SUBJECT: MATHEMATICS

TOPIC: MATRICES AND DETERMINANTS I

CLASS: SSS 3

LESSON OBJECTIVES

1. Define matrix and recognize matrix order and notation.
2. State the different types of matrices.
3. Add and subtract two or more matrices.
4. Multiply matrices by a scalar quantity.
5. Multiply two matrices.

MATRICES AND DETERMINANTS

Definition

A matrix is a set of numbers arranged in rows and columns to form a rectangular pattern or array and enclosed inside curved brackets. The plural of matrix is matrices. The following are examples of matrices. $\begin{pmatrix} 2 & 3 \\ -5 & 1 \end{pmatrix}$, $(1\ 3\ 5)$, $\begin{pmatrix} 7 \\ 0 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 5 & 1 & 3 \\ 2 & 4 & 4 \end{pmatrix}$

Order of a Matrix

Consider the matrix below

$$\begin{array}{cc}
\begin{matrix}\text{1st Column} & \text{2nd Column}\end{matrix} & \\
\begin{pmatrix}4 & 2\\ 1 & 3\\ 0 & -5\end{pmatrix} & \begin{matrix}\text{1st Row}\\ \text{2nd Row}\\ \text{3rd Row}\end{matrix}
\end{array}$$

Each number in a matrix is called an <u>element</u> of that matrix. The above matrix has 6 elements. These elements are 4, 2, 1, 3, 0, -5. The elements of a matrix are often arranged in <u>rows</u> and <u>columns</u>. For example, the matrix above has 3 rows and 2 columns and it is called a <u>matrix</u> of <u>order</u> 3×2.

Remember that when describing the order of a matrix the number of rows is stated first and the number of columns second. In general, a matrix having <u>m</u> rows and <u>n</u> columns has order <u>m</u> x <u>n</u> read as ‘m by n’.

a. $\begin{pmatrix} 2 & 4 \\ 0 & -1 \end{pmatrix}$

Order

2 x 2

It has 4 elements

b. $\begin{pmatrix} 3 & 2 & 4 \end{pmatrix}$

Order

1 x 3

It has 3 elements

c. $\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$

Order

3 x 1

It has 3 elements

d. $\begin{pmatrix} 3 & 2 & 4 \\ 1 & 0 & -2 \end{pmatrix}$

Order

2 x 3

It has 6 elements

Notation

Matrices are usually denoted by bold capital letters, while the elements are denoted by small letters enclosed in bracket.

For example

$$A = \begin{pmatrix} 2 & 4 \\ 0 & -1 \end{pmatrix}$$

Is a 2 x 2 matrix whose elements can be described in terms of row and column positions using double subscripts (or suffixes) notation:

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Where

a_{11} indicates the elements in the 1st row and 1st column (so, $a_{11} = 2$).

a_{12} indicates the element in the 1st row and 2nd column, (so, $a_{12} = 4$),

and so on.

Example 1

If $A = \begin{bmatrix} -2 & 3 & 0 \\ 5 & 6 & 4 \\ 1 & 7 & 2 \end{bmatrix}$

Find:

(i) a_{21} (ii) a_{32} (iii) a_{23} (iv) a_{33}

Solution

(i) $a_{21} = 5$ (ii) $a_{32} = 7$ (iii) $a_{23} = 4$ (iv) $a_{33} = 2$

Example 2

Given that

$$A = \begin{bmatrix} 0 & 4 & 1 & -1 \\ 0 & 3 & -2 & 0 \end{bmatrix} \; and \; B = \begin{bmatrix} 1 & 2 \\ -2 & 4 \end{bmatrix}$$

(a) State the order of A and B

(b) Determine (i) a_{13} (ii) a_{22} (iii) a_{23}

Solution

(a) The matrix A has 2 rows and 4 columns, so its order is 2 x 4.

The matrix B has 2 rows and 2 columns, so its order is 2 x 2.

(b) (i) a_{13} indicates the element in the 1st row and 3rd column for matrix A.

So $a_{13} = 1$

(ii) Similarly $a_{22} = 3$ for matrix A and $a_{22} = 4$ for matrix B

(iii) $a_{23} = -2$ for matrix A.

