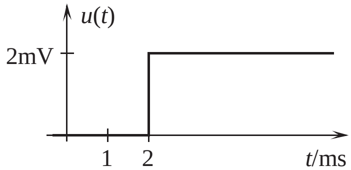
Lösungen

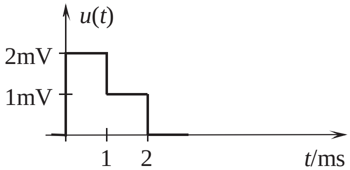
Lösungen Teil I

Lösungen zum Kapitel 3

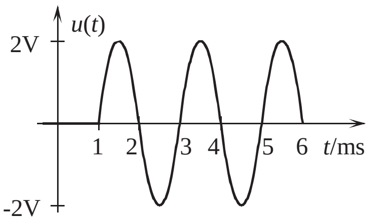
1. a) $u(t) = 2\,\text{mV}\,\varepsilon\,(t - 2\,\text{ms})$



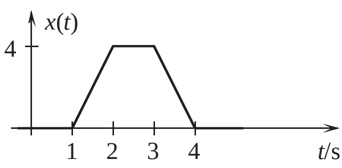
b) $u(t) = 2\,\text{mV}\,\varepsilon(t) - 1\,\text{mV}\,\varepsilon\,(t - 1\,\text{ms}) - 1\,\text{mV}\varepsilon\,(t - 2\,\text{ms})$



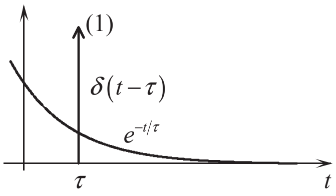
c) $u(t) = -2\,\text{V}\,\varepsilon\,(t - 1\,\text{ms})\sin\left(\frac{\pi}{1\,\text{ms}}t\right)$



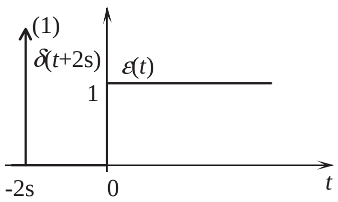
d) $x(t) = \frac{4}{s}r\,(t - 1\,\text{s}) - \frac{4}{s}r\,(t - 2\,\text{s}) - \frac{4}{s}r\,(t - 3\,\text{s}) + \frac{4}{s}r\,(t - 4\,\text{s})$



2. a) $\int\limits_{-\infty}^{\infty} \delta(t-\tau)\, \mathrm{e}^{-t/\tau}\, \mathrm{d}t = e^{-\tau/\tau} = \mathrm{e}^{-1}$



b) $\int\limits_{-\infty}^{\infty} \delta(t+2\,\mathrm{s})\, \varepsilon(t)\, \mathrm{d}t = 0$



3. a) $x(t) = 2\,\frac{\mathrm{V}}{\mathrm{ms}}\, r(t - 2\,\mathrm{ms})$

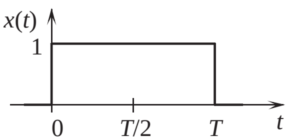
b) $x(t) = \varepsilon(t) \cdot \cos\left(\frac{2\pi}{4s}\, t\right)$

c) $x(t) = r(t+T) - r(t-T) - r(t-2T) + r(t-4T)$

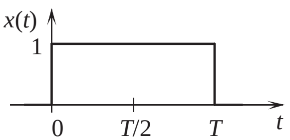
d) $x(t) = 2\varepsilon(t+2T) - 3\varepsilon(t+T) + 3\varepsilon(t-T)$

e) $x(t) = \mathrm{rect}\left(\frac{t-T/2}{3T}\right) + 2\Lambda\left(\frac{t+T/2}{T/2}\right) + 2\Lambda\left(\frac{t-3T/2}{T/2}\right)$

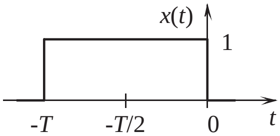
4. a) $x(t) = \mathrm{rect}\left(\frac{T/2-t}{T}\right)$



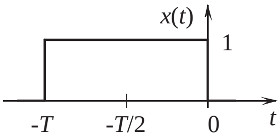
b) $x(t) = \mathrm{rect}\left(\frac{T/2-t}{-T}\right)$



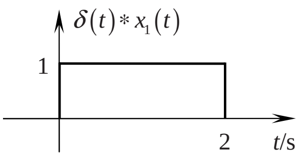
c) $x(t) = \mathrm{rect}\left(\dfrac{T/2 + t}{T}\right)$



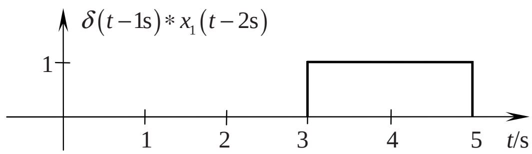
d) $x(t) = \mathrm{rect}\left(\dfrac{T/2 + t}{-T}\right)$



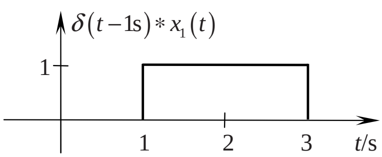
5. a)



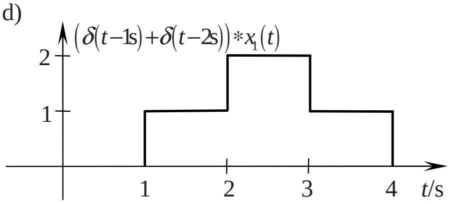
c)



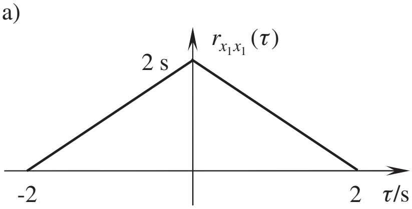
b)

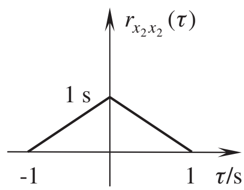


d)



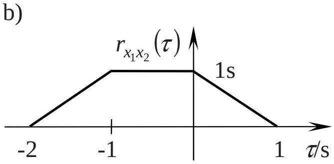
6. a)

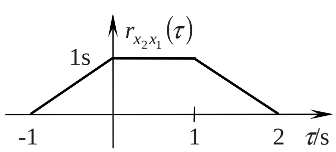




$r_{x_1x_1}(\tau) = 2\,\mathrm{s}\,\Lambda\left(\dfrac{\tau}{2\,\mathrm{s}}\right)$ $\qquad$ $r_{x_2x_2}(\tau) = 1\,\mathrm{s}\,\Lambda\left(\dfrac{\tau}{1\,\mathrm{s}}\right)$

b)

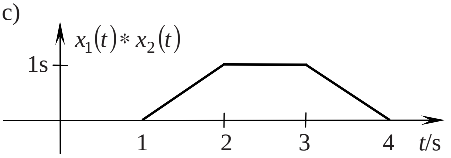




$$r_{x_1x_2}(\tau) = \begin{cases} \tau + 2\,\mathrm{s} & \text{für} \quad -2\,\mathrm{s} \leqq \tau \leqq -1\,\mathrm{s} \\ 1\,\mathrm{s} & \text{für} \quad -1\,\mathrm{s} \leqq \tau \leqq 0 \\ -\tau + 1\,\mathrm{s} & \text{für} \quad 0 \leqq \tau \leqq 1\,\mathrm{s} \end{cases}$$

$$r_{x_2x_1}(\tau) = \begin{cases} \tau + 1\,\mathrm{s} & \text{für} \quad -1\,\mathrm{s} \leqq \tau \leqq 0 \\ 1\,\mathrm{s} & \text{für} \quad 0 \leqq \tau \leqq 1\,\mathrm{s} \\ -\tau + 2\,\mathrm{s} & \text{für} \quad 1\,\mathrm{s} \leqq \tau \leqq 2\,\mathrm{s} \end{cases}$$

c)

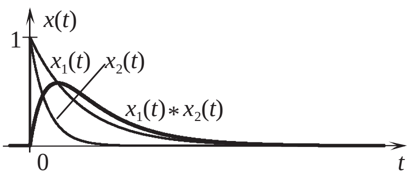


$$x_1(t) * x_2(t) = \begin{cases} t - 1\,\text{s} & \text{für} \quad 1\,\text{s} \leqq t \leqq 2\,\text{s} \\ 1\,\text{s} & \text{für} \quad 2\,\text{s} \leqq t \leqq 3\,\text{s} \\ -t + 4\,\text{s} & \text{für} \quad 3\,\text{s} \leqq t \leqq 4\,\text{s} \end{cases}$$

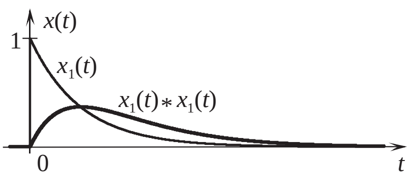
d) $$x_1(t) * x_2(-t) = \begin{cases} t + 2\,\text{s} & \text{für} \quad -2\,\text{s} \leqq t \leqq -1\,\text{s} \\ 1\,\text{s} & \text{für} \quad -1\,\text{s} \leqq t \leqq 0 \\ -t + 1\,\text{s} & \text{für} \quad 0 \leqq t \leqq 1\,\text{s} \end{cases}$$

Korrelation $r_{x_1 x_2}(\tau)$ und Faltung $x_1(t) * x_2(-t)$ sind identisch

7. a) $\left(\varepsilon(t)\,\mathrm{e}^{-t/T_1}\right) * \left(\varepsilon(t)\,\mathrm{e}^{-t/T_2}\right) = \varepsilon(t)\dfrac{T_1\,T_2}{T_1 - T_2}\left(\mathrm{e}^{-t/T_1} - e^{-t/T_2}\right); \quad T_1 \neq T_2$

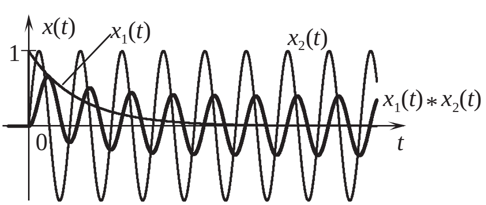


b) $\left(\varepsilon(t)\cdot\,\mathrm{e}^{-t/T_1}\right) * \left(\varepsilon(t)\cdot\,\mathrm{e}^{-t/T_1}\right) = t\cdot\varepsilon(t)\,\mathrm{e}^{-t/T_1}$



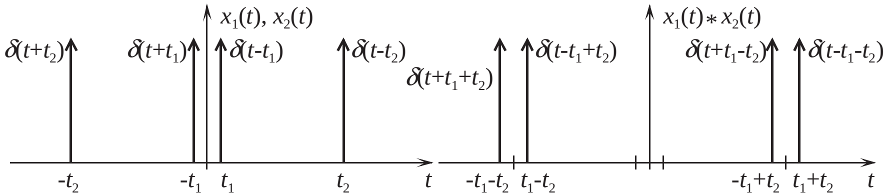
c) $\left(\varepsilon(t)\cdot\,\mathrm{e}^{-t/T_1}\right) * \left(\varepsilon(t)\cdot\sin\left(2\pi f_0 t\right)\right)$

$$= \varepsilon(t)\frac{T_1^2}{1 + \left(2\pi f_0 T_1\right)^2}\left(\frac{1}{T_1}\sin\left(2\pi f_0 t\right) - 2\pi f_0\cos\left(2\pi f_0 t\right) + 2\pi f_0\,\mathrm{e}^{-t/T_1}\right)$$



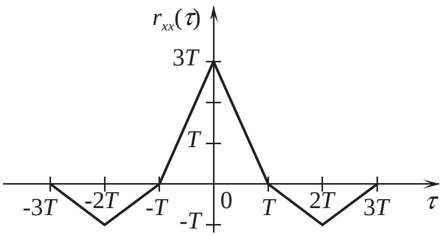
d) $x_1(t) * x_2(t) = \big(\delta(t+t_1) + \delta(t-t_1)\big) * \big(\delta(t+t_2) + \delta(t-t_2)\big)$

$$= \delta(t+t_1+t_2) + \delta(t+t_1-t_2) + \delta(t-t_1+t_2) + \delta(t-t_1-t_2)\,; \quad t_1 \ll t_2$$

