be the lateral load used in design or it may be a single lateral load applied to the top of the frame. The first analysis should be a first-order analysis, the second analysis should be a second-order analysis. The deflection from the second-order analysis should not exceed 2.5 times the deflection from the first-order analysis. If one story is much more flexible than the others the deflection ratio should be computed in that story. The lateral load should be large enough to give deflections of size that can be compared accurately. In unsymmetrical frames which deflect laterally under gravity loads alone, the lateral load should act in the direction for which it will increase the lateral deflections.

When using 10.13.4.2 to compute $\delta_s M_s$, the value of Q evaluated using factored gravity loads should not exceed 0.60. This is equivalent to $\delta_s = 2.5$. The values of V_u and Δ_o used to compute Q can result from assuming any real or arbitrary set of lateral loads provided that V_u and Δ_o are both from the same loading. If Q as computed in 10.11.4.2 is 0.2 or less, the stability check in 10.13.6 is satisfied.

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When $\delta_s M_s$ is computed using Eq. (10-19), an upper limit of 2.5 is placed on δ_s. For higher δ_s values the frame will be very susceptible to variations in EI, foundation rotations, and the like. If δ_s exceeds 2.5 the frame must be stiffened to reduce δ_s. ΣP_u shall include the axial load in all columns and walls including columns which are not part of the lateral load resisting system. The value $\delta_s = 2.5$ is a very high magnifier. It has been chosen to offset the conservatism inherent in the moment magnifier procedure.

The value of β_d should be an overall value for each story calculated as the ratio of the maximum factored sustained axial load in that story to the total factored axial load in that story.

10.13.7 — In sway frames, flexural members shall be designed for the total magnified end moments of the compression members at the joint.

R10.13.7 — The strength of a laterally unbraced frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beam, the structure approaches a mechanism and its axial load capacity is drastically reduced. Section 10.13.7 provides that the designer make certain that the restraining flexural members have the capacity to resist the magnified column moments.

10.14 — Axially loaded members supporting slab system

Axially loaded members supporting a slab system included within the scope of 13.1 shall be designed as provided in Chapter 10 and in accordance with the additional requirements of Chapter 13.

10.15 — Transmission of column loads through floor system

When the specified compressive strength of concrete in a column is greater than 1.4 times that specified for a floor system, transmission of load through the floor system shall be provided by one of the following.

R10.15 — Transmission of column loads through floor system

The requirements of this section are based on a paper on the effect of floor concrete strength on column strength.[10.35] The provisions mean that where the column concrete strength does not exceed the floor concrete strength by more than 40 percent, no special precautions need be taken. For higher column concrete strengths, methods in 10.15.1 or 10.15.2 must be used for corner or edge columns and methods in 10.15.1, 10.15.2, or 10.15.3 for interior columns with adequate restraint on all four sides.

10.15.1 — Concrete of strength specified for the column shall be placed in the floor at the column location. Top surface of the column concrete shall extend 2 ft into the slab from face of column. Column concrete shall be well integrated with floor concrete, and shall be placed in accordance with 6.4.5 and 6.4.6.

R10.15.1 — Application of the concrete placement procedure described in 10.15.1 requires the placing of two different concrete mixes in the floor system. The lower strength mix must be placed while the higher strength concrete is still plastic and must be adequately vibrated to ensure the concretes are well integrated. This requires careful coordination of the concrete deliveries and possible use of retarders. In some cases, additional inspection services will be required when this procedure is used. It is important that the

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higher strength concrete in the floor in the region of the column be placed before the lower strength concrete in the remainder of the floor to prevent accidental placing of the low strength concrete in the column area. It is the designer's responsibility to indicate on the drawings where the high and low strength concretes are to be placed.

With the 1983 code, the amount of column concrete to be placed within the floor is expressed as a simple 2-ft extension from face of column. Since the concrete placement requirement must be carried out in the field, it is now expressed in a way that is directly evident to workers. The new requirement will also locate the interface between column and floor concrete farther out into the floor, away from regions of very high shear.

10.15.2 — Strength of a column through a floor system shall be based on the lower value of concrete strength with vertical dowels and spirals as required.

10.15.3 — For columns laterally supported on four sides by beams of approximately equal depth or by slabs, strength of the column may be based on an assumed concrete strength in the column joint equal to 75 percent of column concrete strength plus 35 percent of floor concrete strength.

10.16 — Composite compression members

R10.16 — Composite compression members

10.16.1 — Composite compression members shall include all such members reinforced longitudinally with structural steel shapes, pipe, or tubing with or without longitudinal bars.

R10.16.1 — Composite columns are defined without reference to classifications of combination, composite, or concrete-filled pipe column. Reference to other metals used for reinforcement has been omitted because they are seldom used with concrete in construction.

10.16.2 — Strength of a composite member shall be computed for the same limiting conditions applicable to ordinary reinforced concrete members.