Types of Matrices

(a) **Row matrix:** This is a matrix having only one row of elements.

For example the matrix $\begin{pmatrix} 1 & 3 & 5 \end{pmatrix}$

(b) **Column matrix:** This is a matrix having only one column of element.

For example the matrix $\begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$

(c) **Zero or Null matrix:** This is a matrix having all its elements zero. For Example $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is a null matrix

(d) **Square matrix:** This is a matrix having equal number of rows and columns.

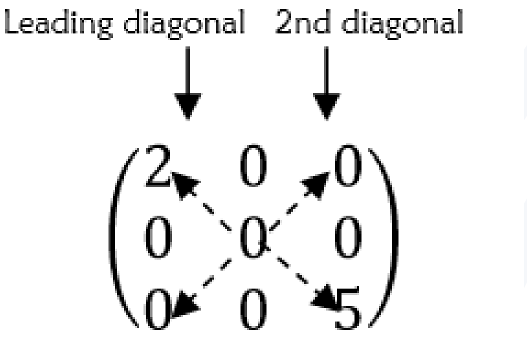
The matrix: $\begin{pmatrix} 3 & -3 \\ 1 & 2 \end{pmatrix}$

Is a square matrix of order 2 x 2 similarly the matrix:

$$\begin{bmatrix} 5 & 0 & 3 \\ 1 & -2 & 2 \\ 4 & 1 & -1 \end{bmatrix}$$

Is a 3 x 3 square matrix.

<u>Diagonal matrix:</u> This is a square matrix in which all the elements are zero <u>except</u> those on the <u>leading</u> (<u>main</u>) diagonal. For example:



Note that on the leading diagonal the elements can be any value including zero.

Unit matrix: This is a diagonal matrix in which the elements on the leading diagonal are all unity. A unit matrix is usually denoted by the letter I.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ADDITION AND SUBTRACTION OF MATRICES

Two matrices can be added or subtracted only if they have the same size or order. In order words, you can add a 2 x 2 matrix to another 2 x 2 matrix. On the other hand you cannot add a 2 x 2 matrix to a 2 x 3 matrix. This is also applicable to subtraction.

Example 1: If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$

Find A + B

Solution $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+3 \\ 3+5 & 4+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 \\ 8 & 6 \end{bmatrix}$$

Example 2: If $A = \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ Find A – B

Solution

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 2-3 \\ 1-1 & 4-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$$

Example 3: Given that

$$A = \begin{bmatrix} 3 & 6 & 4 \\ 2 & 1 & 0 \\ -5 & 2 & 3 \end{bmatrix} \; and \; B = \begin{bmatrix} 2 & 5 & 5 \\ -3 & 0 & 4 \\ -4 & 8 & 2 \end{bmatrix}$$

(a) Determine

(i) A + B (ii) B + A (iii) A – B (iv) B – A

Solution

(i)
$$A = \begin{bmatrix} 3 & 6 & 4 \\ 2 & 1 & 0 \\ -5 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 5 & 5 \\ -3 & 0 & 4 \\ -4 & 8 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 6 & 4 \\ 2 & 1 & 0 \\ -5 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 5 & 5 \\ -3 & 0 & 4 \\ -4 & 8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3+2 & 6+5 & 4+5 \\ 2+(-3) & 1+0 & 0+4 \\ -5+(-4) & 2+8 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 11 & 9 \\ -1 & 1 & 4 \\ -9 & 10 & 5 \end{bmatrix}$$

(ii) $$B + A = \begin{bmatrix} 2 & 5 & 5 \\ -3 & 0 & 4 \\ -4 & 8 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 6 & 4 \\ 2 & 1 & 0 \\ -5 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & 5+6 & 5+4 \\ -3+2 & 0+1 & 4+0 \\ -4+(-5) & 8+2 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 11 & 9 \\ -1 & 1 & 4 \\ -9 & 10 & 5 \end{bmatrix}$$

(iii) $$A - B = \begin{bmatrix} 3 & 6 & 4 \\ 2 & 1 & 0 \\ -5 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 5 & 5 \\ -3 & 0 & 4 \\ -4 & 8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & 6-5 & 4-5 \\ 2-(-3) & 1-0 & 0-4 \\ -5-(-4) & 2-8 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 5 & 1 & -4 \\ -1 & -6 & 1 \end{bmatrix}$$