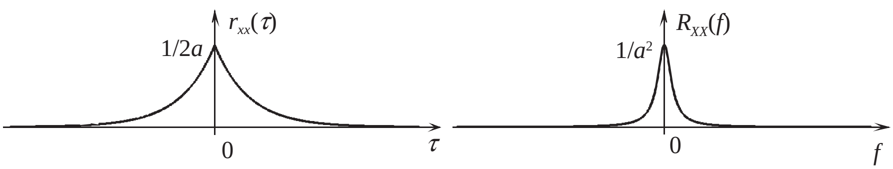


8. $R_{XX}(f) = T^2 \operatorname{si}^2(\pi f T)\left(3 - \mathrm{e}^{\mathrm{j}2\pi f 2T} - \mathrm{e}^{-\mathrm{j}2\pi f 2T}\right)$

$$r_{XX}(\tau) = T\left(3\Lambda\left(\frac{\tau}{T}\right) - \Lambda\left(\frac{\tau+2T}{T}\right) - \Lambda\left(\frac{\tau-2T}{T}\right)\right)$$



9. $r_{XX}(\tau) = \dfrac{1}{2a}\,\mathrm{e}^{-a|\tau|}$ $\qquad R_{XX}(f) = \dfrac{1}{a^2 + (2\pi f)^2}$



10. a) Energiesignal, da Beginn bei $t = 0$ und betragsmäßig exponentiell abfallend

b) $W = \dfrac{T}{1 + 64\pi^2} + T$

c) $W = \dfrac{2T}{3}$

d) $\overline{P} = 0$

11. Energiesignal

$$W = \int_0^{4T} \cos^2\left(\frac{2\pi t}{T}\right) \mathrm{d}t = \frac{1}{2}\int_0^{4T}\left(1 + \cos\left(\frac{4\pi t}{T}\right)\right)\mathrm{d}t = \frac{t}{2}\bigg|_0^{4T} = \underline{\underline{2T}}$$

$\overline{P} = 0$

Lösungen zum Kapitel 4

12. Stellen Sie folgende Signale grafisch dar und geben Sie jeweils $\{x(kT_A)\}$ als Folge und die Bildungsvorschrift der Folge an.

a) $\{x(kT_A)\} = \left\{ \begin{array}{lll} 1+k & \text{für} & k \geqq 0 \\ 0 & \text{für} & k < 0 \end{array} \right\} = \{\, \underline{1};\ 2;\ 3;\ 4;\ \ldots \}$

b) $\{x(kT_A)\} = \left\{ \begin{array}{lll} 2 & \text{für} & \text{k:0, 1} \\ 1 & \text{für} & \text{k:2, 3} \\ 0 & \text{sonst} & \end{array} \right\} = \{\, \underline{2};\ 2;\ 1;\ 1 \,\}$

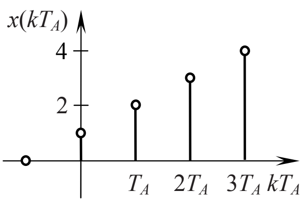
c) $\{x(kT_A)\} = \left\{ \begin{array}{lll} 2^k & \text{für} & k \geqq 0 \\ 0 & \text{für} & k < 0 \end{array} \right\} = \{\, \underline{1};\ 2;\ 4;\ 8;\ \ldots \}$

d) $\{x(kT_A)\} = \left\{ \begin{array}{lll} 4 & \text{für} & k = 2 \\ 0 & \text{für} & k \neq 2 \end{array} \right\} = \{\, \underline{0};\ 0;\ 2;\ \ldots \}$

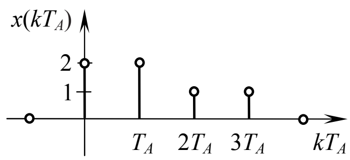
e) $\{x(kT_A)\} = \left\{ \begin{array}{lll} \cos(\pi k/5) & \text{für} & 0 \leqq k \leqq 9 \\ 0 & \text{sonst} & \end{array} \right\}$

$\{x(kT_A)\} = \{\underline{1}; 0{,}8; 0{,}3; -0{,}3; -0{,}8; -1; -0{,}8; -0{,}3; 0{,}3; 0{,}8\}$

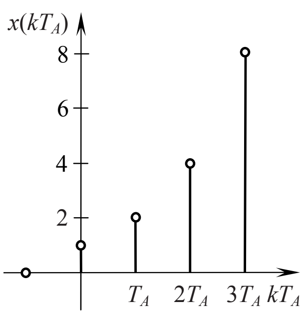
a)



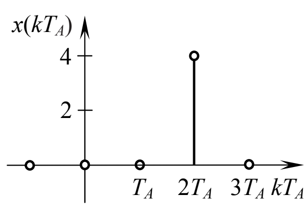
b)



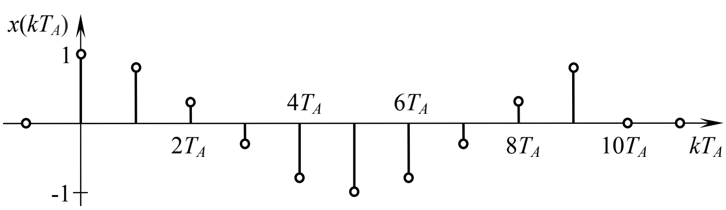
c)



d)



e)

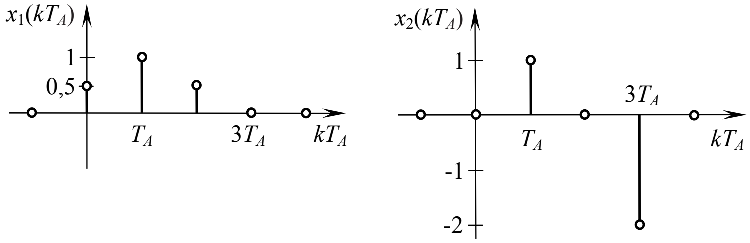


13. a) $6 + 3 - 1 = 8$ Abtastungen

b) $\{x_1(kT_\mathrm{A}) * x_2(kT_\mathrm{A})\} = \{-\underline{3}; 9; -11; 9; -7; 5; -3; 1\}$

c) $\{\varepsilon(kT_\mathrm{A})\} * \{x_2(kT_\mathrm{A})\} = \{\underline{-1}; 1\}$

14. a)



b) $\{r_{x_1x_1}(kT_\mathrm{A})\} = \{0{,}25; 1; \underline{1{,}5}; 1; 0{,}25\}$

$\{r_{x_2x_2}(kT_\mathrm{A})\} = \{-2; 0; \underline{5}; 0; -2\}$

c) $\{r_{x_1x_2}(kT_\mathrm{A})\} = \{-1; -2; -0{,}5; \underline{1}; 0{,}5\}$

$\{r_{x_2x_1}(kT_\mathrm{A})\} = \{0{,}5; \underline{1}; -0{,}5; -2; -1\}$

d) $\{x_1(kT_\mathrm{A})\} * \{x_2(kT_\mathrm{A})\} = \{\underline{0};\ 0{,}5;\ 1;\ -0{,}5;\ -2;\ -1\ \}$

e) $\{x_1(kT_\mathrm{A})\}_\mathrm{P} * \{x_2(kT_\mathrm{A})\}_\mathrm{P} = \{\ldots\ -2; \underline{-2}; -2; -2; \ldots\}$

f) $\{x_1(kT_\mathrm{A})\} * \{x_2(kT_\mathrm{A})\}_\mathrm{P} = \{-\underline{2}; -0{,}5; 1; -0{,}5\}_\mathrm{p}$

g) $\{x_1(kT_\mathrm{A}) * x_2(-kT_\mathrm{A})\} = \{-1; -2; -0{,}5; \underline{1}; 0{,}5\}$

Lösungen zum Kapitel 5

15. a) $\omega_\mathrm{P} = \dfrac{\pi}{\mathrm{ms}} = \pi \cdot 10^3\,\mathrm{s}^{-1}$

$$x_1(t) = \frac{1}{\pi} + \frac{1}{2}\sin(\omega_\mathrm{P} t) - \frac{2}{\pi}\left[\frac{1}{1\cdot 3}\cos(2\omega_\mathrm{P} t) + \frac{1}{3\cdot 5}\cos(4\omega_\mathrm{P} t) + \ldots\right]$$

$$x_1(t) = \frac{1}{\pi} + \frac{1}{2\cdot 2\mathrm{j}}\,\mathrm{e}^{\mathrm{j}\omega_\mathrm{P} t} - \frac{1}{3\pi}\,\mathrm{e}^{\mathrm{j}2\omega_\mathrm{P} t} - \frac{1}{15\pi}\,\mathrm{e}^{\mathrm{j}4\omega_\mathrm{P} t} + \ldots$$
$$- \frac{1}{2\cdot 2\mathrm{j}}\,\mathrm{e}^{-\mathrm{j}\omega_\mathrm{P} t} - \frac{1}{3\pi}\,\mathrm{e}^{-\mathrm{j}2\omega_\mathrm{P} t} - \frac{1}{15\pi}\,\mathrm{e}^{-\mathrm{j}4\omega_\mathrm{P} t} + \ldots$$

$$x_2(t) = x_1(t + 0{,}25\,\mathrm{ms})$$

$$x_2(t) = \frac{1}{\pi} + \frac{1}{2}\sin\left(\omega_\mathrm{P} t + \frac{\pi}{4}\right) - \frac{2}{\pi}\left(\frac{1}{3}\cos\left(2\omega_\mathrm{P} t + \frac{\pi}{2}\right) + \frac{1}{15}\cos(4\omega_\mathrm{P} t + \pi) + \ldots\right).$$

$$x_2(t) = \frac{1}{\pi} + \frac{1}{2\cdot 2\mathrm{j}}\,\mathrm{e}^{\mathrm{j}(\omega_\mathrm{P} t + \pi/4)} - \frac{1}{3\pi}\,\mathrm{e}^{\mathrm{j}(2\omega_\mathrm{P} t + \pi/2)} - \frac{1}{15\pi}\,\mathrm{e}^{\mathrm{j}(4\omega_\mathrm{P} t + \pi)} - \ldots$$
$$- \frac{1}{2\cdot 2\mathrm{j}}\,\mathrm{e}^{-\mathrm{j}(\omega_\mathrm{P} t + \pi/4)} - \frac{1}{3\pi} + \mathrm{e}^{-\mathrm{j}(2\omega_\mathrm{P} t + \pi/2)} - \frac{1}{15\pi} + \mathrm{e}^{-\mathrm{j}(4\omega_\mathrm{P} t + \pi)} - \ldots$$

b) $$x_1(t) = \frac{1}{\pi} + \frac{1}{2}\cos\left(\omega_\mathrm{P} t - \frac{\pi}{2}\right) + \frac{2}{\pi}\left(\frac{1}{3}\cos(2\omega_\mathrm{P} t + \pi) + \frac{1}{15}\cos(4\omega_\mathrm{P} t + \pi) + \ldots\right)$$

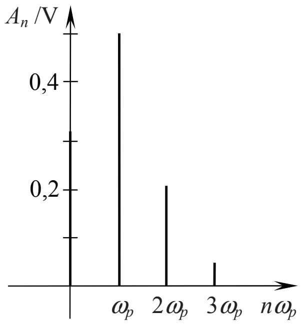
$$x_1(t) = \frac{1}{\pi} + \frac{1}{4}\,\mathrm{e}^{-\mathrm{j}\pi/2}\,\mathrm{e}^{\mathrm{j}\omega_\mathrm{P} t} + \frac{1}{3\pi}\,\mathrm{e}^{-\mathrm{j}\pi}\,\mathrm{e}^{\mathrm{j}2\omega_\mathrm{P} t} + \frac{1}{15\pi}\,\mathrm{e}^{-\mathrm{j}\pi}\,\mathrm{e}^{\mathrm{j}4\omega_\mathrm{P} t} + \ldots$$
$$+ \frac{1}{4}\,\mathrm{e}^{\mathrm{j}\pi/2}\,\mathrm{e}^{-\mathrm{j}\omega_\mathrm{P} t} + \frac{1}{3\pi}\,\mathrm{e}^{\mathrm{j}\pi}\,\mathrm{e}^{-\mathrm{j}2\omega_\mathrm{P} t} + \frac{1}{15\pi}\,\mathrm{e}^{\mathrm{j}\pi}\,\mathrm{e}^{-\mathrm{j}4\omega_\mathrm{P} t} + \ldots$$

$$x_2(t) = \frac{1}{\pi} + \frac{1}{2}\cos\left(\omega_P t - \frac{\pi}{4}\right) + \frac{2}{\pi}\left(\frac{1}{3}\cos\left(2\omega_P t - \frac{\pi}{2}\right) + \frac{1}{15}\cos(4\omega_P t) + \ldots\right)$$

