Homework Assignment **Trigonometry: SOHCAHTOA & Radians**

Name:

Date:

Though this homework is short, it requires an in-depth understanding of the same underlying concepts within trigonometry. Homework needs to be scanned and emailed to me as a PDF. I suggest using CamScanner, which is an app on iOS and android devices that allows you to scan and email documents quickly and easily. If you have any questions, please email me or message me on VT. My email is Woodhead.seth2000@gmail.com. We will go over the answers during our next class. *Good luck studying!*

Test Yourself on SAT Trigonometry Questions!

Practice #1

In triangle DCE, the measure of angle C is 90°, $\mathrm{DC} = 5$ and $\mathrm{CE} = 12$. What is the value of $\sin\left(D\right)$?

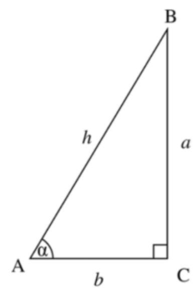
Practice #2

In a right triangle, $\cos\left(\frac{\pi}{2} - x\right) = \frac{6}{8}$. What is $\sin\left(x\right)$?

Practice #3

In circle O, central angle AOB has a measure of $\frac{3\pi}{4}$ radians. The area of the sector formed by central angle AOB is what fraction of the area of the circle?

Although trigonometry makes up **less than 5% of all math questions**, you still want to get those questions right, and you won't be able to answer any trigonometry questions correctly without knowing the following formulas:



Find the **sine of an angle** given the measures of the sides of the triangle.

$$\sin(x) = \frac{(\textit{Measure of the opposite side to the angle})}{(\textit{Measure of the hypotenuse})}$$

In the figure above, the sine of the labeled angle would be $\frac{a}{h}$

Find the **cosine of an angle** given the measures of the sides of the triangle.

$$\cos(x) = \frac{(\textit{Measure of the adjacent side to the angle})}{(\textit{Measure of the hypotenuse})}$$

In the figure above, the cosine of the labeled angle would be $\frac{b}{h}$.

Find the **tangent of an angle** given the measures of the sides of the triangle.

$$\tan(x) = \frac{(\textit{Measure of the opposite side to the angle})}{(\textit{Measure of the adjacent side to the angle})}$$

In the figure above, the tangent of the labeled angle would be $\frac{a}{b}$.

A helpful memory trick is an acronym: **SOHCAHTOA.**

Sine equals **O**pposite over **H**ypotenuse

Cosine equals **A**djacent over **H**ypotenuse

Tangent equals **O**pposite over **A**djacent

You should also know the **complementary angle relationship** for sine and cosine, which is $\sin(x°) = \cos(90° - x°)$.

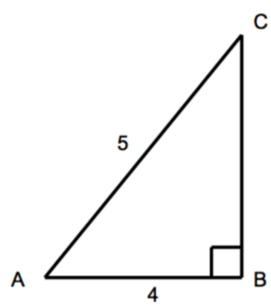
How to Apply Trigonometry Skills on SAT Math

There are **two main trigonometry questions types** you'll see on the test. I'll teach you how to address each.

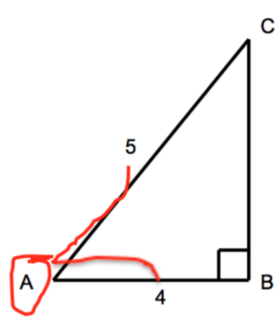
Question type 1 will ask you to find the sine, cosine, or tangent and using the measures of the sides of the triangle. In order to answer these questions, you will need to use a diagram (that means drawing one if it's not given to you). Let's walk through this example:

> Triangle ABC is a right triangle where angle B measures 90°; the hypotenuse is 5 and side AB is 4. What is cosine A?

First, set up this triangle using the given information:



Then, identify the information you need. In this case, the question asked for the cosine A. We know, based on the previous formulas that $\cos\left(A\right) = \frac{\left(Measure\ of\ the\ adjacent\ side\ to\ the\ angle\right)}{\left(Measure\ of\ the\ hypotenuse\right)}$. Identify the pieces you need: the angle, the adjacent side to the angle, and the hypotenuse:

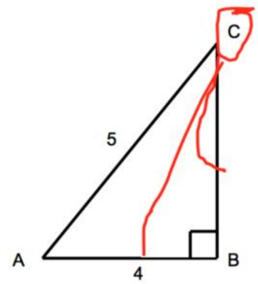


We have all the information we need, so we just need to put it into the formula:

$\cos\left(A\right) = \frac{\left(Measure\ of\ the\ adjacent\ side\ to\ the\ angle\right)}{\left(Measure\ of\ the\ hypotenuse\right)} = \frac{4}{5}$.

$\frac{4}{5}$ is the answer.

A slightly harder version of this question might ask you for tangent C instead of sine C. If you look back at the diagram, you'll notice we don't know what the measure of the adjacent side to angle C is (which is what we need to find tan A).