(iv) $$B - A = \begin{bmatrix} 2 & 5 & 5 \\ -3 & 0 & 4 \\ -4 & 8 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 4 \\ 2 & 1 & 0 \\ -5 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 5-6 & 5-4 \\ -3-2 & 0-1 & 4-0 \\ -4+5 & 8-2 & 2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & 1 \\ -5 & -1 & 4 \\ 1 & 6 & -1 \end{bmatrix}$$

Multiplication of Matrices

Multiplication of a matrix by a scalar

To multiply a matrix by a number i.e. a scalar simply multiply each element of the matrix by the number. For example:

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and k be a scalar

Example 1

Let $X = \begin{bmatrix} 4 & -11 \\ -8 & 3 \end{bmatrix}, Y = \begin{bmatrix} -5 & 1 \\ 0 & 6 \end{bmatrix}$

Find:

(a) 2x (b) -3y (c) 4x – 2y

Solution

(a) $$2x = 2\begin{bmatrix} 4 & -11 \\ -8 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 22 \\ -16 & 6 \end{bmatrix}$$

(b) $$-3y = -3\begin{bmatrix} -5 & 1 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 15 & -3 \\ 0 & -18 \end{bmatrix}$$

(c) $$4x - 2y = 4\begin{bmatrix} 4 & -11 \\ -8 & 3 \end{bmatrix} - 2\begin{bmatrix} -5 & 1 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -44 \\ -32 & 12 \end{bmatrix} - \begin{bmatrix} -10 & 2 \\ 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & -46 \\ -32 & 0 \end{bmatrix}$$

Multiplication of two matrices

Two matrices can be multiplied together only if the number of columns in the 1st matrix is equal to the number of rows in the 2nd matrix.

Example 1

If $A = \begin{bmatrix} 3 & -2 \\ 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 7 \\ 6 & -5 \end{bmatrix}$

Find:

(a) AB (b) BA

Solution

(a) $$AB = \begin{pmatrix} 3 & -2 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} 3 & 7 \\ 6 & -5 \end{pmatrix}$$

$$= \begin{bmatrix} (3 \times 3) + (-2 \times 6) & (3 \times 7) + (-2 \times -5) \\ (5 \times 3) + (0 \times 6) & (5 \times 7) + (0 \times -5) \end{bmatrix}$$

$$= \begin{bmatrix} 9 + (-12) & 21 + 10 \\ 15 + 0 & 35 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 - 12 & 31 \\ 15 & 35 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 31 \\ 15 & 35 \end{bmatrix}$$

(b) $$BA = \begin{bmatrix} 3 & 7 \\ 6 & -5 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (3 \times 3) + (7 \times 5) & (3 \times -2) + (7 \times 0) \\ (6 \times 3) + (-5 \times 5) & (6 \times -) + (-5 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} 9 + 35 & -6 + 0 \\ 18 - 25 & -12 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 44 & -6 \\ -7 & -12 \end{bmatrix}$$

EVALUATION

1. Determine the orders if the following matrices

(a) $\begin{pmatrix} 4 & -3 & 2 \\ 1 & 0 & -2 \\ 7 & 6 & -4 \\ -2 & 5 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 4 & 1 \\ -2 & 3 & 1 \\ 0 & 4 & -2 \end{pmatrix}$

2. If $A = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, B = \begin{pmatrix} 4 & -2 \\ 9 & 8 \end{pmatrix}$ $and\ c = \begin{pmatrix} 3 & 12 \\ 6 & -9 \end{pmatrix}$

Calculate:

(a) 5A (b) 3B (c) $\frac{2}{3}$ C (d) $\frac{1}{2}B + \frac{1}{3}C$

3. Let $A = \begin{pmatrix} 1 & 4 \\ 2 & -3 \end{pmatrix}, B = \begin{pmatrix} 5 & 0 \\ 6 & 1 \end{pmatrix}$ $and\ C = \begin{pmatrix} 3 & 3 \\ 2 & -1 \end{pmatrix}$

Find (i) AB (ii) AC

THANK YOU FOR WATCHING!!