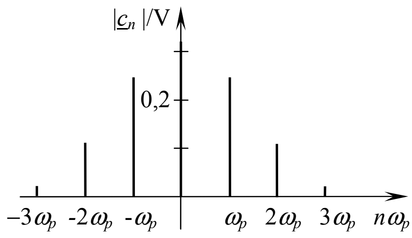
$$x_2(t) = \frac{1}{\pi} + \frac{1}{4}\,e^{-j\pi/4}\,e^{j\omega_P t} + \frac{1}{3\pi}\,e^{-j\pi/2}\,e^{j2\omega_P t} + \frac{1}{15\pi}\,e^{j0}\,e^{j4\omega_P t} + \ldots$$
$$+\frac{1}{4}\,e^{j\pi/4}\,e^{-j\omega_P t} + \frac{1}{3\pi}\,e^{j\pi/2}\,e^{-j2\omega_P t} + \frac{1}{15\pi}\,e^{j0}\,e^{-j4\omega_P t} + \ldots$$

Amplitudenspektren der Signale $x_1(t)$ und $x_2(t)$

aus reeller Form der FR

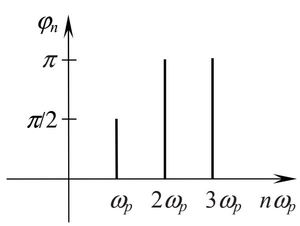


aus komplexer Form der FR

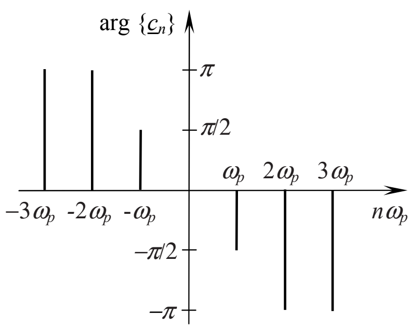


Phasenspektren des Signals $x_1(t)$

aus reeller Form der FR

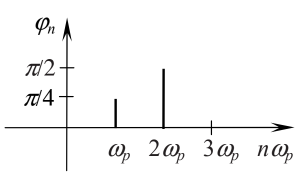


aus komplexer Form der FR

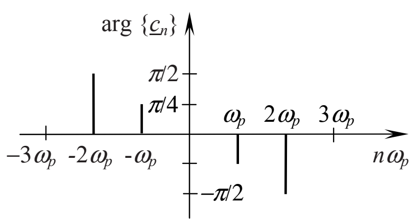


Phasenspektren des Signals $x_2(t)$

aus reeller Form der FR



aus komplexer Form der FR



16. a) $\underline{X_1}(f) = F\{x_1(t)\} = \int\limits_{-0{,}5\,\mathrm{s}}^{0{,}5\,\mathrm{s}} 1 \cdot \mathrm{e}^{-\mathrm{j}2\pi f t}\,\mathrm{d}t = 1\,\mathrm{s} \cdot \mathrm{si}\left(\pi f T\right); \quad T = 1\,\mathrm{s}$

$$\left|\underline{X_1}(f)\right| = \left|1\,\mathrm{s} \cdot \mathrm{si}\left(\pi f T\right)\right|$$

$$\arg\{\underline{X_1}(f)\} = \begin{cases} 0 & \text{für} \quad \mathrm{si}\left(\pi f T\right) \geqq 0 \\ \pm\pi & \text{für} \quad \mathrm{si}\left(\pi f T\right) < 0 \end{cases}$$

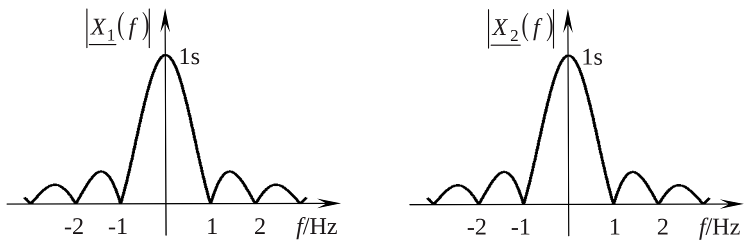
$$\underline{X_2}(f) = F\{x_2(t)\} = \int\limits_{1\,\mathrm{s}}^{2\,\mathrm{s}} 1 \cdot \mathrm{e}^{-\mathrm{j}2\pi f t}\,\mathrm{d}t = 1\,\mathrm{s} \cdot \mathrm{si}\left(\pi f \cdot \mathrm{s}\right)\,\mathrm{e}^{-\mathrm{j}\pi f \cdot 3\,\mathrm{s}}$$

$$\left|\underline{X_2}(f)\right| = \left|1\,\mathrm{s} \cdot \mathrm{si}\left(\pi f T\right)\right|$$

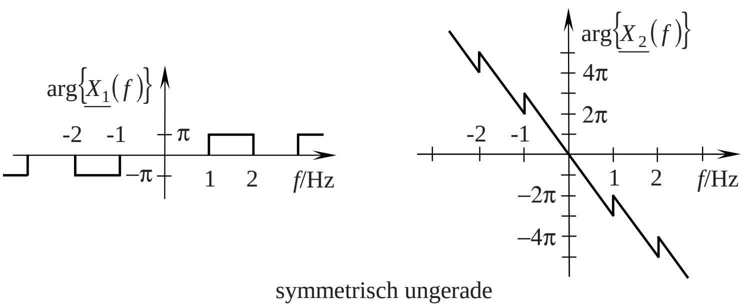
$$\arg\{\underline{X_2}(f)\} = \begin{cases} -\pi f \cdot 3\,\mathrm{s} & \text{für} \quad \mathrm{si}\left(\pi f T\right) \geqq 0 \\ -\pi f \cdot 3\,\mathrm{s} \pm \pi & \text{für} \quad \mathrm{si}\left(\pi f T\right) < 0 \end{cases}$$

b) Amplituden- und Phasenspektren

c) Symmetrieeigenschaften



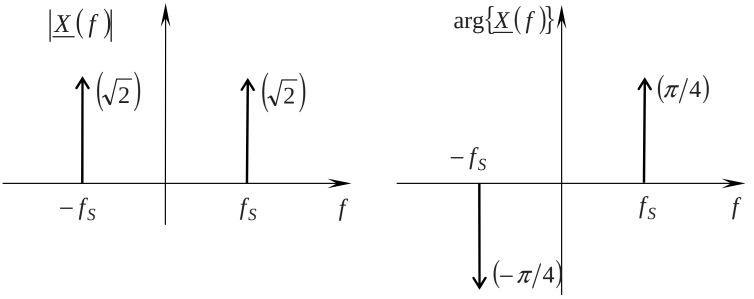
symmetrisch gerade



symmetrisch ungerade

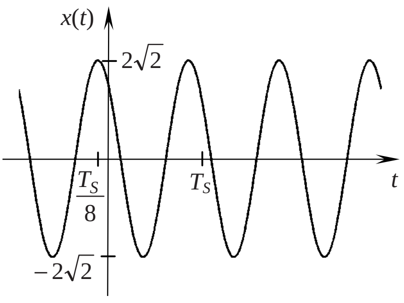
17. a) $\underline{X}(f) = \delta\left(f + f_S\right) + \delta\left(f - f_S\right) + j\left(-\delta\left(f + f_S\right) + \delta\left(f - f_S\right)\right)$

b) $\underline{X}(f) = \sqrt{2}\,e^{-j\pi/4}\delta\left(f + f_S\right) + \sqrt{2}\,e^{j\pi/4}\delta\left(f - f_S\right)$



c) $x(t) = 2\cos\left(2\pi f_S t\right) - 2\sin\left(2\pi f_S t\right)$

$x(t) = 2\sqrt{2}\cos\left(2\pi f_S t + \pi/4\right) = 2\sqrt{2}\cos\left(2\pi f_S\left(t + T_S/8\right)\right)$



18. a) Moduliertes Signal

$$u_{AM}(t) = \hat{U}_{NF}\cos\left(2\pi\cdot 2\,\text{kHz}\cdot t\right)\hat{U}_T\cos\left(2\pi\cdot 1\,\text{MHz}\cdot t\right)\frac{1}{\hat{U}_T}$$

$$u_{AM}(t) = \frac{\hat{U}_{NF}}{2}\left(\cos\left(2\pi\cdot 1{,}002\,\text{MHz}\cdot t\right) + \cos\left(2\pi\cdot 0{,}998\,\text{MHz}\cdot t\right)\right)$$

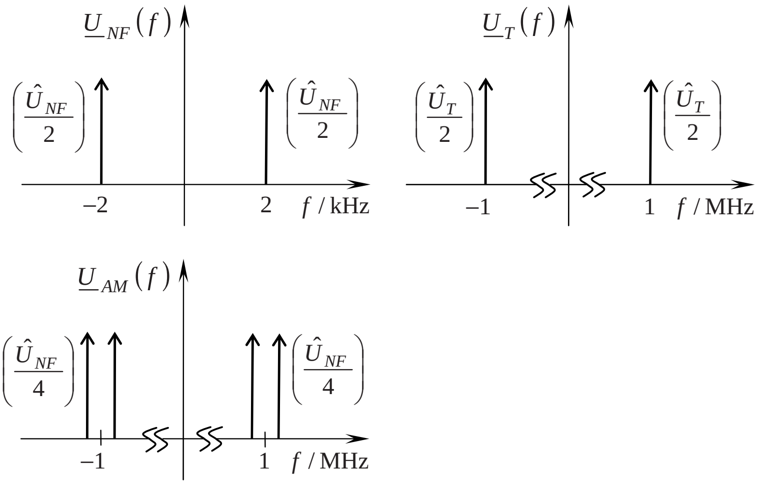
b) Fourier-transformierte Signale

$$\underline{U}_{NF}(f) = \frac{\hat{U}_{NF}}{2}\delta\left(f + 2\,\text{kHz}\right) + \frac{\hat{U}_{NF}}{2}\delta\left(f - 2\,\text{kHz}\right)$$

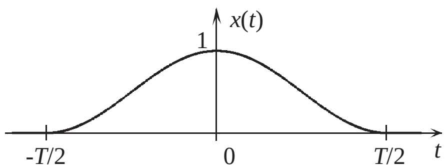
$$\underline{U}_T(f) = \frac{\hat{U}_T}{2}\delta\left(f + 1\,\text{MHz}\right) + \frac{\hat{U}_T}{2}\delta\left(f - 1\,\text{MHz}\right)$$

$$\underline{U}_{AM}(f) = \frac{\hat{U}_{NF}}{4}\delta\left(f + 1{,}002\,\text{MHz}\right) + \frac{\hat{U}_{NF}}{4}\delta\left(f - 1{,}002\,\text{MHz}\right) + \frac{\hat{U}_{NF}}{4}\delta\left(f + 0{,}998\,\text{MHz}\right) + \frac{\hat{U}_{NF}}{4}\delta\left(f - 0{,}998\,\text{MHz}\right)$$

c) Spektren der drei Signale

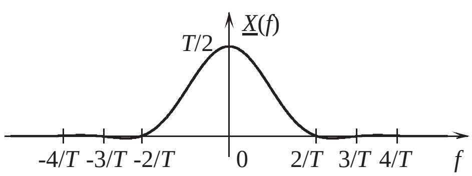


19. a)

