SUBJECT: MATHEMATICS

CLASS: SS3

TOPIC: SURDS

OBJECTIVES OF LESSON

At the end of the lesson ,learners should be able to:

- Identify surds and similar surds
- Simplify surds into their basic forms
- Add, subtract, multiply and divide in surds
- Rationalize surds using the conjugate

SURDS

- A number which can be expressed as a quotient m/n of two integers, (provided that n is not equal to 0) is called a rational number. Any real number which is not rational is irrational. Irrational numbers which are in the form of roots are called SURDS. number.

- In general, SURDS ARE IRRATIONAL NUMBERS of the form $a\sqrt{b}$,where a is a rational number and $\sqrt{b}$ is a irrational. Irrational numbers are $\sqrt{2}$,$\sqrt{3}$,$\sqrt{5}$, π,etc. We can also have surds as $\sqrt[3]{5}$, $\sqrt[5]{27}$, and so on, but we shall deal with surds which are SQUARE ROOTS.

Rules for surds

1. $\sqrt{mn} = \sqrt{m} \times \sqrt{n}$ eg. $\sqrt{2 \times 6} = \sqrt{2} \times \sqrt{6} = \sqrt{12}$
2. $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$ eg. $\frac{\sqrt{35}}{\sqrt{5}} = \sqrt{\frac{35}{5}} = \sqrt{7}$
3. $a\sqrt{m} + b\sqrt{m} = (a + b)\sqrt{m}$ eg. $2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$
4. $a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$ eg. $\sqrt{3} - 4\sqrt{3} = -3\sqrt{3}$
5. $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$ eg. $2\sqrt{3} \times 4\sqrt{5} = 8\sqrt{15}$
6. $\sqrt{m} \times \sqrt{m} = \sqrt{m^2} = m$ eg. $\sqrt{3} \times \sqrt{3} = 3$

SIMILAR SURDS OR LIKE SURDS

- SURDS are similar if they have the same radical sign and radicand. Given the $\sqrt{3}$, the radical sign is the square root and 3 is the radicand. example of similar surds are $\sqrt{2}$,$2\sqrt{2}$, $-4\sqrt{2}$ etc. Similar surds can be added or subtracted. mixed surds are unlike surds, such as $2\sqrt{2}$,$2\sqrt{7}$,$3\sqrt{3}$, etc. mixed surds cannot be added or subtracted, that is they cannot be simplified further.

EXAMPLE 1

Simplify $3\sqrt{2} + \sqrt{5} + 3\sqrt{2} + 2\sqrt{5}$

Solution

Collect like surds

$$3\sqrt{2} + 3\sqrt{2} + \sqrt{5} + 2\sqrt{5} = 6\sqrt{2} + 3\sqrt{5}$$

EXAMPLE 2

simplify $3\sqrt{7} - 4\sqrt{7} - 3$

Solution

(3-4)$\sqrt{7}$ -3 = $-\sqrt{7} - 3$

NOTE: The surd must be reduced to their basic forms before adding or subtracting the similar surds.

REDUCTION TO BASIC FORM

$\sqrt{n}$ is in basic form , if n does not contain a factor which is a perfect square.

Thus, $\sqrt{5\,,}\ \sqrt{19}\ and\ \sqrt{10}$ are in basic form and can be simplified no further.$\sqrt{20}$, $\sqrt{50}$, $\sqrt{108}$ are not in basic form as they can be reduced further through simplification .

Examples: Simplify the following (a) $\sqrt{18}$ (b) $\sqrt{108}$ (c) $\sqrt{80}$

SOLUTION

1. $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

2. $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$

3. $\sqrt{108} = \sqrt{36 \times 3} = 6\sqrt{3}$

ADDITION AND SUBTRACTION OF SURDS

In surds or roots, we can add or subtract like surds in the same manner as like terms in algebraic letters .Like surds are surds that have the same number under the square root sign.

For example, $2\sqrt{3}$, $\sqrt{3}$, $3\sqrt{3}$, $\frac{\sqrt{3}}{5}$

Examples: Simplify the following

(a) $4\sqrt{18} - \sqrt{200} + 3\sqrt{50}$

(b) $\sqrt{48} + 4\sqrt{90} - 2\sqrt{40} + \sqrt{27}$

SOLUTIONS

a) $4\sqrt{9 \times 2} - \sqrt{100 \times 2} + 3\sqrt{25 \times 2}$

$$4 \times 3\sqrt{2} \quad - 10\sqrt{2} \;\; + 3 \times 5\sqrt{2}$$
$$12\sqrt{2} - 10\sqrt{2} \;\; + 15\sqrt{2}$$
$$17\sqrt{2}$$

(b) $\sqrt{16 \times 3} + 4\sqrt{9 \times 10} - 2\sqrt{4 \times 10} \;\; + \sqrt{9 \times 3}$

$$= 4\sqrt{3} + 4 \times 3\sqrt{10} - 2 \times 2\sqrt{10} + 3\sqrt{3}$$

$collecting\ like\ surds$

$$= 4\sqrt{3} + 3\sqrt{3} + 12\sqrt{10} - 4\sqrt{10}$$

$$= 7\sqrt{3} + 8\sqrt{10}$$

MULTIPLICATION OF SURDS

To multiply surds;

