Plastic Design vs Elastic Design

The American steel code, the AISC and the Canadian steel code, CSA S16 recognise steel sections based on the degree of compactness and symmetry. They are given class values, ranging from Class 1 to Class 4, with Class 1 being the most compact and symmetrical and Class 4 being the least.

For Class 1 and 2 sections the moment resistance is predicated on the plastic section modulus, Zx, and for Class 3 and 4 sections it is predicated on the elastic section modulus Sx. The higher moment capacity reflects this robustness. The more 'stocky' section is able to develop full strength without any local buckling.

In addition, both codes permit the analysis of a structure using either elastic or plastic design. Elastic design is based on the elastic stresses developed with a given design load. Plastic design is something else. It is based on the structure failing and becoming a mechanism. Using the increase in flexural capacity due to the robust section is not truly 'plastic design', IMHO. Plastic design permits the forces in the structure to exceed those that would be determined by an elastic failure. Plastic design requires the use of 'stocky' sections, in general, those that are Class 1 only. There is an added benefit to using the higher moment determined by using the plastic section modulus, Zx in lieu of Sx. The failure mechanism load determined by plastic design is generally greater than the elastic load capacity. The improvement is two fold. The increase in strength using Zx in lieu of Sx and the increase in strength considering the structure as a mechanism.

For example, a steel beam with fixed ends and a uniform load fails elastically at wl^2/12 when the end moments reach their elastic moment capacity. For a Class 1 section, the code allows you to use the resisting moment determined using the plastic section section modulus. This gives you an increase of about 10% or 15% based on the increase of the plastic section modulus compared to the elastic section modulus. The ratio of the plastic section modulus to the elastic section modulus is called the 'shape factor'. For W sections this is approximately 15%. Actual failure due to a UDL occurs when the end moments become plastic, and the mid span moment becomes plastic. This occurs at wl^2/16 for both the end moments and midspan moment. This mechanism gives a further load increase of 33% for failure to occur. As noted, the actual failure does not occur until a plastic moment develops at both the endspans and at midspan. This is called the plastic failure load.

There are a couple of issues that the designer must be aware of. Plastic design does not address instabilities. There are generally three types of buckling: local, distortional and overall buckling. Local buckling of the section is avoided by using stocky sections. Failure can occur at a load less than the plastic failure load due to elastic instability. In addition, there are stress issues related to residual stresses from previous loading that have to be considered. This is referred to as 'shakedown' analysis. It's not always 'just a walk in the park'. Once a structure with a given loading condition has developed a plastic moment and the load is released, there are internal stresses that remain. This is what permits shakedown to occur with a different loading regime. If the same loading that produced plastic hinges is applied, the structure remains elastic up to that load due to these internal stresses.

Plastic design is less conservative. It is not generally used and, contrary to common perception, it is generally much less costly. Members tend to be smaller and there are fewer pieces to handle. There may be a reduction in economy because true plastic design is limited to Class 1 sections and there may not be a 'least weight' section available for a given depth. In addition, splices with plastic design have to be developed for both shear and moment. This type of connection accommodates alternate span loading. The reduction in member size generally offsets the added cost for the connections. In addition, design for deflections is normally not an issue. Deflections are approximately 1/3 of that produced by a simple span beam spanning the same distance given the same member size.

Elastic design lends itself to automatic computer analysis and design. With fully automated software, a complete structure can be elastically designed; some software can actually provide detailed shop drawings from the output.

For statically determinate structures, one plastic hinge is sufficient to cause a failure mechanism. Thus, comparison of the maximum bending moment from an elastic analysis to the plastic flexural strength is consistent and the results are the same.

For statically indeterminate structures, one plastic hinge won't cause a collapse mechanism. Use of an elastic anaylsis for this type of construction is conservative. After elastic ‘failure’, the strucure can normally be loaded to a load exceeding the elastic failure load, until a mechanism is formed. This it the true plastic design failure load. Design using this method is still safe, but the design is less conservative. It should be pointed out that using service loads, the total structure is generally completely in the elastic region. The lateral-torsional buckling bracing requirements are more complicated and generally a little more stringent than those used for the elastic approach. The sections, however, are generally stockier and less prone to buckling.