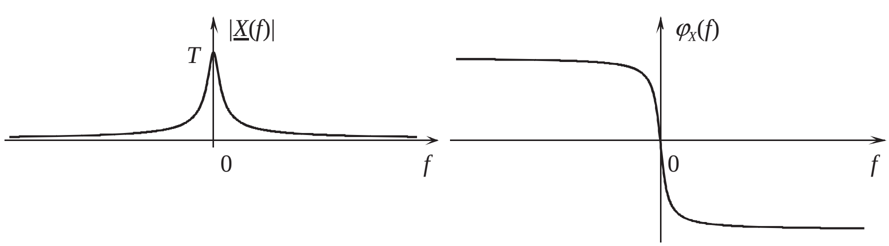


b) $x(t) = \frac{1}{2}\,\mathrm{rect}\left(\frac{t}{T}\right) + \frac{1}{4}\,\mathrm{rect}\left(\frac{t}{T}\right)\,\mathrm{e}^{\mathrm{j}2\pi t/T} + \frac{1}{4}\,\mathrm{rect}\left(\frac{t}{T}\right)\,\mathrm{e}^{-\mathrm{j}2\pi t/T}$

$\circ\!\!-\!\!\bullet\ \underline{X}(f) = \frac{T}{2}\,\frac{\mathrm{si}(\pi f T)}{1-(fT)^2}$



20. a) $\underline{X}(f) = \frac{T}{1+\mathrm{j}2\pi f T}, \qquad |\underline{X}(f)| = \frac{T}{\sqrt{1+(2\pi f T)^2}}, \qquad \varphi_X(f) = -\arctan(2\pi f T)$



b) $\underline{X}(f) = \frac{1}{2}\delta(f) + \frac{1}{\mathrm{j}2\pi f}$

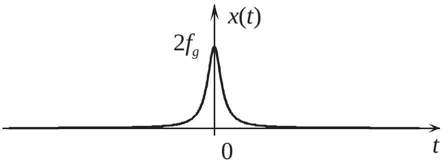
21. a) Reelles und symmetrisch gerades Zeitsignal

b) $x(t) = \dfrac{2 f_g}{1 + (2\pi f_g t)^2}$

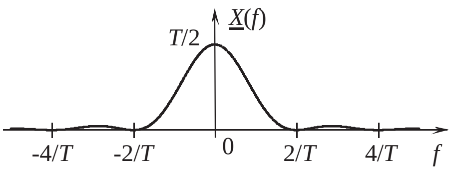
c) $\left(\dfrac{1}{2}\delta(f) + \dfrac{1}{j2\pi f}\right) \underline{X}(f) = \dfrac{1}{2}\delta(f) + \dfrac{1}{j2\pi f}\,\mathrm{e}^{-|f/f_g|}$

d) $\underline{X}(f) = 1 \bullet\!\!-\!\!\circ\; x(t) = \delta(t)$

e)



22. $\underline{X}(f) = T \cdot \mathrm{si}^2\left(\pi f T\right)$



23. $-4T \leqq \tau < 0$: $\quad r^{\mathrm{E}}_{xx}(\tau) = \dfrac{T}{4\pi}\sin\left(2\pi\dfrac{4T+\tau}{T}\right) + \dfrac{4T+\tau}{2}\cos\left(2\pi\dfrac{\tau}{T}\right)$

$0 \leqq \tau \leqq 4T$: $\quad r^{\mathrm{E}}_{xx}(\tau) = \dfrac{T}{4\pi}\sin\left(2\pi\dfrac{4T-\tau}{T}\right) + \dfrac{4T-\tau}{2}\cos\left(2\pi\dfrac{\tau}{T}\right)$

$r_{xx}(\tau) = \dfrac{T}{4\pi}\sin\left(2\pi\dfrac{4T-|\tau|}{T}\right) + \dfrac{T}{2}\Lambda\left(\dfrac{\tau}{4T}\right)\cos\left(2\pi\dfrac{\tau}{T}\right)$

$R_{XX} = 4T^2 \cdot \mathrm{si}^2\left(\pi\left(f + \dfrac{1}{T}\right)4T\right) + 4T^2 \cdot \mathrm{si}^2\left(\pi\left(f - \dfrac{1}{T}\right)4T\right) - \dfrac{T^2}{2\pi^2}\,\dfrac{\sin^2\left(\pi f 4T\right)}{1-\left(fT\right)^2}$

Lösungen zum Kapitel 6

24. a) $f_A \geqq 28\,\mathrm{MHz}$

b) Beim theoretischen Minimum würde als Rekonstruktionsfilter ein idealer Tiefpass benötigt, der nicht realisierbar ist.

25. $f_A \geqq 6{,}8\,\mathrm{kHz}$

26. a) $\dfrac{2 \cdot 1{,}05\,\mathrm{MHz}}{m} \leqq \dfrac{2 \cdot 0{,}75\,\mathrm{MHz}}{m-1} \Rightarrow (m-1) \cdot 1{,}05\,\mathrm{MHz} \leqq m \cdot 0{,}75\,\mathrm{MHz}$

$m \cdot 1{,}05\,\mathrm{MHz} - 1{,}05\,\mathrm{MHz} \leqq m \cdot 0{,}75\,\mathrm{MHz} \Rightarrow m \leqq \dfrac{1{,}05\,\mathrm{MHz}}{0{,}3\,\mathrm{MHz}} = 3$

b) $\dfrac{2 \cdot 1{,}05\,\text{MHz}}{m} \leqq f_\text{A} \leqq \dfrac{2 \cdot 0{,}75\,\text{MHz}}{m-1}$

$m = 1$: $\quad 2 \cdot 1{,}05\,\text{MHz} = 2{,}1\,\text{MHz} \quad \leqq f_\text{A} \leqq \infty$

$m = 2$: $\quad \dfrac{2 \cdot 1{,}05\,\text{MHz}}{2} = 1{,}05\,\text{MHz} \leqq f_\text{A} \leqq \dfrac{2 \cdot 0{,}75\,\text{MHz}}{1} = 1{,}5\,\text{MHz}$

$m = 3$: $\quad \dfrac{2 \cdot 1{,}05\,\text{MHz}}{3} = 0{,}70\,\text{MHz} \leqq f_\text{A} \leqq \dfrac{2 \cdot 0{,}75\,\text{MHz}}{2} = 0{,}75\,\text{MHz}$

c) $f_\text{A} = \dfrac{4}{2m-1}\,\dfrac{f_\text{gu} + f_\text{go}}{2} = \dfrac{4}{2m-1} \cdot 0{,}9\,\text{MHz}$

$m = 1$: $\quad f_\text{A} = \dfrac{4}{1} \cdot 0{,}9\,\text{MHz} = 3{,}6\,\text{MHz}$

$m = 2$: $\quad f_\text{A} = \dfrac{4}{3} \cdot 0{,}9\,\text{MHz} = 1{,}2\,\text{MHz}$

$m = 3$: $\quad f_\text{A} = \dfrac{4}{5} \cdot 0{,}9\,\text{MHz} = 0{,}72\,\text{MHz}$

27. a) Spektren

$$X_1\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\text{A}}\right) = 2\sin\left(2\pi f/f_\text{A}\right)\,\mathrm{e}^{-\mathrm{j}\,(4\pi f/f_\text{A} - \pi/2)}$$

$$\left|X_1\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\text{A}}\right)\right| = \left|2\sin\left(2\pi f/f_\text{A}\right)\right|$$

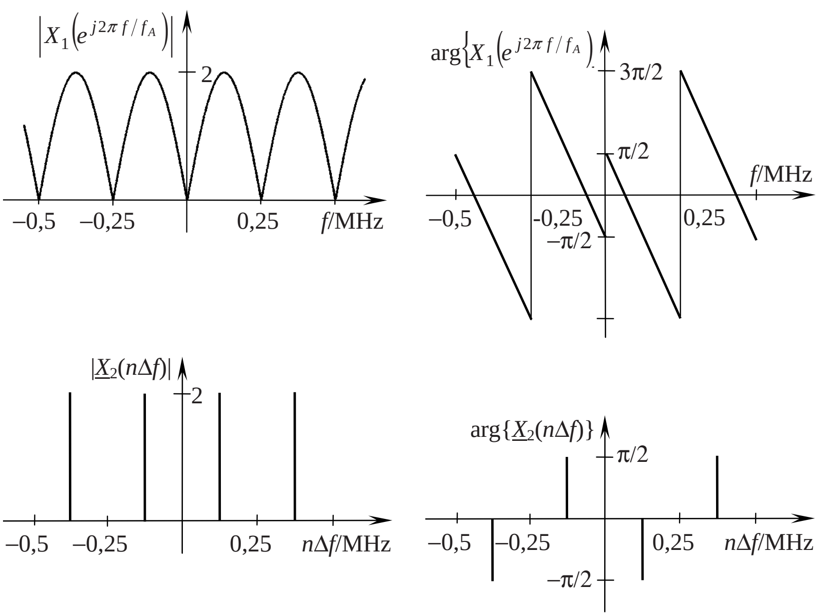
$$\arg\left\{X_1\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\text{A}}\right)\right\} = \begin{cases} -4\pi f/f_\text{A} + \pi/2 & \text{für} \quad \sin\left(2\pi f/f_\text{A}\right) \geqq 0 \\ -4\pi f/f_\text{A} + \pi/2 \pm \pi & \text{für} \quad \sin\left(2\pi f/f_\text{A}\right) < 0 \end{cases}$$

$$\left\{\underline{X}_2\left(n\Delta f\right)\right\} = \{\underline{0}; -2\,\mathrm{j}; 0; 2\,\mathrm{j}\}_\text{p}$$

$$\left\{\left|\underline{X}_2\left(n\Delta f\right)\right|\right\} = \{\underline{0}; 2; 0; 2\}_\text{p}$$

$$\arg\left\{\underline{X}_2\left(n\Delta f\right)\right\} = \{\underline{0}; -\pi/2; 0; \pi/2\}_\text{p}$$

b) Amplituden- und Phasenspektren



c) $X_1\left(\mathrm{e}^{\mathrm{j}\,2\pi n f_\mathrm{A}/4/f_\mathrm{A}}\right) = X_1\left(\mathrm{e}^{\mathrm{j}\,\pi n/2}\right) = 2\sin\left(\pi n/2\right)\ \mathrm{e}^{-\mathrm{j}\,(\pi n - \pi/2)}$

$\left\{X_1\left(\mathrm{e}^{\mathrm{j}\,\pi n/2}\right)\right\} = \{0; -2\,\mathrm{j}; 0; 2\,\mathrm{j}\} = \left\{\underline{X}_2\left(n\Delta f\right)\right\}$

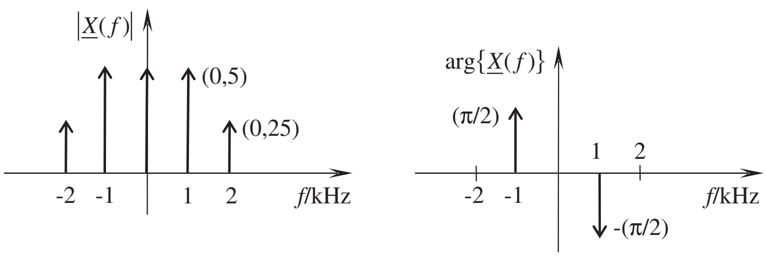
28. $\left\{\underline{X}_1\left(n\Delta f_1\right)\right\} = \{\underline{2};\ -0{,}25 - j0{,}433;\ -0{,}25 - j0{,}433\}_\mathrm{P}\,;\qquad \Delta f_1 = f_\mathrm{A}/3$

$\left\{\underline{X}_2\left(n\Delta f_2\right)\right\} = \{-\underline{1}; -\mathrm{j}3; 1; \mathrm{j}3\}_\mathrm{P}\,;\qquad \Delta f_2 = f_\mathrm{A}/4$

$\left\{\underline{X}_1\left(n\Delta f_1\right)\right\} \cdot \left\{\underline{X}_2\left(n\Delta f_2\right)\right\} = \left\{\underline{X}\left(n f_\mathrm{A}\right)\right\} = \{-2\}_\mathrm{P}$

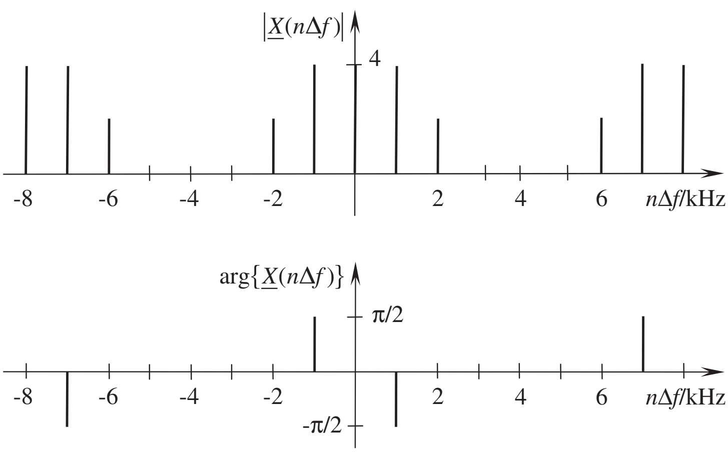
Für $f \neq n f_\mathrm{A}$ ist das Produkt nicht definiert.