In that case, we need to use the Pythagorean theorem (or our knowledge of 3-4-5 right triangles find the measure of the adjacent side to angle C (BC).

$$BC = \sqrt{(5^2) - (4^2)} = \sqrt{(25) - (16)} = \sqrt{9} = 3$$

Know that we know that side BC is 3, we just need to put it into the formula:

$$\tan(C) = \frac{(Measure\ of\ the\ opposite\ side\ to\ the\ angle)}{(Measure\ of\ the\ adjacent\ side\ to\ the\ angle)} = \frac{4}{3}$$

Now that we know how to apply the necessary formulas to tackle trig questions, let's try to appl them to some real SAT practice problems.

SAT Trigonometry Practice Problems

Example #1

In triangle ABC, the measure of $\angle B$ is $90°$, $BC = 16$, and $AC = 20$. Triangle DEF is similar to triangle ABC, where vertices D, E, and F correspond to vertices A, B, and C, respectively, and each side of triangle DEF is $\frac{1}{3}$ the length of the corresponding side of triangle ABC. What is the value of $\sin F$?

Answer Explanation: Triangle ABC is a right triangle with its right angle at B. Therefore, AC is the hypotenuse of right triangle ABC, and AB and BC are the legs of right triangle ABC. According to the Pythagorean theorem,

$$AB = \sqrt{(202) - (162)} = \sqrt{(400) - (256)} = \sqrt{144} = 12$$

Since triangle DEF is similar to triangle ABC, with vertex F corresponding to vertex C, the measure of angle F equals the measure of angle C. Therefore, $\sin F = \sin C$. From the side lengths of triangle ABC, $\sin C = \frac{(Measure\ of\ the\ opposite\ side\ to\ the\ angle)}{(Measure\ of\ the\ hypotenuse)} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{12}{20} = \frac{3}{5}$. Therefore, $\sin F = \frac{3}{5}$.

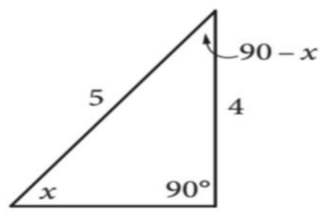
The final answer is $\frac{3}{5}$ or .6.

Example #2

In a right triangle, one angle measures $x°$, where $\sin x° = \frac{4}{5}$. What is $\cos(90° - x°)$?

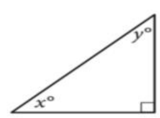
Answer Explanation: There are two ways to solve this. The quicker way is if you know the complementary angle relationship for sine and cosine, which is $\sin(x°) = \cos(90° - x°)$. Therefore, $\cos(90° - x°) = \frac{4}{5}$ or 0.8.

However, you can also solve this problem by constructing a diagram using the given information. It's a right triangle (which it has to be to use sine/cosine), and the sine of angle x is $\frac{4}{5}$ if $\text{sine} = \frac{(\textit{opposite side})}{\text{hypotenuse}}$ then the opposite side is 4 long, and the hypotenuse is 5 long:



Since two of the angles of the triangle are of measure x° and 90°, the third angle must have the measure $180° - 90° - x° = 90° - x°$. From the figure, $\cos(90° - x°)$, which is equal the $\frac{\textit{adjacent side}}{\textit{the hypotenuse}}$, is also $\frac{4}{5}$ or 0.8.

Example #3

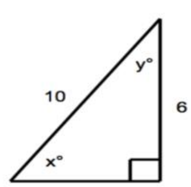


In the triangle above, the sine of $x°$ is 0.6. What is the cosine of $y°$?

Answer Explanation: Similarly to the other trigonometry problem, there are two ways to solve this problem.

The quicker way is to realize that x and y are complementary angles (add up to 90°). Then, using the complementary angle relationship for sine and cosine, which is $\sin(x°) = \cos(90° - x°)$, you realize that $\cos(y°) = 0.6$.

However, you can also solve this problem by constructing a diagram using the given information. It's a right triangle (which it has to be to use sine/cosine), and the sine of angle x is 0.6. Therefore, the ratio of the side opposite the x° angle to the hypotenuse is .6.



The side opposite the x° angle is the side adjacent to the y° angle.
$\cos(y°) = \frac{(\textit{the side adjacent to the } y° \textit{ angle})}{(\textit{the hypotenuse})} = \frac{6}{10}$, is equal to .6.

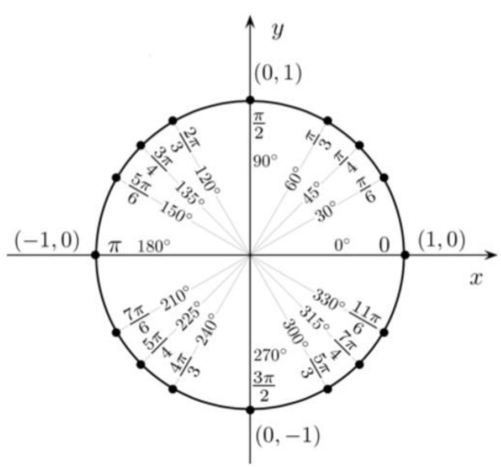
Radians

Radians will **only account for a small portion (around 5%) of SAT math questions**, but you still want to get those questions right! Radians are one of the trickier concepts. What do you need to know about radian measure?