A) Simplify the surds, if possible

B) Group the number together, surds together and then multiply

C) Simplify further if possible

Examples: Simplify the following

A) $\sqrt{18} \times \sqrt{32}$

B) $\left(\sqrt{3} + \sqrt{5}\right)(2\sqrt{3} + 3\sqrt{5})$

C) $(\sqrt{2} - 2\sqrt{3})^2$

SOLUTION

a) $\sqrt{18} \times \sqrt{32} \quad = \quad \sqrt{9 \times 2} \times \sqrt{16 \times 2}$

$$= 3\sqrt{2} \times 4\sqrt{2}$$

Multiply the numbers and then multiply the surds.

$$= 3 \times 2 \times \sqrt{2} \times \sqrt{2}$$

$$= 12 \times 2 \; = 24$$

Example B :-

$$(\sqrt{3}+\sqrt{5})\left(2\sqrt{3}+3\sqrt{5}\right) = \sqrt{3}\left(2\sqrt{3}+3\sqrt{5}\right)+\sqrt{5}(2\sqrt{3}+3\sqrt{5})$$

$$= 2\times 3+3\sqrt{15}+2\sqrt{15}+3\times 5$$

$$= 6+5\sqrt{15}+15$$

$$= 6+15+5\sqrt{15}$$

$$= 21+5\sqrt{15}$$

Example c:-

$$(\sqrt{2} - 2\sqrt{3})(\sqrt{2} - 2\sqrt{3}) \quad = \quad \sqrt{2}(\sqrt{2} - 2\sqrt{3}) - 2\sqrt{3}(\sqrt{2} - 2\sqrt{3})$$

$$= 2 - 2\sqrt{6} - 2\sqrt{6} + 4 \times 3$$

$$= 2 + 12 - 4\sqrt{6}$$

$$= 14 - 4\sqrt{6}$$

DIVISION OF SURDS

To divide surds:

Simplify the fraction if necessary

If the denominator has a surd, then rationalize it. To rationalize the denominator means to eliminate the surd in the denominator by multiplying both the numerator and the denominator.

Simplify further if possible

Example: Simplify the following

$(a)\quad \dfrac{3}{\sqrt{5}}$

$(b)\quad \dfrac{5}{\sqrt{12}}$

$(c)\quad \dfrac{5\sqrt{2}}{2\sqrt{5}}$

SOLUTION

a) $\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}$

b) $\frac{5}{\sqrt{12}} = \frac{5}{\sqrt{4\times 3}} = \frac{5}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{10\sqrt{3}}{4\times 3} = \frac{10\sqrt{3}}{12} = \frac{5\sqrt{3}}{6}$

c) $\frac{5\sqrt{2}}{2\sqrt{5}} = \frac{5\sqrt{2}}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{10}}{2\sqrt{25}} = \frac{5\sqrt{10}}{2\times 5} = \frac{5\sqrt{10}}{10} = \frac{\sqrt{10}}{2}$

CONJUGATE SURDS

Given the surd expression $a+\sqrt{b}$,then $a-\sqrt{b}$ is called its conjugate.

Simplify the following by rationalizing the denominators:

(a) $2/3\sqrt{5}+4$ (b) $2\sqrt{3}+2/2\sqrt{3}-2$

SOLUTION

A) $$= \frac{2}{3\sqrt{5}+4} \times \frac{3\sqrt{5}-4}{3\sqrt{5}-4}$$

$$= \frac{2(3\sqrt{5}-4)}{3\sqrt{5}\left(3\sqrt{5}-4\right)4(3\sqrt{5}-4)}$$

$$\frac{6\sqrt{5}-8}{9\times5-12\sqrt{5}+12\sqrt{5}-16}$$

$$\frac{6\sqrt{5}-8}{45-16}$$

$$\frac{6\sqrt{5}-8}{29}$$

B)

$$\frac{2\sqrt{3}+2}{2\sqrt{3}-2} \times \frac{2\sqrt{3}+2}{2\sqrt{3+2}}$$

$$\frac{2\sqrt{3}\left(2\sqrt{3}+2\right)+2(2\sqrt{3}+2)}{2\sqrt{3}\,(2\sqrt{3}+2)-2(2\sqrt{3}+2}$$

$$\frac{4\times 3+4\sqrt{3}+4\sqrt{3}+4}{4\times 3+4\sqrt{3}-4\sqrt{3}-4}$$

$$\frac{12+4+8\sqrt{3}}{12-4}$$

$$\frac{16+8\sqrt{3}}{8}$$

$$\frac{16}{8}+\frac{8\sqrt{3}}{8}$$

$$2+\sqrt{3}$$

EVALUATION

1) Simplify the following;

a) $\sqrt{128} - 2\sqrt{72} + \sqrt{32} + \sqrt{8}$

b) $3\sqrt{48} + \sqrt{192} + 3\sqrt{12} - \sqrt{147}$

2) Simplify the following by rationalizing the denominators:

a) $\frac{2\sqrt{5}}{\sqrt{3}}$

b) $\frac{2}{\sqrt{11}}$

3) Simplify the following:

a) $(\sqrt{3}-4)(\sqrt{3}+3)$

b) $(5\sqrt{2}+2\sqrt{3})(5\sqrt{2}-2\sqrt{3})$

4) Express the following in terms the form $a+b\sqrt{c}$:

a) $\frac{\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

$b)$ $\frac{\sqrt{3}-3\sqrt{2}}{2\sqrt{3}-2\sqrt{2}}$

c) $\frac{3\sqrt{3}+2}{3\sqrt{3}-2}$

THANK YOU
FOR WATCHING