29. a) $$\underline{X}(f) = \frac{1}{2}\Big(\delta(f) + \mathrm{j}\left(\delta\left(f + 1\,\mathrm{kHz}\right) - \delta\left(f - 1\,\mathrm{kHz}\right)\right) + \frac{1}{2}\left(\delta\left(f + 2\,\mathrm{kHz}\right) + \delta\left(f - 2\,\mathrm{kHz}\right)\right)\Big)$$



b) 8 Abtastwerte

c) $\Delta f = 1\,\mathrm{kHz}, \quad f_{\mathrm{A\,min}} = 4\,\mathrm{kHz}$



Lösungen Teil II

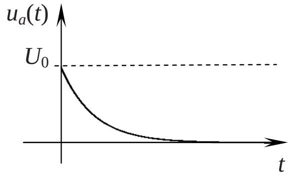
Lösungen zu den Kapiteln 9 bis 13

1. a) $\dot{u}_a(t)RC + u_a(t) = \dot{u}_e(t)RC$

   b) $u_a(t) = U_0\,e^{-t/RC} \cdot \varepsilon(t)$

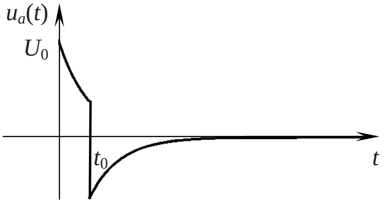
   c) $G(p) = \dfrac{RCp}{1 + RCp}$

   d) $u_a(t) = U_0\,e^{-t/RC} \cdot \varepsilon(t)$

   e) 

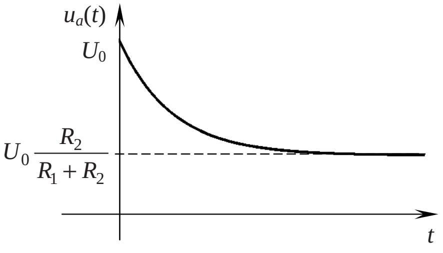
2. $$G(p) = \frac{RCp}{1 + RCp} = \frac{RCp + 1}{1 + RCp} - \frac{1}{1 + RCp} \;\bullet\!\!-\!\!\circ\; g(t) = \delta(t) - \frac{1}{RC}\,e^{-t/RC} \cdot \varepsilon(t)$$

$$u_a(t) = \begin{cases} U_0\,e^{-t/RC} & \text{für} \quad 0 \leqq t < t_0 \\ U_0\,e^{-t/RC}\left(1 - e^{t_0/RC}\right) & \text{für} \quad t > t_0 \end{cases}$$



3. a) $G(p) = \dfrac{R_1R_2Cp + R_2}{R_1R_2Cp + R_1 + R_2}$

   b) $u_a(t) = U_0\left(\dfrac{R_2}{R_1 + R_2} + \dfrac{R_1}{R_1 + R_2}\,e^{-\frac{R_1+R_2}{CR_1R_2}\cdot t}\right) \cdot \varepsilon(t)$



   c) $p_{01} = -\dfrac{1}{R_1C}$, $\quad p_{\infty 1} = -\dfrac{R_1 + R_2}{R_1R_2C}$, $\quad$ System ist stabil

4. a) $G(p) = \dfrac{K}{p}$

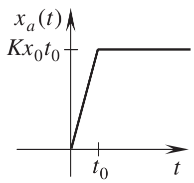
   b) $x_{e1}(t) = x_0 \varepsilon(t) \circ\!\!-\!\!\bullet\, X_{e1}(p) = \dfrac{x_0}{p}$

   $x_{e2}(t) = -x_0 \varepsilon(t - t_0) \circ\!\!-\!\!\bullet\, X_{e2}(p) = -\dfrac{x_0}{p}\,\mathrm{e}^{-pt_0}$

   c) $x_{a1}(t) = K x_0 r(t), \quad x_{a2}(t) = -K x_0 r\,(t - t_0)$

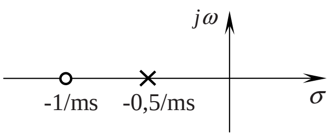
   $$x_{a1}(t) = \begin{cases} K x_0 r(t) & \text{für} \quad 0 \leqq t \leqq t_0 \\ K x_0 t_0 & \text{für} \quad t \geqq t_0 \end{cases}$$

   d)



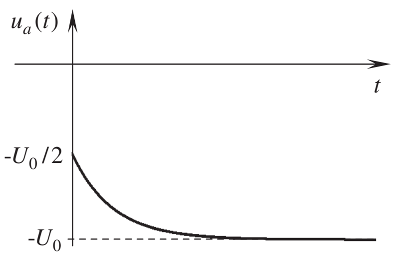
5. a) $G(p) = -\dfrac{R_2}{R_1} \cdot \dfrac{R_1 C_1 p + 1}{R_2 C_2 p + 1}$

   System ist stabil



   b) $u_a(t) = -U_0 \left(1 - \dfrac{1}{2}\,\mathrm{e}^{-t/R_2C_2}\right) \cdot \varepsilon(t) = -U_0 \left(1 - \dfrac{1}{2}\,\mathrm{e}^{-t/2\,\mathrm{ms}}\right) \cdot \varepsilon(t)$

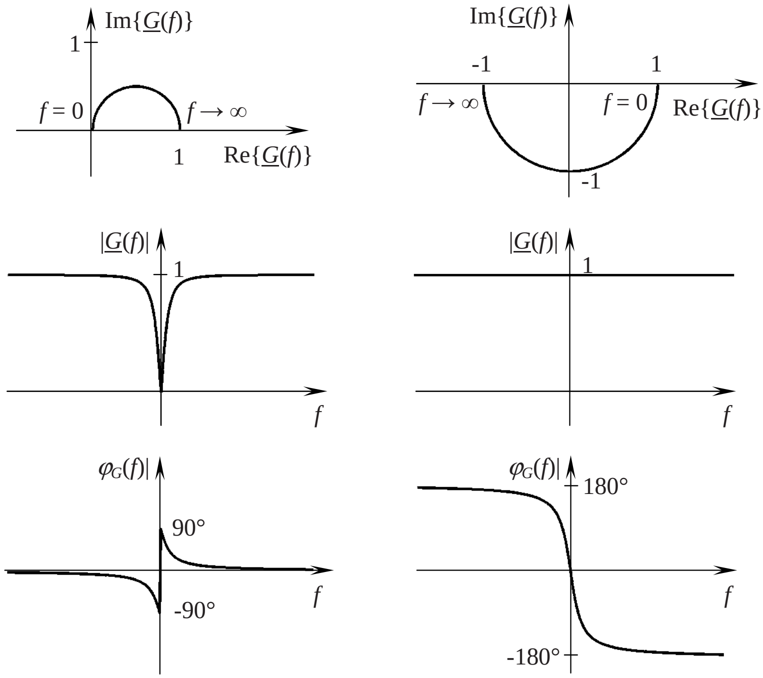
   c)



6. a) $\underline{G}(f) = \dfrac{\mathrm{j}2\pi f T}{1 + \mathrm{j}2\pi f T} = \sqrt{\dfrac{(2\pi f T)^2}{1 + (2\pi f T)^2}} \cdot \mathrm{e}^{\mathrm{j}\,(90^\circ \cdot \mathrm{sgn}(f) - \arctan(2\pi f T))}$

   $$\underline{G}(f) = \frac{1 - \mathrm{j}2\pi f T}{1 + \mathrm{j}2\pi f T} = \frac{\sqrt{1 + (2\pi f T)^2} \cdot \mathrm{e}^{-\mathrm{j}\arctan(2\pi f T)}}{\sqrt{1 + (2\pi f T)^2} \cdot \mathrm{e}^{\mathrm{j}\arctan(2\pi f T)}} = 1 \cdot \mathrm{e}^{-2\mathrm{j}\arctan(2\pi f T)}$$

b)



c) Amplitudengänge sind symmetrisch gerade Funktionen.
Phasengänge sind symmetrisch ungerade Funktionen.

7. a) $\underline{G}(f) = \mathrm{e}^{-\mathrm{j}2\pi f \cdot 1\,\mu\mathrm{s}}$

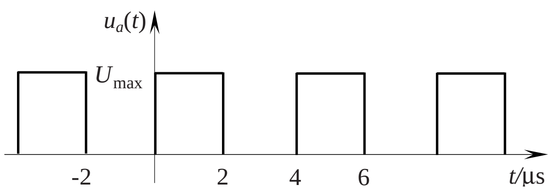
b) $u_\mathrm{e}(t) = \dfrac{U_\mathrm{max}}{2}\Big(\;\; 1 + 0{,}637\,\mathrm{e}^{\mathrm{j}2\pi f_\mathrm{P} t} \;\; - 0{,}212\,\mathrm{e}^{\mathrm{j}3\cdot 2\pi f_\mathrm{P} t} \;\; + - \ldots$
$\qquad \ldots + 0{,}637\,\mathrm{e}^{-\mathrm{j}2\pi f_\mathrm{P} t} - 0{,}212\,\mathrm{e}^{-\mathrm{j}3\cdot 2\pi f_\mathrm{P} t} + - \ldots \Big)$

mit $f_\mathrm{P} = 0{,}25\,\mathrm{MHz}$

c) $u_\mathrm{a}(t) = \dfrac{U_\mathrm{max}}{2}\Big(\;\; 1 + 0{,}637\,\mathrm{e}^{\mathrm{j}2\pi f_\mathrm{P}(t-1\,\mu\mathrm{s})} \;\; - 0{,}212\,\mathrm{e}^{\mathrm{j}3\cdot 2\pi f_\mathrm{P}(t-1\,\mu\mathrm{s})} \;\; + - \ldots$
$\qquad \ldots + 0{,}637\,\mathrm{e}^{-\mathrm{j}2\pi f_\mathrm{P}(t-1\,\mu\mathrm{s})} - 0{,}212\,\mathrm{e}^{-\mathrm{j}3\cdot 2\pi f_\mathrm{P}(t-1\,\mu\mathrm{s})} + - \ldots \Big)$

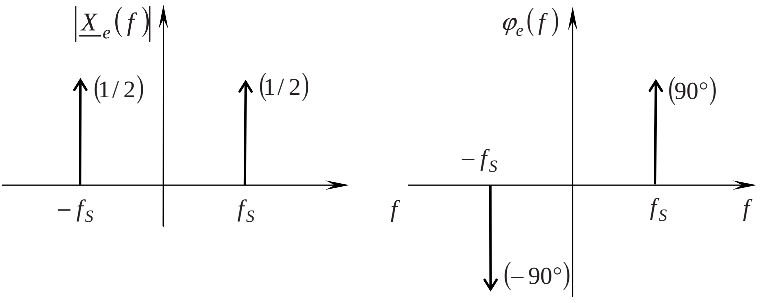
mit $f_\mathrm{P} = 0{,}25\,\mathrm{MHz}$

d)

