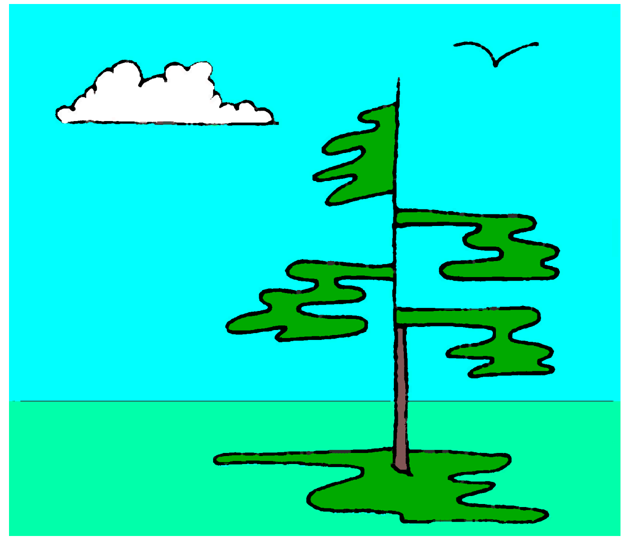
NorthWoods Software



Program Name: Steel-Bollard

Project Number: -

Project Description: -

Project Designer: Dik

Last Revised (yy-mm-dd): 21.10.19

Reference: HSS Bollards, Jeff Packer, UofT, CSA S16

Disclaimer:

Created using SMath Studio, a MathCAD workalike from https://en.smath.info/view/SMathStudio. The User is responsible to verify data and calculations using an alternative method

Menu:

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.... Sum / For	Red Important Note	Gray Temporary Variables	

Defined Units:

$grav := 9.80665 \frac{m}{sec^2}$ — Acceleration

$K := kip$ — Force

$K\_ft := K\ ft$ $\quad kN\_m := kN\ m$ $\quad K\_in := K\ in$ $\quad kN\_mm := kN\ mm$ $\quad lb\_in := lbf\ in$ — Moment

$kNmpm := \frac{kN\_m}{m}$ $\quad iKpi := \frac{K\_in}{in}$ — Moment per Unit Length

$pcf := \frac{lbf}{ft^3}$ $\quad kNpcm := \frac{kN}{m^3}$ $\quad kgpcm := \frac{kg}{m^3}$ — Density

$Klf := \frac{K}{ft}$ $\quad plf := \frac{lbf}{ft}$ $\quad kNpm := \frac{kN}{m}$ $\quad Kpi := \frac{K}{in}$ $\quad kNpmm := \frac{kN}{mm}$ — Force/Unit Length

$psf := \frac{lbf}{ft^2}$ $\quad Ksf := \frac{K}{ft^2}$ $\quad Ksi := \frac{K}{in^2}$ $\quad kNpsm := \frac{kN}{m^2}$ $\quad psi := \frac{lbf}{in^2}$ — Pressure

$Npsmm := \frac{N}{mm^2}$ — Pressure

$pci := \frac{lbf}{in^3}$ — Subgrade Modulus

$psfpf := \frac{psf}{ft}$ $\quad kPapm := \frac{kPa}{m}$ — Pressure per Depth

$pmcf := \frac{lb}{ft^3}$ $\quad lb := lbf$ — Force

$mph := \frac{mi}{hr}$ $\quad kph := \frac{km}{hr}$ $\quad mps := \frac{m}{sec}$ — Velocity/Speed

$ispf := \frac{in^2}{ft}$ $\quad mmspm := \frac{mm^2}{m}$ — Area per Unit Length

$ppf := \frac{lbf}{ft}$ $\quad Npm := \frac{N}{m}$ — Stiffness

c b a

Input Data:

Material Property Factors:

$\varphi_s := 0.90$ Rolled Sections $\varphi_s = 0.90$

$\varphi_c := 0.65$ Rolled Sections $\varphi_c = 0.65$

Load Factors:

$\alpha_D := 1.25$ $\alpha_D = 1.25$

$\alpha_L := 1.5$ $\alpha_L = 1.50$

$Check\left(\alpha_D \ge 1.25\right) = \text{"...OK"}$

$Check\left(\alpha_L \ge 1.5\right) = \text{"...OK"}$

Steel Properties:

Rail Properties $stl_{NDX} := 1$

NDX	des	fy	Fu
1	"G40.21-350W"	50 Ksi	65 Ksi
2	"G40.21-300W"	44 Ksi	65 Ksi
3	"A36"	36 Ksi	58 Ksi
4	"A53 Gr B"	33 Ksi	60 Ksi