R10.16.2 — The same rules used for computing the load-moment interaction strength for reinforced concrete sections can be applied to composite sections. Interaction charts for concrete-filled tubing would have a form identical to those of ACI SP-7[10.36] and the *Design Handbook*, V.2, Columns,[10.29] but with γ slightly greater than 1.0.

10.16.3 — Any axial load strength assigned to concrete of a composite member shall be transferred to the concrete by members or brackets in direct bearing on the composite member concrete.

10.16.4 — All axial load strength not assigned to concrete of a composite member shall be developed by direct connection to the structural steel shape, pipe, or tube.

R10.16.3 and R10.16.4 — Direct bearing or direct connection for transfer of forces between steel and concrete can be developed through lugs, plates, or reinforcing bars welded to the structural shape or tubing before the concrete is cast. Flexural compressive stress need not be considered a part of direct compression load to be developed by bearing. A concrete encasement around a structural steel shape may stiffen the shape, but it would not necessarily increase its strength.

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10.16.5 — For evaluation of slenderness effects, radius of gyration of a composite section shall be not greater than the value given by

$$r = \sqrt{\frac{(E_c I_g/5) + E_s I_t}{(E_c A_g/5) + E_s A_t}} \quad \text{(10-21)}$$

and, as an alternative to a more accurate calculation, ***EI*** in Eq. (10-11) shall be taken either as Eq. (10-12) or

$$EI = \frac{(E_c I_g/5)}{1 + \beta_d} + E_s I_t \quad \text{(10-22)}$$

10.16.6 — Structural steel encased concrete core

10.16.6.1 — For a composite member with concrete core encased by structural steel, thickness of the steel encasement shall be not less than

$$b\sqrt{\frac{f_y}{3E_s}} \text{ for each face of width } b$$

nor

$$h\sqrt{\frac{f_y}{8E_s}} \text{ for circular sections of diameter } h$$

10.16.6.2 — Longitudinal bars located within the encased concrete core shall be permitted to be used in computing A_t and I_t.

10.16.7 — Spiral reinforcement around structural steel core

A composite member with spirally reinforced concrete around a structural steel core shall conform to the following.

10.16.7.1 — Specified compressive strength of concrete f_c' shall be not less than 2500 psi.

10.16.7.2 — Design yield strength of structural steel core shall be the specified minimum yield strength for grade of structural steel used but not to exceed 50,000 psi.

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R10.16.5 — Eq. (10-21) is given because the rules of 10.11.2 for estimating the radius of gyration are overly conservative for concrete filled tubing and are not applicable for members with enclosed structural shapes.

In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the steel thus increasing the steel stresses. In the case of lightly reinforced columns, this load transfer may cause the compression steel to yield prematurely, resulting in a loss in the effective ***EI***. Accordingly, both the concrete and steel terms in Eq. (10-12) are reduced to account for creep. For heavily reinforced columns or for composite columns in which the pipe or structural shape makes up a large percentage of the cross section, the load transfer due to creep is not significant. Accordingly, Eq. (10-22) was revised in the 1980 code supplement so that only the ***EI*** of the concrete is reduced for sustained load effects.

R10.16.6 — Structural steel encased concrete core

Steel encased concrete sections should have a metal wall thickness large enough to attain longitudinal yield stress before buckling outward.

R10.16.7 — Spiral reinforcement around structural steel core

Concrete that is laterally contained by a spiral has increased load-carrying strength, and the size of spiral required can be regulated on the basis of the strength of the concrete outside the spiral by means of the same reasoning that applies for columns reinforced only with longitudinal bars. The radial pressure provided by the spiral ensures interaction between concrete, reinforcing bars, and steel core such that longitudinal bars will both stiffen and strengthen the cross section.

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10.16.7.3 — Spiral reinforcement shall conform to 10.9.3.

10.16.7.4 — Longitudinal bars located within the spiral shall be not less than 0.01 nor more than 0.08 times net area of concrete section.

10.16.7.5 — Longitudinal bars located within the spiral shall be permitted to be used in computing A_t and I_t.

10.16.8 — Tie reinforcement around structural steel core

A composite member with laterally tied concrete around a structural steel core shall conform to the following.

10.16.8.1 — Specified compressive strength of concrete f_c' shall be not less than 2500 psi.

10.16.8.2 — Design yield strength of structural steel core shall be the specified minimum yield strength for grade of structural steel used but not to exceed 50,000 psi.

10.16.8.3 — Lateral ties shall extend completely around the structural steel core.

10.16.8.4 — Lateral ties shall have a diameter not less than $^1/_{50}$ times the greatest side dimension of composite member, except that ties shall not be smaller than No. 3 and are not required to be larger than No. 5. Welded wire fabric of equivalent area shall be permitted.