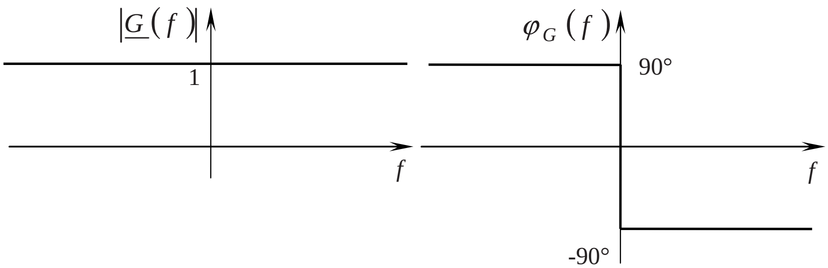


8. a) $T_S = 2\,\text{ms}, \quad f_S = 0{,}5\,\text{kHz}, \quad \omega_S = \pi \cdot 10^3\,\text{s}^{-1}, \quad x_e(t) = -\sin\left(2\pi f_S t\right)$

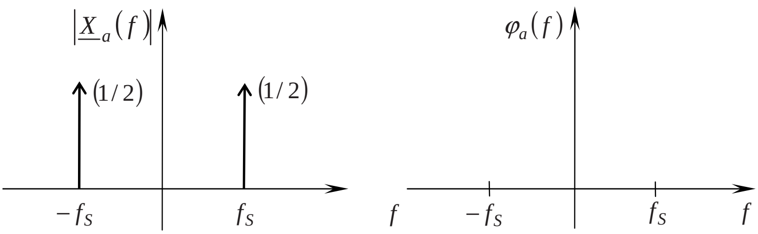
b) $x_e(t) = -\sin\left(2\pi f_S t\right) \circ\!\!-\!\!\bullet\ \underline{X}_e(f) = -\dfrac{1}{2}\,\text{j}\,\left(\delta\left(f + f_S\right) - \delta\left(f - f_S\right)\right)$



c)



d)



e) $x_a(t) = \cos\left(2\pi f_S t\right)$

9. a) Ordnung 2, da 2 unabhängige Energiespeicher (L, C)

b) $G(p) = \dfrac{1 + p\left(L/R + RC\right) + p^2 LC}{2 + p\left(L/R + RC\right) + p^2 LC}$

Polstellen:
Für $R = 30\,\Omega$, $L = 1\,\text{mH}$, $C = 1\,\mu\text{F}$ treten komplexe Pole mit negativem Realteil auf.

$$p_{\infty 1,2} = -\frac{1}{2}\left(\frac{R}{L} + \frac{1}{RC}\right) \pm \text{j}\sqrt{\frac{2}{LC} - \frac{1}{4}\left(\frac{R}{L} + \frac{1}{RC}\right)^2} = \left(-31\,667 \pm \text{j}\,31\,579\right)\ \text{s}^{-1}$$

Nullstellen:
Für $R = 30\,\Omega$, $L = 1\,\text{mH}$, $C = 1\,\mu\text{F}$ treten zwei einfache reelle Nullstellen auf.

$$p_{01,2} = -\frac{1}{2}\left(\frac{R}{L} + \frac{1}{RC}\right) \pm \sqrt{\frac{1}{4}\left(\frac{R}{L} + \frac{1}{RC}\right)^2 - \frac{1}{LC}}, \qquad p_{01} = -30\,000\,\text{s}^{-1},$$

$p_{02} = -33\,333\,\text{s}^{-1}$

c) Das System ist stabil. Alle Polstellen des Systems haben einen negativen Realteil.

d) $U_a(p) = 1\,\text{V}\dfrac{(p-p_{01})\,(p-p_{02})}{p\,(p-p_{\infty1})\,(p-p_{\infty2})}$

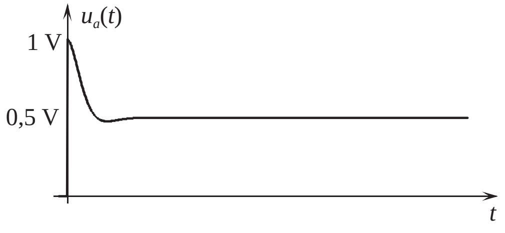
e) $U_a(p) = \dfrac{K_1}{p} + \dfrac{K_2}{p-p_{\infty1}} + \dfrac{K_3}{p-p_{\infty2}}$

$$K_1 = 1\,\text{V}\frac{p_{01}p_{02}}{p_{\infty1}p_{\infty2}} = 0{,}5\,\text{V}, \quad K_2 = 1\,\text{V}\frac{(p_{\infty1}-p_{01})\,(p_{\infty1}-p_{02})}{p_{\infty1}\,(p_{\infty1}-p_{\infty2})} = (0{,}25+\text{j}0{,}25)\ \text{V},$$

$$K_3 = 1\,\text{V}\frac{(p_{\infty2}-p_{01})\,(p_{\infty2}-p_{02})}{p_{\infty2}\,(p_{\infty2}-p_{\infty1})} = (0{,}25-\text{j}0{,}25)\ \text{V} = K_2^*$$

$$u_a(t) = \varepsilon(t)\left(K_1 + 2\,|K_2|\ \mathrm{e}^{\mathrm{Re}\{p_{\infty1}\}t}\cos\left(\mathrm{Im}\{p_{\infty1}\}\,t + \varphi_{K_2}\right)\right)$$

$$\text{mit } \varphi_{p\infty1} = \arctan\left(\frac{\mathrm{Im}\{K_2\}}{\mathrm{Re}\{K_2\}}\right) = 45^\circ$$



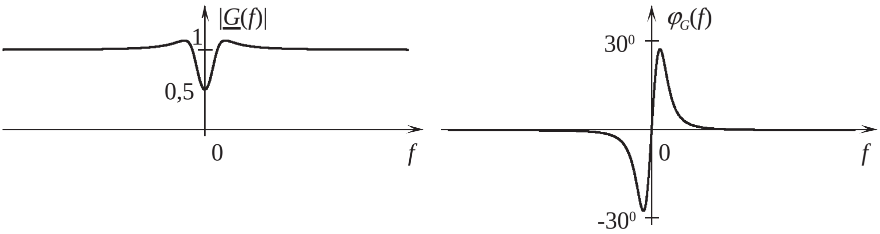
f) $\underline{G}(f) = \dfrac{1+\text{j}2\pi f\,(L/R+RC)-(2\pi f)^2\,LC}{2+\text{j}2\pi f\,(L/R+RC)-(2\pi f)^2\,LC}$

g) $|\underline{G}(f)| = \sqrt{\dfrac{\left(1-(2\pi f)^2\,LC\right)^2 + \left(2\pi f\,(L/R+RC)\right)^2}{\left(2-(2\pi f)^2\,LC\right)^2 + \left(2\pi f\,(L/R+RC)\right)^2}}$

$$\underline{G}(f) = \frac{1+\left(1-(2\pi f)^2\,LC\right)^2 + (2\pi f)^2\left(\left(\frac{L}{R}\right)^2 + LC + (RC)^2\right) + \text{j}2\pi f\left(\frac{L}{R}+RC\right)}{\left(2-(2\pi f)^2\,LC\right)^2 + (2\pi f)^2\left(\frac{L}{R}+RC\right)^2}$$

Da der Realteil des Frequenzgangs für alle Frequenzen positiv ist, lautet der Phasengang:

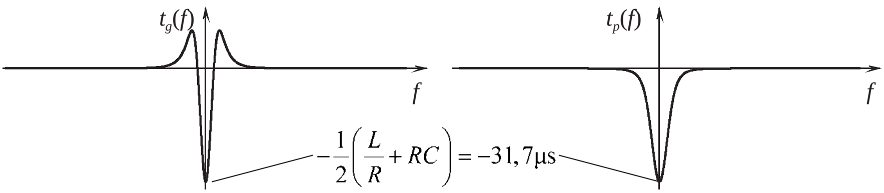
$$\varphi_G(f) = \arctan\left(\frac{2\pi f\,(L/R+RC)}{1+\left(1-(2\pi f)^2\,LC\right)^2 + (2\pi f)^2\left((L/R)^2 + LC + (RC)^2\right)}\right).$$



h) Gruppen- und Phasenlaufzeit

$$t_{\mathrm{g}}(f)=\frac{\left(\frac{L}{R}+RC\right)\left(-2-2\,(2\pi f)^2\,LC+3\left((2\pi f)^2\,LC\right)^2+(2\pi f)^2\left(\left(\frac{L}{R}\right)^2+LC+(RC)^2\right)\right)}{\left(2\pi f\left(\frac{L}{R}+RC\right)\right)^2+\left(1+\left(1-(2\pi f)^2LC\right)^2+(2\pi f)^2\left(\left(\frac{L}{R}\right)^2+LC+(RC)^2\right)\right)^2}$$

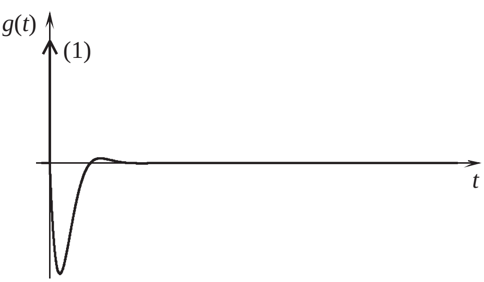
$$t_{\mathrm{p}}(f)=-\frac{1}{2\pi f}\arctan\left(\frac{2\pi f\,(L/R+RC)}{1+\left(1-(2\pi f)^2\,LC\right)^2+(2\pi f)^2\left((L/R)^2+LC+(RC)^2\right)}\right)$$



i) Impulsantwort

$$g(t)=\delta(t)+\frac{(p_{\infty 1}-p_{01})\,(p_{\infty 1}-p_{02})}{p_{\infty 1}-p_{\infty 2}}\varepsilon(t)\,\mathrm{e}^{p_{\infty 1}t}-\frac{(p_{\infty 2}-p_{01})\,(p_{\infty 2}-p_{02})}{p_{\infty 1}-p_{\infty 2}}\varepsilon(t)\,\mathrm{e}^{p_{\infty 2}t}$$

$$g(t)=\delta(t)-\varepsilon(t)\frac{1}{LC}\,\frac{\mathrm{e}^{p_{\infty 1}t}-\mathrm{e}^{p_{\infty 2}t}}{p_{\infty 1}-p_{\infty 2}}=\delta(t)-\varepsilon(t)\frac{\mathrm{e}^{\mathrm{Re}\{p_{\infty 1}\}t}}{LC\cdot\mathrm{Im}\{p_{\infty 1}\}}\sin\left(\mathrm{Im}\{p_{\infty 1}\}\,t\right)$$



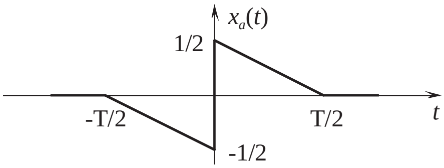
j) $$\underline{U}_{\mathrm{a}}(f)=\frac{1\,\mathrm{V}}{4}\delta(f)+\frac{1\,\mathrm{V}}{\mathrm{j}2\pi f}\cdot\frac{1+\mathrm{j}2\pi f\,(L/R+RC)-(2\pi f)^2\,LC}{2+\mathrm{j}2\pi f\,(L/R+RC)-(2\pi f)^2\,LC}$$

10. a) Nicht kausal da $g(t)$ zum Zeitpunkt $-T/2$ beginnt

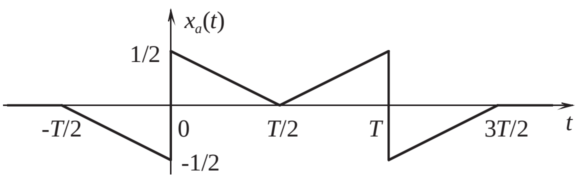
b) Stabil, da $\int\limits_{-\infty}^{\infty} |g(t)| \,\mathrm{d}t = \int\limits_{-\infty}^{-\infty} \delta(t) \,\mathrm{d}t + \frac{1}{T}\int\limits_{-T/2}^{T/2} \mathrm{rect}\left(\frac{t}{T}\right) \mathrm{d}t = 1 + 1 < \infty$

c) $\underline{G}(f) = 1 - \mathrm{si}\left(\pi f T\right)$

d) $x_\mathrm{a}(t) = \varepsilon(t) - \frac{1}{T} r\left(t + T/2\right) + \frac{1}{T} r\left(t - T/2\right)$



e) $x_\mathrm{a}(t) = \mathrm{rect}\left(\frac{t - T/2}{T}\right) - \Lambda\left(\frac{t - T/2}{T}\right)$