$stl :=$ (matrix above)

$desM_1 := stl_{stl_{NDX}\ 2}$ $f_{y1} := stl_{stl_{NDX}\ 3}$ $\nu := 0.3$

$F_{u1} := stl_{stl_{NDX}\ 4}$ $E_s := 29000\ \text{Ksi}$ $G_s := \dfrac{E_s}{2 \cdot (1+\nu)}$

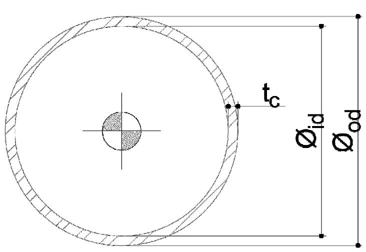
$\gamma_s := 489\ \text{pcf}$

Section Properties-Bollard: $ci_{NDX} := 1$

NDX	desI	esM	d	t
1	"HSS 8.625x0.313"	"HSS 219x8.0"	8.625 in	0.313 in

$ci :=$ (row above) $desI_c := ci_{ci_{NDX}\ 2}$

$desM_c := ci_{ci_{NDX}\ 3}$ $\phi_{od} := ci_{ci_{NDX}\ 4}$ $t_c := ci_{ci_{NDX}\ 5}$



$desI_c = \text{"HSS 8.625x0.313"}$ Imperial Designation

$desM_c = \text{"HSS 219x8.0"}$ Metric Designation

$\phi_{od} = 8.63\ \text{in}$ $\phi_{od} = 219.1\ \text{mm}$ Bollard Outer ϕ

$t_c = 0.31\ \text{in}$ $t_c = 8\ \text{mm}$ Bollard Wall Thicknesss

$\phi_{id} := \phi_{od} - 2 \cdot t_c$ $\phi_{id} = 7.999\ \text{in}$ $\phi_{id} = 203.2\ \text{mm}$ Bollard Inner ϕ

$A_c := \pi \cdot \dfrac{\left(\phi_{od}^{\,2} - \phi_{id}^{\,2}\right)}{4}$ $A_c = 8.1733\ \text{in}^2$ $A_c = 5273\ \text{mm}^2$ HSSc Area

$Sx_c := \dfrac{\pi \cdot \left(\phi_{od}^{\,4} - \phi_{id}^{\,4}\right)}{32 \cdot \phi_{od}}$ $Sx_c = 16.3911\ \text{in}^3$ $Sx_c = 3 \cdot 10^5\ \text{mm}^3$ HSSc Section Modulus

$Zx_c := \dfrac{\left(\phi_{od}^{\,3} - \phi_{id}^{\,3}\right)}{6}$ $Zx_c = 21.6352\ \text{in}^3$ $Zx_c = 4 \cdot 10^5\ \text{mm}^3$ Pipe Plastic Section

Section Class:

$$\text{if } \frac{\phi_{od}}{t_c} \le \frac{13000}{\frac{f_{y1}}{MPa}}: \quad Class := 1$$
$$\text{else if } \frac{\phi_{od}}{t_c} \le \frac{18000}{\frac{f_{y1}}{MPa}}: \quad Class := 2$$
$$\text{else if } \frac{\phi_{od}}{t_c} \le \frac{66000}{\frac{f_{y1}}{MPa}}: \quad Class := 3$$
$$\text{else}: \quad Class := 4$$

$Class = 1$ — Class of Section

$$\text{if } Class \le 2: \quad M_r := \varphi_s \cdot f_{y1} \cdot Zx_c$$
$$\text{else}: \quad M_r := \varphi_s \cdot f_{y1} \cdot Sx_c$$

$M_r = 81.1$ K_ft — $M_r = 110.0$ kN_m — Moment Resistance

Concrete Properties:

Concrete Strength: $co_{NDX} := 5$

$co :=$

NDX	f'c
1	20 MPa
2	25 MPa
3	30 MPa
4	35 MPa
5	40 MPa
6	45 MPa
7	50 MPa

$f'_c := co_{co_{NDX}\,2}$

$f'_c = 5.80$ Ksi — $f'_c = 40.00$ MPa — Initial Concrete Strength

Concrete Strength Override

$fc' := 6$ Ksi — $fc' = 6$ Ksi — $fc' = 41$ MPa — Concrete Strength Override

$$\text{if } fc' = 0 \text{ Ksi}: \quad f'_c := f'_c$$
$$\text{else}: \quad f'_c := fc'$$