Definition of Radian Measure

The bare bones definition: Radian is a measure of an angle (just as degree is a measure of angle).

The in-depth/conceptual version: Radian is a measure of an angle that is based on the length of the arc that the angle intercepts on the unit circle. That sounds like gibberish I know. Let me break it down. A unit circle is a circle with a radius of 1 unit. See picture:



Gustavb/Wikimedia

The circumference (or length around) this unit circle is 2π, since $C = 2\pi r$, and r=1.

If the measure of an angle were 360°, the radian measure would be 2π since the length of the arc that the 360° angle intercepts on the unit circle would be the whole circumference of the circle (which we already established was 2π). Here are some good basic radian measures to have memorized:

Degrees	Radians (exact)
30°	$\frac{\pi}{6}$
45°	$\frac{\pi}{4}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$

How to Convert Between Angle Measure in Degrees and Radians

To go from degrees to radians, you need to multiply by π, divide by 180°. Here is how to convert 90° to radians:

$$\frac{90°\pi}{180°}$$
$$= \frac{\pi}{2}$$

To go from radians to degrees, you need to multiply by 180°, divide by π. Here is how to convert to degrees:

$$\frac{\left(\frac{\pi}{4}\right)\left(180°\right)}{\pi}$$
$$= \frac{\left(\frac{180°\pi}{4}\right)}{\pi}$$
$$= 45°$$

How to Evaluate Trigonometric Functions at Benchmark Angle Measures

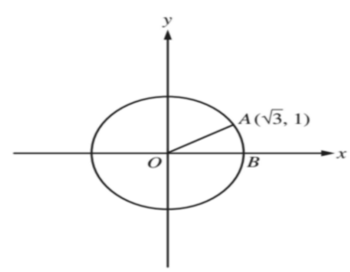
The benchmark angle measures (as defined by the College Board) are 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ radians which ar equal to the angle measures 0°, 30°, 45°, 60°, and 90°, respectively.

You need to be able to use these with the trigonometric functions described in the above trigonometry section (sine, cosine, and tangent). You will not be asked for values of trigonometric functions that require a calculator.

Remember, the complementary angle relationship for sine and cosine, which is $\sin\left(x°\right) = \cos\left(90° - x°\right)$ will be $\sin\left(x\right) = \cos\left(\frac{\pi}{2} - x\right)$ when converted into radians.

SAT Radians Practice Problems

Example #1



In the xy-plane above, O is the center of the circle, and the measure of $\angle AOB$ is $\frac{\pi}{a}$ radians. What is the value of a ?

Answer Explanation: The correct answer is 6. By the distance formula, the length of radius OA is $\sqrt{\left(\left(\sqrt{3}\right)^2\right) + \left(1^2\right)} = \sqrt{3+1} = \sqrt{4} = 2$. Thus, $\sin\left(\angle AOB\right) = \frac{1}{2}$.

Therefore ∠AOB is 30°, which is equal to $30\left(\frac{\pi}{180}\right) = \frac{\pi}{6}$ radians. Hence, the value of a is 6.

Example #2

In a circle with center O, central angle AOB has a measure of $\frac{5\pi}{4}$ radians. The area of the sector formed by central angle AOB is what fraction of the area of the circle?

Answer Explanation: A complete rotation around a point is 360° or 2π radians. Since the central angle AOB has measure $\frac{5\pi}{4}$ radians, it represents $/ 2\pi = \frac{5}{8}$ of a complete rotation around point O. Therefore, the sector formed by central angle AOB has area equal to $\frac{5}{8}$ the area of the entire circle The answer is $\frac{5}{8}$ or in decimal form .625.

Example #3

Which of the following is equivalent to $\cos\left(\frac{3\pi}{10}\right)$?

A) -$cos\left(\frac{\pi}{5}\right)$
B) $\sin\left(\frac{7\pi}{10}\right)$
C) -$sin\left(\frac{\pi}{5}\right)$
D) $\sin\left(\frac{\pi}{5}\right)$

Answer Explanation: To answer this question correctly, you need to both understand trigonometry and radians. Sine and cosine are related by the equation $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$.

In order to find out what the equivalent to $\cos\left(\frac{3\pi}{10}\right)$ is, you need to change $\frac{3\pi}{10}$ into the form $\frac{\pi}{2} - x$. To do that, you need to set up an equation:

$$\frac{3\pi}{10} = \frac{\pi}{2} - x$$

Then, solve for x.

$$\frac{3\pi}{10} - \frac{\pi}{2} = -x$$

$$\frac{3\pi}{10} - \frac{5\pi}{10} = -x$$

$$-\frac{2\pi}{10} = -x$$

$$\frac{2\pi}{10} = x$$

$$\frac{\pi}{5} = x$$

Therefore, $\cos\left(\frac{3\pi}{10}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \sin\left(\frac{\pi}{5}\right)$. D is the correct answer.