11. a) Tiefpass, da Frequenzen größer $3 f_\mathrm{g}$ unterdrückt

b) Der reelle und symmetrisch gerade Frequenzgang korrespondiert zu einer reellen und symmetrisch geraden Impulsantwort, die somit Anteile bei Zeiten $t < 0$ enthält. $\Rightarrow$ nicht kausal und somit nicht technisch realisierbar

c) $\underline{G}(f) = 3\Lambda\left(\frac{f}{3 f_\mathrm{g}}\right) - 2\Lambda\left(\frac{f}{2 f_\mathrm{g}}\right)$

d) $g(t) = 9 f_\mathrm{g}\,\mathrm{si}^2\left(3\pi f_\mathrm{g} t\right) - 4 f_\mathrm{g}\,\mathrm{si}^2\left(2\pi f_\mathrm{g} t\right)$

e) $g\left(2t\right) = 9 f_\mathrm{g}\,\mathrm{si}^2\left(6\pi f_\mathrm{g} t\right) - 4 f_\mathrm{g}\,\mathrm{si}^2\left(4\pi f_\mathrm{g} t\right) \circ\!\!-\!\!\bullet \frac{1}{2}\underline{G}\left(\frac{f}{2}\right) = \frac{3}{2}\Lambda\left(\frac{f}{6 f_\mathrm{g}}\right) - \Lambda\left(\frac{f}{4 f_\mathrm{g}}\right)$

12. a) $X(p) = \frac{\left(p + \tau^{-1} + \mathrm{j}2\pi f_0\right) + \left(p + \tau^{-1} - \mathrm{j}2\pi f_0\right)}{2\left(p + \tau^{-1} - \mathrm{j}2\pi f_0\right)\left(p + \tau^{-1} + \mathrm{j}2\pi f_0\right)} = \frac{p + \tau^{-1}}{\left(p + \tau^{-1}\right)^2 + \left(2\pi f_0\right)^2}$

b) $\ddot{x}(t) \circ\!\!-\!\!\bullet \frac{p^2\left(p + \tau^{-1}\right)}{\left(p + \tau^{-1}\right)^2 + \left(2\pi f_0\right)^2} - p x(0) - \dot{x}(0), \quad x(0) = 1, \quad \dot{x}(0) = -\tau^{-1}$

$\ddot{x}(t) \circ\!\!-\!\!\bullet \frac{p^2\left(p + \tau^{-1}\right)}{\left(p + \tau^{-1}\right)^2 + \left(2\pi f_0\right)^2} - p + \tau^{-1}$

c) $x(t) \circ\!\!-\!\!\bullet \underline{X}(f) = \frac{\tau^{-1} + \mathrm{j}2\pi f}{\left(\tau^{-1} + \mathrm{j}2\pi f\right)^2 + \left(2\pi f_0\right)^2}$

$\ddot{x}(t) \circ\!\!-\!\!\bullet \underline{X}(f) = -\left(2\pi f\right)^2 \frac{\tau^{-1} + \mathrm{j}2\pi f}{\left(\tau^{-1} + \mathrm{j}2\pi f\right)^2 + \left(2\pi f_0\right)^2}$

Lösungen zu den Kapiteln 14 bis 18

13. a) nichtlinear, da $e^{x_{e1}(kT_A)} + e^{x_{e2}(kT_A)} \neq e^{x_{e1}(kT_A)+x_{e2}(kT_A)}$

    b) linear, da $cx_{e1}\left((k-1)T_A\right) + cx_{e2}\left((k-1)T_A\right) = c\left(x_{e1}\left((k-1)T_A\right) + x_{e2}\left((k-1)T_A\right)\right)$

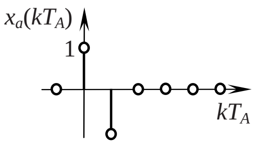
14. a) zeitinvariant, da

$$x_a\left(kT_A\right) = x_e\left(kT_A\right) - x_e\left((k-1)T_A\right) \rightarrow$$
$$x_a\left((k-n)T_A\right) = x_e\left((k-n)T_A\right) - x_e\left((k-n-1)T_A\right)$$

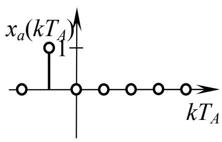
    b) zeitvariant, da

$$x_a\left(kT_A\right) = x_e\left(-kT_A\right) \rightarrow$$
$$x_a\left((k-n)T_A\right) = x_e\left(-(k-n)T_A\right) = x_e\left((n-k)T_A\right)$$

15. a) kausal



    b) nichtkausal



16. a) System 1: Lösung der DZGL mit Ansatzverfahren

    Da als Eingangssignal der Einheitsimpuls anliegt, ergibt sich für die eine der zwei benötigten Anfangsbedingungen $x_a(0) = 1$. Die zweite Anfangsbedingung $x_a(-T_A) = 0$ resultiert aus der Tatsache, dass das System kausal ist. Da das vollständige Eingangssignal schon beim Festlegen der Anfangsbedingung berücksichtigt wird, ist eine partikuläre Lösung $x_{api}\left(kT_A\right) = 0$.

$$x_a\left(kT_A\right) = \frac{1}{\lambda_1 - \lambda_2}\left(\lambda_1^{k+1} - \lambda_2^{k+1}\right) \cdot \varepsilon\left(kT_A\right) \text{ mit } \lambda_{1,2} = -\frac{1}{2} \pm j\frac{1}{2}\sqrt{3}$$

$$x_a\left(kT_A\right) = \frac{2}{\sqrt{3}}\sin\left(\frac{2}{3}\pi\left(k+1\right)\right) \cdot \varepsilon\left(kT_A\right)$$

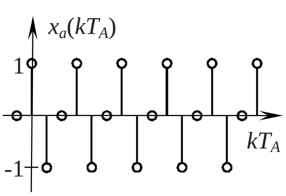
    System 1: Lösung der DZGL mit Rekursionsverfahren

$$\left\{x_a\left(kT_A\right)\right\} = \{\underline{1};\ -1;\ 0;\ 1;\ -1;\ 0;\ 1;\ -1;\ 0;\ \ldots\}$$

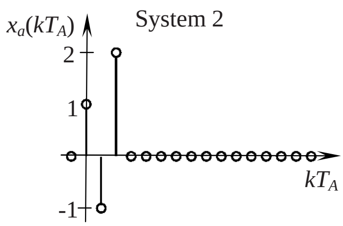
    System 2: Lösung der DZGL mit Rekursionsverfahren und Einsetzverfahren

$$\left\{x_a\left(kT_A\right)\right\} = \left\{\delta\left(kT_A\right)\right\} - \left\{\delta\left((k-1)T_A\right)\right\} + \left\{2\delta\left((k-2)\ T_A\right)\right\} = \{\underline{1}; -1; 2\}$$

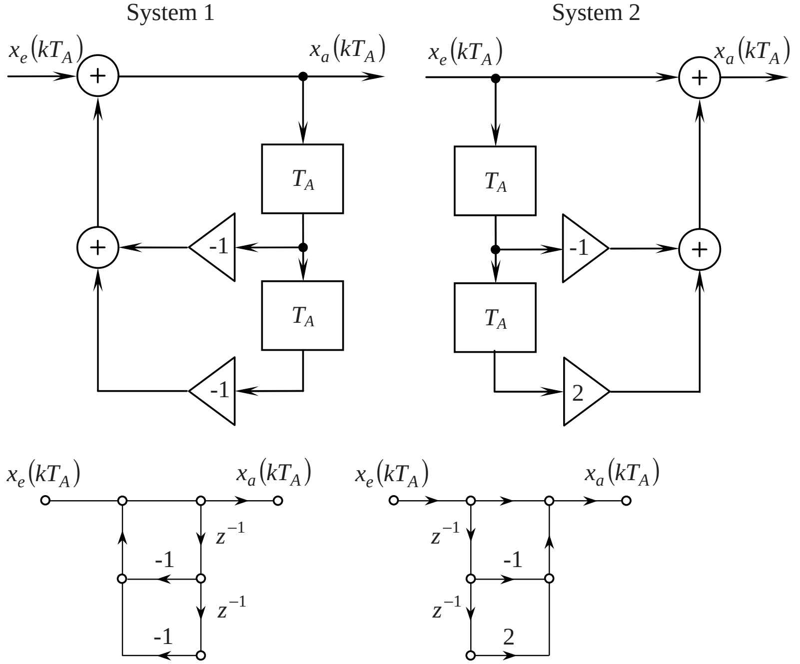
System 1



System 2



b)



c) System 1: $G(z) = \dfrac{1}{1 + z^{-1} + z^{-2}}$, $\quad z_{\infty 1,2} = -0{,}5 \pm \mathrm{j}\,0{,}866 = 1 \cdot \mathrm{e}^{\pm \mathrm{j}\,120^\circ}$, $\quad z_{01,2} = 0$

Alle Polstellen liegen auf dem Einheitskreis, das System ist instabil.

System 2: $G(z) = 1 - z^{-1} + 2z^{-2}$, $\quad z_{\infty 1,2} = 0$, $\quad z_{01,2} = -0{,}5 \pm \mathrm{j}\,1{,}3229$

Alle Polstellen liegen im Einheitskreis, das System ist stabil.

17. a) $x_\mathrm{a}\left(kT_\mathrm{A}\right) - a_1 x_\mathrm{a}\left((k-1)T_\mathrm{A}\right) = x_\mathrm{e}\left(kT_\mathrm{A}\right) + b_1 x_\mathrm{e}\left((k-1)T_\mathrm{A}\right)$, $\quad G(z) = \dfrac{1 + b_1 z^{-1}}{1 - a_1 z^{-1}}$

b) $|\, a_1 \,| < 1$ und b_1 beliebig

c) $\left\{g\left(kT_\mathrm{a}\right)\right\} = \left\{\varepsilon\left(kT_\mathrm{a}\right)\right\}$

18. a) $\left\{x_\mathrm{e}\left(kT_\mathrm{a}\right)\right\} = \{\underline{1}; 1; 1\}$

$$= \left\{\delta\left(kT_\mathrm{a}\right) + \delta\left((k-1)T_\mathrm{a}\right) + \delta\left((k-1)T_\mathrm{a}\right)\right\} \circ\!\!-\!\!\bullet\; X_\mathrm{e}(z) = 1 + z^{-1} + z^{-2}$$

$$\left\{x_\mathrm{a}\left(kT_\mathrm{a}\right)\right\} = \{\underline{0}; 1; 0; -1\} = \left\{\delta\left((k-1)T_\mathrm{a}\right) - \delta\left((k-3)T_\mathrm{a}\right)\right\} \circ\!\!-\!\!\bullet\; X_\mathrm{a}(z) = z^{-1} - z^{-3}$$

b) $G(z) = \dfrac{z^{-1} - z^{-3}}{1 + z^{-1} + z^{-2}}$

c) $x_\mathrm{a}\left(kT_\mathrm{a}\right) + x_\mathrm{a}\left((k-1)T_\mathrm{a}\right) + x_\mathrm{a}\left(\left(k-2\right)T_\mathrm{a}\right) = x_\mathrm{e}\left((k-1)T_\mathrm{a}\right) - x_\mathrm{e}\left((k-3)T_\mathrm{a}\right)$

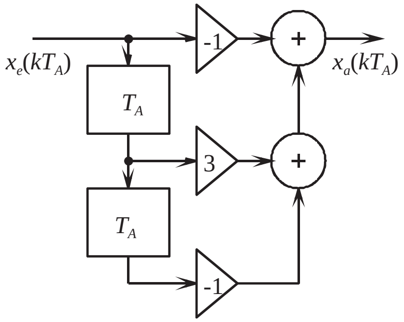
19. a) $x_\mathrm{a}\left(kT_\mathrm{a}\right) + x_\mathrm{a}\left((k-1)T_\mathrm{a}\right) - x_\mathrm{a}\left((k-2)T_\mathrm{a}\right) = x_\mathrm{e}\left((k-1)T_\mathrm{a}\right)$

b) $\left\{g\left(kT_\mathrm{a}\right)\right\} = \{0;\ 1;\ -1;\ 2;\ -3;\ 5;\ -8;\ \ldots\}$

c) $G(z) = \dfrac{z^{-1}}{1 + z^{-1} - z^{-2}}$, $\quad z_{\infty 1} = -1{,}618$, $\quad z_{\infty 2} = 0{,}618$, $\quad z_{01} = 0$

Eine Polstelle liegt außerhalb des Einheitskreises, das System ist instabil.