$fc' = 6$ Ksi — $fc' = 41$ MPa — Design Concrete Strength

Vehicle Data:

Type: $ve_{NDX} := 3$

$ve :=$

NDX	Class	min	max
1	"Small Car"	2200 lbm	3300 lbm
2	"Minivan"	3300 lbm	6600 lbm
3	"SUV"	4400 lbm	7700 lbm
4	"Mini-Bus"	6600 lbm	12000 lbm
5	"Bus"	22000 lbm	40000 lbm
6	"Two-Axle Truck"	13000 lbm	33000 lbm
7	"Three-Axle Truck"	20000 lbm	55000 lbm
8	"Four-Axle Truck"	22000 lbm	66000 lbm

$des_v := ve_{ve_{NDX}\,2}$

$M_{min} := ve_{ve_{NDX}\,3}$

$M_{max} := ve_{ve_{NDX}\,4}$

$M_{max} = 7700$ lbm — $M_{max} = 3493$ kg — Initial Maximum Mass

Maximum Vehicle Mass Override:

$M' := 6600\ \text{lbm}$ — $M' = 6600\ \text{lbm}$ — $M' = 2994\ \text{kg}$ — Maximum Mass Override

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if  M' = 0 lbm
    M_max := M_max
else
    M_max := M'
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$M_{max} = 6600\ \text{lbm}$ — $M_{max} = 2994\ \text{kg}$ — Design Maximum Mass

Vehicle Stiffness:

$k := 300000\ \text{Npm}$ — $k = 20556.53\ \text{ppf}$ — $k = 3.00 \cdot 10^{5}\ \text{Npm}$ — Vehicle Stiffness

Speed:

$sp := 15\ \text{mph}$ — $sp = 15.00\ \text{mph}$ — $sp = 6.71\ \text{mps}$ — Vehicle Speed

Impact Moment:

$h_v := 1.64\ \text{ft}$ — $h_v = 1.64\ \text{ft}$ — $h_v = 0.500\ \text{m}$ — Point of Impact

$F_v := sp \cdot \sqrt{k \cdot M_{max}}$ — $F_v = 45176.93\ \text{lb}$ — $F_v = 200.96\ \text{kN}$ — Vehicle Impact Force

$M_s := F_v \cdot h_v$ — $M_s = 74.09\ \text{K\_ft}$ — $M_s = 100.45\ \text{kN\_m}$ — Factored Design Moment

$M_f := F_v \cdot h_v \cdot \alpha_L$ — $M_f = 111.14\ \text{K\_ft}$ — $M_f = 150.68\ \text{kN\_m}$ — Factored Design Moment

AISC Composite Moment Capacity: **Source to be Confirmed**

$K_c := f'_c \cdot \phi_{id}^{2}$ — $K_c = 383.90\ \text{K}$ — $K_c = 1707.69\ \text{kN}$

$K_s := f_{y1} \cdot \left(\frac{\phi_{od} - t_c}{2}\right) \cdot t_c$ — $K_s = 65.04\ \text{K}$ — $K_s = 289.32\ \text{kN}$

$$\theta := \frac{0.0260 \cdot K_c - 2 \cdot K_s}{0.0848 \cdot K_c} + \frac{\sqrt{\left(0.0260 \cdot K_c + 2 \cdot K_s\right)^{2} + 0.857 \cdot K_c \cdot K_s}}{0.0848 \cdot K_c}$$

$\theta = 2.53\ \text{rad}$ — $\theta = 145.07\ °$ — Angular Coefficient

$Z_{cB} := \frac{\phi_{id}^{3}}{6} \cdot \left(\sin\left(\frac{\theta}{2}\right)\right)^{3} \cdot \frac{\theta}{2}$ — $Z_{cB} = 93.73\ \text{in}^{3}$ — $Z_{cB} = 1.54 \cdot 10^{6}\ \text{mm}^{3}$ — Section Modulus for Concrete

$Z_{sB} := \frac{\phi_{od}^{3} - \phi_{id}^{3}}{6} \cdot \sin\left(\frac{\theta}{2}\right)$ — $Z_{sB} = 20.64\ \text{in}^{3}$ — $Z_{sB} = 3.38 \cdot 10^{5}\ \text{mm}^{3}$ — Section Modulus for Steel

$M_B := f_{y1} \cdot Z_{sB} + 0.95 \cdot f'_c \cdot \frac{Z_{cB}}{2}$ — $M_B = 108.25\ \text{K\_ft}$ — $M_B = 146.77\ \text{kN\_m}$ — Composite Moment Resistance

CSA S16; 18.2.3 (Simplified):

