

- $Total\ Internal\ Work = \frac{V^2 h}{2E} \left[\left(\frac{1}{wL \sin^2 \alpha \cos^2 \alpha} \right) + \left(\frac{1}{2A_c \tan^2 \alpha} \right) + \left(\frac{h \tan^2 \alpha}{LA_b} \right) \right] \text{ (equation 1).}$

- Final form: $\frac{\left[\frac{2}{(wL)} + \frac{1}{A_c} \right]}{\left[\frac{2}{(wL)} + \frac{2h}{A_b L} \right]} = (\tan^4 \alpha) \text{ (equation 2).}$

Where V, h, Ac, L and Ab are material properties and can be considered as constants.

***Differentiate the function with respect to alpha (α) and set it to 0 to find the maximum value.
Using double angle identities and writing the functions in terms of sin and cos:***

- $\frac{d}{dx} \left(\frac{1}{wL \sin^2 \alpha \cos^2 \alpha} \right) = \frac{-8 \sin(4\alpha) \csc^4 2\alpha}{wL} = \frac{2}{wL \sin \alpha \cos^3 \alpha} - \frac{2}{wL \cos \alpha \sin^3 \alpha}$
- $\frac{d}{dx} \left(\frac{1}{2A_c \tan^2 \alpha} \right) = \frac{-\cot \alpha \csc^2 \alpha}{A_c} = \frac{-\cos \alpha}{\sin^3 \alpha A_c}$
- $\frac{d}{dx} \left(\frac{h \tan^2 \alpha}{LA_b} \right) = \frac{2h \tan \alpha \sec^2 \alpha}{A_b L} = \frac{2h \sin \alpha}{\cos^3 \alpha A_b L}$
- $\frac{d}{dx} \left(\frac{h^4 \tan^2 \alpha}{360L^2 I_c} \right) = \frac{h^4 \tan \alpha \sec^2 \alpha}{180I_c L^2} = \frac{h^4 \sin \alpha}{180I_c L^2 \cos^3 \alpha}$

$\therefore$

- $0 = \frac{2}{wL \sin \alpha \cos^3 \alpha} - \frac{2}{wL \cos \alpha \sin^3 \alpha} - \frac{\cos \alpha}{\sin^3 \alpha A_c} + \frac{2h \sin \alpha}{\cos^3 \alpha A_b L}$
- $\frac{2h \sin \alpha}{\cos^3 \alpha A_b L} + \frac{2}{wL \sin \alpha \cos^3 \alpha} = \frac{2}{wL \cos \alpha \sin^3 \alpha} + \frac{\cos \alpha}{\sin^3 \alpha A_c}$

Factoring out $\frac{1}{\sin^3 \alpha}$ and $\frac{1}{\cos^3 \alpha}$ from RHS and LHS respectively:

- $(\frac{1}{\cos^3 \alpha})[\frac{2h \sin \alpha}{A_b L} + \frac{2}{wL \sin \alpha}] = (\frac{1}{\sin^3 \alpha})[\frac{2}{wL \cos \alpha} + \frac{\cos \alpha}{A_c}]$
- $(\tan^3 \alpha)[\frac{2h \sin \alpha}{A_b L} + \frac{2}{wL \sin \alpha}] = [\frac{2}{wL \cos \alpha} + \frac{\cos \alpha}{A_c}]$
- $(\tan^3 \alpha)(\sin \alpha)[\frac{2h}{A_b L} + \frac{2}{wL \sin^2 \alpha}] = (\cos \alpha)[\frac{2}{wL \cos^2 \alpha} + \frac{1}{A_c}]$

Finally, rearranging like terms:

$$\frac{\left[\frac{2}{(wL) \cos^2 \alpha} + \frac{1}{A_c} \right]}{\left[\frac{2}{(wL) \sin^2 \alpha} + \frac{2h}{A_b L} \right]} = (\tan^4 \alpha)$$