20. a) $x_\text{a}\left(kT_\text{a}\right) = -x_\text{e}\left(kT_\text{a}\right) + 3x_\text{e}\left((k-1)T_\text{a}\right) - x_\text{e}\left((k-2)T_\text{a}\right)$



b) $\{x_\text{a}(kT_\text{A})\} = \{-\underline{1}; 2; 1; 1; 1; \ldots\}$

c) $\{x_\text{a}(kT_\text{A})\} = \{-\underline{1}; 2; 1; 1; 1; 2; -1; 0; 0; 0; \ldots\}$

d) $\{x_\text{a}(kT_\text{A})\} = \{-\underline{1}; 2{,}5; 0; 0; 0; 0; \ldots\} + \left\{\varepsilon\left((k-2)T_\text{A}\right) 0{,}5^k\right\}$

e) $X_\text{a}(z) = \dfrac{-z^2+3z-1}{z^2} \cdot \dfrac{z}{z-1} = -\dfrac{z}{z-1} + 3z^{-1}\dfrac{z}{z-1} - z^{-2}\dfrac{z}{z-1}$ b)

$\bullet\!\!-\!\!\circ \; \{x_\text{a}\left(kT_\text{A}\right)\} = -\{\varepsilon\left(kT_\text{A}\right)\} + 3\{\varepsilon\left((k-1)T_\text{A}\right)\} - \{\varepsilon\left((k-2)T_\text{A}\right)\}$

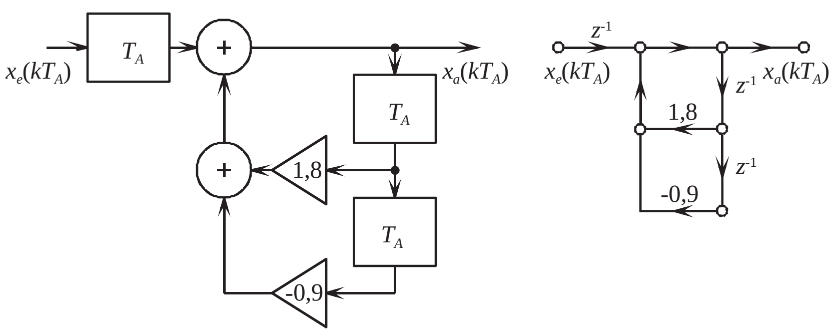
$X_\text{a}(z) = \dfrac{-z^2+3z-1}{z^2} \cdot \dfrac{z}{z-1}\left(1 - z^{-5}\right)$ c)

$\bullet\!\!-\!\!\circ \; \left\{x_\text{a}\left(kT_\text{A}\right)\right\} = -\left\{\text{rect}_5\left(kT_\text{A}\right)\right\} + 3\left\{\text{rect}_5\left((k-1)T_\text{A}\right)\right\} - \left\{\text{rect}_5\left((k-2)T_\text{A}\right)\right\}$

$X_\text{a}(z) = \dfrac{-z^2+3z-1}{z^2} \cdot \dfrac{z}{z-0{,}5} = -\dfrac{z}{z-0{,}5} + 3z^{-1}\dfrac{z}{z-0{,}5} - z^{-2}\dfrac{z}{z-0{,}5}$ d)

$\bullet\!\!-\!\!\circ \; \left\{x_\text{a}\left(kT_\text{A}\right)\right\} = -\left\{\varepsilon\left(kT_\text{A}\right) 0{,}5^k\right\} + 3\left\{\varepsilon\left((k-1)T_\text{A}\right) 0{,}5^{k-1}\right\} - \left\{\varepsilon\left((k-2)T_\text{A}\right) 0{,}5^{k-2}\right\}$

21. a)



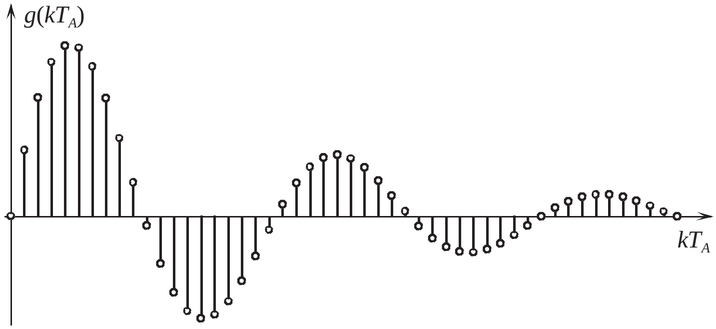
b) $X_\text{a}(z) = \dfrac{z}{z^2 - 1{,}8z + 0{,}9} X_\text{e}(z)$

c) $G(z) = \dfrac{z}{z^2 - 1{,}8z + 0{,}9}$

d) $z_{\infty 1,2}^2 - 1{,}8z_{\infty 1,2} + 0{,}9 = 0 \Rightarrow z_{\infty 1,2} = 0{,}9 \pm \sqrt{0{,}9^2 - 0{,}9} = 0{,}9 \pm \text{j}0{,}3$

Stabil, da $|\,z_{\infty 1,2}\,| < 1$

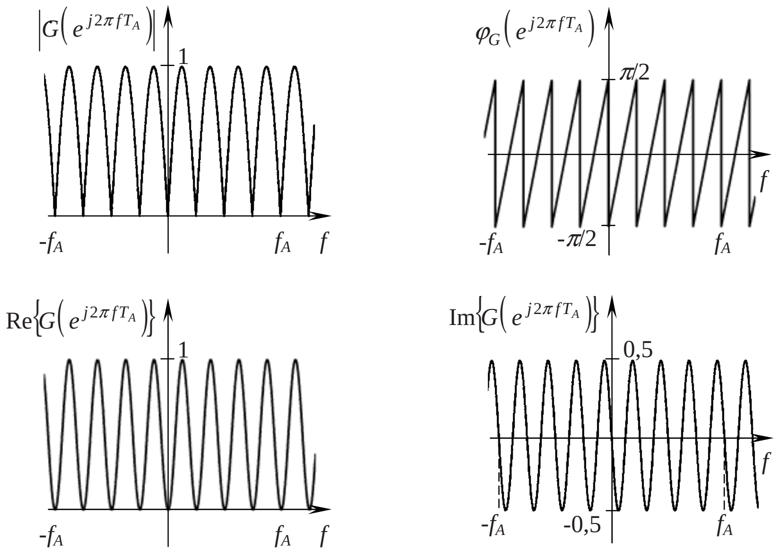
e) $\left\{g\left(kT_\mathrm{A}\right)\right\} = \left\{\varepsilon\left((k-1)T_\mathrm{A}\right)\dfrac{z_{\infty 1}^{k} - z_{\infty 2}^{k}}{z_{\infty 1} - z_{\infty 2}}\right\}$



f) $\left\{x_\mathrm{a}\left(kT_\mathrm{A}\right)\right\} = -\left\{\varepsilon\left((k-1)T_\mathrm{A}\right)\dfrac{z_{\infty 1}^{k} - z_{\infty 2}^{k}}{z_{\infty 1} - z_{\infty 2}}\right\}$

$$+2\left\{\varepsilon\left((k-2)T_\mathrm{A}\right)\frac{z_{\infty 1}^{k-1} - z_{\infty 2}^{k-1}}{z_{\infty 1} - z_{\infty 2}}\right\} - \left\{\varepsilon\left((k-3)T_\mathrm{A}\right)\frac{z_{\infty 1}^{k-2} - z_{\infty 2}^{k-2}}{z_{\infty 1} - z_{\infty 2}}\right\}$$

22. a)



b) Amplitudengang und Realteil des Frequenzgangs sind symmetrisch gerade Funktionen. Phasengang und Imaginärteil des Frequenzgangs sind symmetrisch ungerade Funktionen.

23. a) $X_\mathrm{e}\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}}\right) = 1 + \mathrm{e}^{-\mathrm{j}2\pi f/f_\mathrm{A}} + \mathrm{e}^{-\mathrm{j}4\pi f/f_\mathrm{A}} + \mathrm{e}^{-\mathrm{j}6\pi f/f_\mathrm{A}}$

b) $X_\mathrm{a}\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}}\right) = 0{,}5\left(1 + 2e^{-\mathrm{j}2\pi f/f_\mathrm{A}} + 2e^{-\mathrm{j}4\pi f/f_\mathrm{A}} + 2e^{-\mathrm{j}6\pi f/f_\mathrm{A}} + \mathrm{e}^{-\mathrm{j}8\pi f/f_\mathrm{A}}\right)$

$$X_\mathrm{a}\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}}\right) = \left(0{,}5\cdot\mathrm{e}^{\mathrm{j}4\pi f/f_\mathrm{A}} + \mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}} + 1 + \mathrm{e}^{-\mathrm{j}2\pi f/f_\mathrm{A}} + 0{,}5\cdot\mathrm{e}^{-\mathrm{j}4\pi f/f_\mathrm{A}}\right)\mathrm{e}^{-\mathrm{j}4\pi f/f_\mathrm{A}}$$

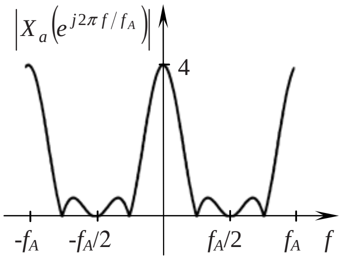
$$X_\mathrm{a}\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}}\right) = \left(1 + \cos\left(4\pi f/f_\mathrm{A}\right) + 2\cos\left(2\pi f/f_\mathrm{A}\right)\right)\mathrm{e}^{-\mathrm{j}4\pi f/f_\mathrm{A}}$$

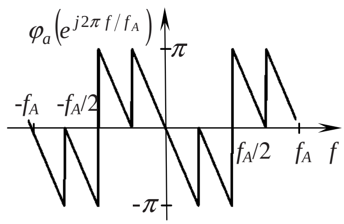
c) Amplitudenspektrum des Ausgangssignals

$$\left|X_\mathrm{a}\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}}\right)\right| = \left|1 + \cos\left(4\pi f/f_\mathrm{A}\right) + 2\cos\left(2\pi f/f_\mathrm{A}\right)\right|$$

Phasenspektrum des Ausgangssignals

$$\varphi_\mathrm{a}\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}}\right) = \begin{cases} -4\pi f/f_\mathrm{A} & \text{für}\quad 1 + \cos\left(4\pi f/f_\mathrm{A}\right) + 2\cos\left(2\pi f/f_\mathrm{A}\right) \geqq 0 \\ -4\pi f/f_\mathrm{A} \pm \pi & \text{für}\quad 1 + \cos\left(4\pi f/f_\mathrm{A}\right) + 2\cos\left(2\pi f/f_\mathrm{A}\right) < 0 \end{cases}$$





d) $$\mathrm{IDTFT}\left\{X_\mathrm{a}\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}}\right)\right\} = x_\mathrm{a}\left(kT_\mathrm{A}\right) = \frac{1}{f_\mathrm{A}}\int\limits_{-f_\mathrm{A}/2}^{f_\mathrm{A}/2} X_\mathrm{a}\left(\mathrm{e}^{\mathrm{j}2\pi f/f_\mathrm{A}}\right)\mathrm{e}^{\mathrm{j}k2\pi f/f_\mathrm{A}}\,\mathrm{d}f$$

$$\left\{x_\mathrm{a}\left(kT_\mathrm{A}\right)\right\} = \{\underline{1}; 2; 2; 2; 1\}$$