$\alpha_1 := \max\left(\left[0.85 - 0.0015 \cdot \frac{f'_c}{\text{MPa}} \quad 0.73\right]\right)$

$\alpha_1 = 0.79$ — Concrete Coefficient

$$h_n := \frac{1.18 \cdot \alpha_1 \cdot \varphi_c \cdot \pi \cdot \frac{\phi_{id}^{2}}{4} \cdot f'_c}{2.36 \cdot \phi_{od} \cdot \alpha_1 \cdot \varphi_c \cdot f'_c + 4 \cdot t_c \cdot \left(2 \cdot \varphi_s \cdot f_{y1} - 1.18 \cdot \alpha_1 \cdot \varphi_c \cdot f'_c\right)}$$

$h_n = 1.07\ \text{in}$ — $h_n = 27.12\ \text{mm}$ — Numeric Coefficient

$$M_{rc} := \left(Zx_c - 2 \cdot t_c \cdot h_n^{2}\right) \cdot \varphi_s \cdot f_{y1} + \left(\frac{2 \cdot \left(0.5 \cdot \phi_{od} - t_c\right)^{3}}{3} - \left(0.5 \cdot \phi_{od} - t_c\right) \cdot h_n^{2}\right) \cdot \left(1.18 \cdot \alpha_1 \cdot \varphi_c \cdot f'_c\right)$$

$M_{rc} = 90.0\ \text{K\_ft}$ — $M_{rc} = 122.0\ \text{kN\_m}$ — Composite Moment Capacity

CSA S16; 18.2.3 (Detailed):

$\beta' := 1\ \text{rad}$ — Recursive Calculation

for $c \in [1..3]$

$$\beta := \frac{\varphi_s \cdot A_c \cdot f_{y1} + 0.295 \cdot \alpha_1 \cdot \varphi_c \cdot \phi_{od}^{\ 2} \cdot f'_c \cdot \left(\sin\left(\frac{\beta'}{2}\right) - \left(\sin\left(\frac{\beta'}{2}\right)\right)^2 \cdot \tan\left(\frac{\beta'}{4}\right)\right)}{0.148 \cdot \alpha_1 \cdot \varphi_c \cdot \phi_{od}^{\ 2} \cdot f'_c + \varphi_s \cdot \phi_{od} \cdot t_c \cdot f_{y1}}$$

$\beta' := \beta$

$\beta = 2.50\ \text{rad}$ — Angular Coefficient

$\beta_f := \beta$ — Pre-Calculated Substitution

$b_c := \phi_{od} \cdot \sin\left(\frac{\beta_f}{2}\right)$ — $b_c = 8.18\ \text{in}$ — $b_c = 207.90\ \text{mm}$ — Numeric Coefficient

$a := \frac{b_c}{2} \cdot \tan\left(\frac{\beta_f}{4}\right)$ — $a = 2.95\ \text{in}$ — $a = 74.99\ \text{mm}$ — Numeric Coefficient

$C_r := \varphi_s \cdot f_{y1} \cdot \beta_f \cdot \phi_{od} \cdot \frac{t_c}{2}$ — $C_r = 151.85\ \text{K}$ — $C_r = 675.45\ \text{kN}$ — Steel Force

$$C'_r := 1.18 \cdot \alpha_1 \cdot \varphi_c \cdot f'_c \cdot \left(\frac{\beta_f \cdot \phi_{od}^{\ 2}}{8} - \frac{b_c}{2} \cdot \left(\frac{\phi_{od}}{2} - a\right)\right)$$

$C'_r = 64.11\ \text{K}$ — $C'_r = 285.18\ \text{kN}$ — Concrete Force

$ee := b_c \cdot \left(\frac{1}{(2 \cdot \pi - \beta_f)} + \frac{1}{\beta_f}\right)$ — $ee = 5.44\ \text{in}$ — $ee = 138.11\ \text{mm}$ — Steel Moment Arm

$$ee' := b_c \cdot \left(\frac{1}{(2 \cdot \pi - \beta_f)} + \frac{b_c^{\ 2}}{1.5 \cdot \beta_f \cdot \phi_{od}^{\ 2} - 6 \cdot b_c \cdot (0.5 \cdot \phi_{od} - a)}\right)$$

$ee' = 4.75\ \text{in}$ — $ee' = 120.60\ \text{mm}$ — Concrete Moment Arm

$M'_{rc} := C_r \cdot ee + C'_r \cdot ee'$ — $M'_{rc} = 94.2\ \text{K\_ft}$ — $M'_{rc} = 127.7\ \text{kN\_m}$ — Composite Moment Capacity c b a

