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Nonlinear Fokker-Planck Acceleration for Forward-Peaked Transport Problems in Slab Geometry

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1. INTRODUCTION

Nonlinear High-Order/Low-Order (HOLO) acceleration techniques provide substantial performance gains for several transport-related single and multiphysics problems [1–4]. These scale-bridging algorithms accelerate the convergence of slowly converging physics, like scattering source, to reduce the number of iterations required. They also allow us to isolate the High-Order equation from the coupled system (for multiphysics problems) by introducing a consistent coarser-scale Low-Order (LO) equation [5]. However, most of these nonlinear methods cater to isotropic or weakly anisotropic problems [6], with few exceptions addressing highly anisotropic problems [7].

In this summary, we develop a nonlinear Fokker-Planck acceleration algorithm for highly anisotropic, forward-peaked transport problems. The Nonlinear Fokker-Planck Acceleration (NFPA), like the Nonlinear Diffusion Acceleration (NDA) [3], introduces a consistent LO equation that can be coupled with other physics in nonlinear solves while keeping transport sweeps isolated. Our LO equation is based on the Fokker-Planck equation [8]. The linear version of this method, Fokker-Planck Synthetic acceleration [9], has shown significant improvement in the convergence rate for certain classes of problems. The nonlinear method introduced in this work also shows significant improvement in convergence rate for a variety of problems.

This summary contains the description of the NFPA proposed method and it is organized as follows. In Section 2, we introduce the idea of NFPA accelerated transport solve. Then, we present a preliminary result and conclusions in Section 3.

2. NONLINEAR FOKKER-PLANCK ACCELERATION

In this section we present a brief derivation of the nonlinear Fokker-Planck acceleration for anisotropic, monoenergetic transport in slab geometry. We consider the transport equation

$$\mu\frac{\partial}{\partial x}\psi(x,\mu)+\sigma_t(x)\psi(x,\mu)=\sum_{l=0}^{L}(2l+1)P_l(\mu)\sigma_{s,l}(x)\phi_l(x)+Q(x,\mu), \tag{1}$$

where ψ represents the angular flux, ϕ_l is the l^{th} Legendre moment of the angular flux, σ_t is the total macroscopic cross-section, $\sigma_{s,l}$ is the l^{th} scattering cross-section moment, P_l is the l^{th}-order Legendre polynomial, and Q is an internal source. For simplicity, we will drop the notation (x, μ) in the remainder of this summary. The solution to Eq. (1) converges asymptotically to the solution of the following Fokker-Planck (FP) equation in the forward-peaked limit [8]:

$$\mu\frac{\partial\psi}{\partial x} + \sigma_a\psi = \frac{\sigma_{tr}}{2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial\psi}{\partial\mu} + Q\,, \tag{2}$$

where σ_{tr} is the momentum transfer cross-section. In order to make the FP equation consistent with the transport equation, we introduce an additive term to the FP equation such that the modified FP equation has the following form:

$$\mu\frac{\partial\psi}{\partial x} + \sigma_a\psi = \frac{\sigma_{tr}}{2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial\psi}{\partial\mu} + \hat{D}_F\psi + Q\,. \tag{3}$$

Here, $\hat{D}_F\psi$ is the additive term that makes the transport and FP equations consistent. Equation (3) is called the Fokker-Planck-plus-consistency-term equation. We expect this modified equation to take large-angle scattering into account through the consistency term, unlike the standard Fokker-Planck equation. Subtracting Eq. (3) from Eq. (1) and rearranging the terms, we obtain:

$$\hat{D}_F = \frac{\sum_{l=0}(2l+1)P_l\sigma_l\phi_l - \frac{\sigma_{tr}}{2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial\psi}{\partial\mu}}{\psi} - \sigma_{s,0}\,. \tag{4}$$

The NFPA method uses Eqs. (1) and (3) where high order (HO), low order (LO), and closure terms are given by

$$\text{HO}: \mu\frac{\partial\psi_{HO}}{\partial x} + \sigma_t\psi_{HO} = \sum_{l=0}(2l+1)P_l\sigma_l\phi_{l,LO} + Q\,, \tag{5a}$$

$$\text{LO}: \mu\frac{\partial\psi_{LO}}{\partial x} + \sigma_a\psi_{LO} = \frac{\sigma_{tr}}{2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial\psi_{LO}}{\partial\mu} + \hat{D}_F\psi_{LO} + Q\,, \tag{5b}$$

$$\text{Closure}: \hat{D}_F = \frac{\sum_{l=0}(2l+1)P_l\sigma_l\phi^m_{l,HO} - \frac{\sigma_{tr}}{2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial\psi_{HO}}{\partial\mu}}{\psi_{HO}} - \sigma_{s,0}\,. \tag{5c}$$

The nonlinear HOLO-plus-closure system can be solved using any nonlinear solution technique [10].