10.16.8.5 — Vertical spacing of lateral ties shall not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or $^1/_2$ times the least side dimension of the composite member.

10.16.8.6 — Longitudinal bars located within the ties shall be not less than 0.01 nor more than 0.08 times net area of concrete section.

10.16.8.7 — A longitudinal bar shall be located at every corner of a rectangular cross section, with other longitudinal bars spaced not farther apart than one-half the least side dimension of the composite member.

10.16.8.8 — Longitudinal bars located within the ties shall be permitted to be used in computing A_t for strength but not in computing I_t for evaluation of slenderness effects.

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R10.16.8 — Tie reinforcement around structural steel core

Concrete that is laterally contained by tie bars is likely to be rather thin along at least one face of a steel core section, and complete interaction between the core, the concrete, and any longitudinal reinforcement should not be assumed. Concrete will probably separate from smooth faces of the steel core. To maintain the concrete around the structural steel core, it is reasonable to require more lateral ties than needed for ordinary reinforced concrete columns. Because of probable separation at high strains between the steel core and the concrete, longitudinal bars will be ineffective in stiffening cross sections even though they would be useful in sustaining compression forces. Finally, the yield strength of the steel core should be limited to that which exists at strains below those that can be sustained without spalling of the concrete. It has been assumed that axially-compressed concrete will not spall at strains less than 0.0018. The yield strength of 0.0018 x 29,000,000, or 52,000 psi, represents an upper limit of the useful maximum steel stress.

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10.17 — Bearing strength

10.17.1 — Design bearing strength on concrete shall not exceed ϕ **(0.85$f_c'A_1$)**, except when the supporting surface is wider on all sides than the loaded area, design bearing strength on the loaded area shall be permitted to be multiplied by $\sqrt{A_2/A_1}$ but not more than 2.

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R10.17 — Bearing strength

R10.17.1 — This section deals with bearing strength on concrete supports. The permissible bearing stress of **0.85f_c'** is based on tests reported in Reference 10.37. (See also 15.8).

When the supporting area is wider than the loaded area on all sides, the surrounding concrete confines the bearing area, resulting in an increase in bearing strength. No minimum depth is given for a supporting member. The minimum depth of support will be controlled by the shear requirements of 11.11.

When the top of the support is sloped or stepped, advantage may still be taken of the condition that the supporting member is larger than the loaded area, provided the supporting member does not slope at too great an angle. Fig. R10.17 illustrates the application of the frustum to find A_2. The frustum should not be confused with the path by which a

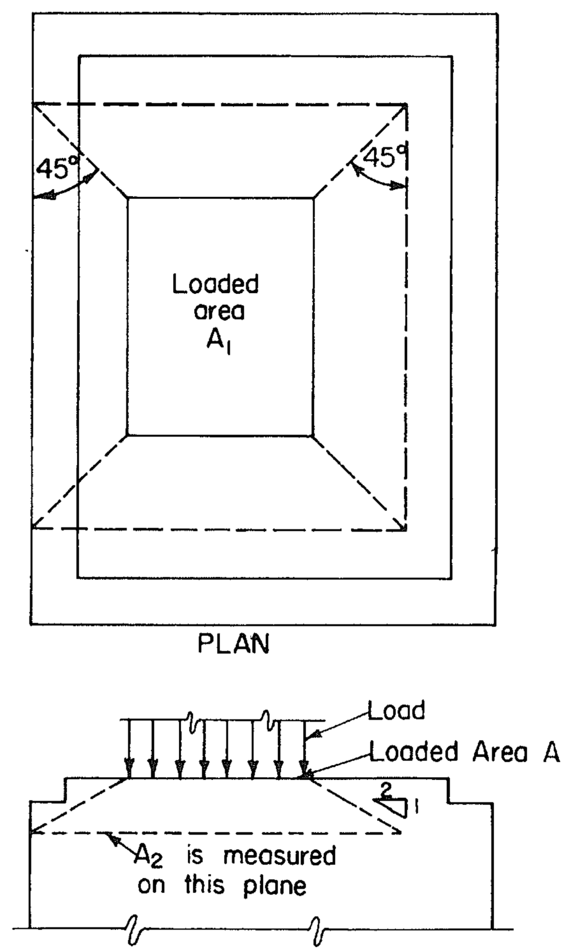

Fig. R10.17—Application of frustum to find A_2 *in stepped or sloped supports*

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load spreads out as it travels downward through the support. Such a load path would have steeper sides. However, the frustum described has somewhat flat side slopes to ensure that there is concrete immediately surrounding the zone of high stress at the bearing. A_1 is the loaded area but not greater than the bearing plate or bearing cross-sectional area.

10.17.2 — Section 10.17 does not apply to post-tensioning anchorages.

R10.17.2 — Post-tensioning anchorages are normally laterally reinforced, in accordance with 18.13.