Summary:

Material Property Factors:

$\varphi_s = 0.90$ Rolled Sections

$\varphi_c = 0.65$ Concrete

Dead Load Factors $\alpha_D = 1.25$

Live Load Factors $\alpha_L = 1.50$

Steel		
Yield Strength	$f_{y1} = 50.00$ Ksi	$f_{y1} = 344.74$ MPa
Ultimate Strength	$F_{u1} = 65.00$ Ksi	$F_{u1} = 448.16$ MPa
Bollard Properties		
Imperial Designation	$desI_c =$ "HSS 8.625x0.313"	
Metric Designation	$desM_c =$ "HSS 219x8.0"	
Bollard Outer ϕ	$\phi_{od} = 8.63$ in	$\phi_{od} = 219.1$ mm
Bollard Wall Thicknesss	$t_c = 0.31$ in	$t_c = 8$ mm
Bollard Inner ϕ	$\phi_{id} = 7.999$ in	$\phi_{id} = 203.2$ mm
HSSc Area	$A_c = 8.1733\ in^2$	$A_c = 5273\ mm^2$
HSSc Section Modulus	$Sx_c = 16.3911\ in^3$	$Sx_c = 3 \cdot 10^5\ mm^3$
HSSc Plastic Section	$Zx_c = 21.6352\ in^3$	$Zx_c = 4 \cdot 10^5\ mm^3$
Class of Section	$Class = 1$	
Moment Resistance	$M_r = 81.1$ K_ft	$M_r = 110.0$ kN_m
Concrete Properties		
Concrete Strength	$f'_c = 6.00$ Ksi	$f'_c = 41.37$ MPa
Vehicle Data		
Vehicle Designation	$des_v =$ "SUV"	
Approx Minimum Mass	$M_{min} = 4400$ lbm	$M_{min} = 1996$ kg
Approx Maximum Mass	$M_{max} = 6600$ lbm	$M_{max} = 2994$ kg
Maximum Mass Override	$M' = 6600$ lbm	$M' = 2994$ kg
Design Maximum Mass	$M_{max} = 6600$ lbm	$M_{max} = 2994$ kg
Vehicle Stiffness	$k = 20557$ ppf	$k = 3.00 \cdot 10^5$ Npm
Vehicle Speed	$sp = 15.00$ mph	$sp = 6.71$ mps
Vehicle Impact Force	$F_v = 45177$ lb	$F_v = 201$ kN
Point of Impact	$h_v = 1.64$ ft	$h_v = 0.500$ m
Design Forces		
Service Design Moment	$M_s = 74.1$ K_ft	$M_s = 100.5$ kN_m
Factored Design Moment	$M_f = 111.1$ K_ft	$M_f = 150.7$ kN_m

AISC Bollard Composite Properties

Concrete Coefficient	$K_c = 383.9\ K$	$K_c = 1707.7\ kN$
Steel Coefficient	$K_s = 65.0\ K$	$K_s = 289.3\ kN$
Theta Coefficient	$\theta = 2.53\ rad$	$\theta = 145\ °$
Section Modulus for Concrete	$Z_{cB} = 93.7\ in^3$	$Z_{cB} = 1.54 \cdot 10^6\ mm^3$
Section Modulus for Steel	$Z_{sB} = 20.6\ in^3$	$Z_{sB} = 3.38 \cdot 10^5\ mm^3$
Composite Moment Resistance	$M_B = 108.2\ K\_ft$	$M_B = 146.8\ kN\_m$

$Check\left(M_B \ge M_s\right) = "...OK"$

CISC Bollard Composite Properties

CSA S16 Moment Resistance; 18.2.3 (Simplified):	$M_{rc} = 90.0\ K\_ft$	$M_{rc} = 122.0\ kN\_m$

$Check\left(M_{rc} \ge M_f\right) = "...NG"$

Steel Force	$C_r = 151.8\ K$	$C_r = 675.5\ kN$
Concrete Force	$C'_r = 64.1\ K$	$C'_r = 285.2\ kN$
Steel Moment Arm	$ee = 5.44\ in$	$ee = 138\ mm$
Concrete Moment Arm	$ee' = 4.75\ in$	$ee' = 121\ mm$
CSA S16 Moment Resistance; 18.2.3 (Detailed):	$M'_{rc} = 94.2\ K\_ft$	$M'_{rc} = 127.7\ kN\_m$

$Check\left(M'_{rc} \ge M_f\right) = "...NG"$