3. PRELIMINARY RESULT AND DISCUSSION

We consider a uniform medium with thickness equals to 100 mean-free-paths and anisotropic scattering. The cross-sections, including the scattering cross-section moments, are presented in Table I [11]. We use the standard diamond-difference/discrete ordinates scheme for space/angle discretization [12]. The angular Laplacian in the modified Fokker-Planck equation employs Morel's weighted finite difference scheme [13]. We use 1000 spatial nodes along with 16 angles for our discretization. An isotropic internal source $Q = 10.0$ is assumed. The tolerance is set to 10^{-4}. A comparison of the solution with stand-alone source iteration and NLFP accelerated solution is presented in Fig. 1. We observe that, for this specific problem, NLFP improved

Table I: Cross-sections

Cross-section	Value	Cross-section	Value
σ_t	13762.62153659	σ_a	2.0
$\sigma_{s,0}$	13760.62153659	$\sigma_{s,8}$	13665.50901853
$\sigma_{s,1}$	13756.65496695	$\sigma_{s,9}$	13645.69235018
$\sigma_{s,2}$	13749.82123209	$\sigma_{s,10}$	13624.53334647
$\sigma_{s,3}$	13740.60473474	$\sigma_{s,11}$	13602.07403769
$\sigma_{s,4}$	13729.19245164	$\sigma_{s,12}$	13578.42235004
$\sigma_{s,5}$	13715.84644502	$\sigma_{s,13}$	13553.59782372
$\sigma_{s,6}$	13700.68597485	$\sigma_{s,14}$	13527.69278088
$\sigma_{s,7}$	13683.86957645	$\sigma_{s,15}$	13500.71882667

the iteration count by three orders of magnitude. This is, however, problem dependent; the method's efficacy must be tested for a wide range of problems.

In this summary, we have introduced a nonlinear Fokker-Planck acceleration algorithm for highly anisotropic transport problems. The consistency term for the NFPA algorithm is solved from the subtraction of the HO and LO equations. To our knowledge, this is the first time a nonlinear Fokker-Planck-based acceleration has been proposed. This method is expected to improve the convergence of slowly converging forward-peaked transport problems. Moreover, NFPA is expected to return a modified Fokker-Planck equation that will account for large angle scattering. In the full version of this work, we will present results for a wide variety of problems and discuss the method's stability.

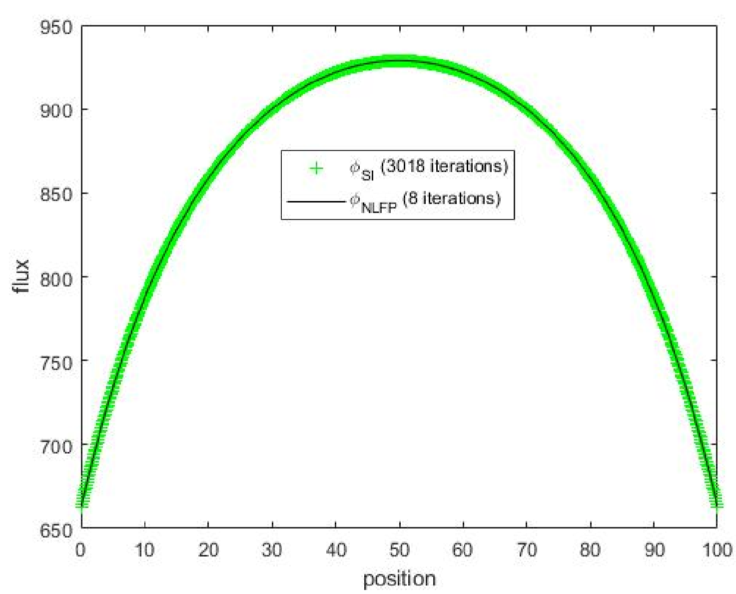

Figure 1: Solver Comparison

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