

## **One-way Coupled Benchmark for Combined-Hyperthermia-Radiotherapy Treatment in Slab Geometry**

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Radiation dosimetry has traditionally been treated as single physics [1]. However, several applications could benefit from the coupling of dose deposition with other physics to synchronously model the underlying system. One such application is the modeling of tumor response to combined-hyperthermia-radiotherapy (CHR) treatment. Several different tumor microenvironments undergo constitutive changes upon exposure to heat [2]. These changes result in increased radiosensitivity in tumor tissue making tumors more susceptible to treatment via radiation [3]. CHR treatments combine heat and radiation to maximize tumor cell kill while reducing damage to the surrounding normal tissue.

We developed a multiphysics-based one-way coupled model to predict the response of localized tumors to CHR in [4]. It combines radiation transport (dosimetry) with heat transfer (hyperthermia) and cell population dynamics (tumor dynamics) to model a highly simplified slab-geometry system. We used the following mono-energetic, slab geometry transport equation with isotropic scattering to model transport [5]:

$$\frac{1}{v} \frac{\partial \psi}{\partial t} + \mu \frac{\partial \psi}{\partial x} + \sigma_t \psi = \frac{\sigma_s}{2} \phi + \frac{Q}{2}, \quad (1)$$

where,  $\psi = \psi(x, \mu, t)$  is the angular flux at position  $x$ , along angular cosine  $\mu$ , at time  $t$ ;  $\sigma_t$  and  $\sigma_s$  are the macroscopic total and scattering cross-sections;  $Q$  is an isotropic internal source;  $v$  is the particle speed; and  $\phi = \int_{-1}^1 \psi d\mu$  is the scalar flux. The cell survival probability was calculated using the following set of equations [6]:

$$D = \left( \frac{\mu_{en}}{\rho} \right) \phi \Delta t E, \quad (2a)$$

$$D_E = \alpha D + \beta D^2, \quad (2b)$$

$$S = \exp(-D_E), \quad (2c)$$

where,  $D$ ,  $D_E$ , and  $S$  are the dose, effective dose, and the cell survival probability at position  $x$  and time  $t$ , respectively. Moreover,  $\alpha$  and  $\beta$  are radiobiology parameters. Hyperthermia is modeled using the following heat conduction equation [7]:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \kappa \frac{\partial T}{\partial x} + q, \quad (3)$$

where  $\rho$ ,  $c_p$ , and  $\kappa$  are the specific heat capacity, the material density, and the thermal conductivity of tissue, respectively;  $q$  is the volumetric heat source; and  $T$  is the temperature at position  $x$  and time  $t$ . The tumor dynamics is modeled as a diffusion process using the following equation [8]:

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x} I \frac{\partial c}{\partial x} + pc - Rc, \quad (4)$$

where  $c$  is the tumor cell concentration at position  $x$  and time  $t$ ;  $I$  is the coefficient representing motility of tumor cells; and  $p$  is the proliferation rate. Here,  $R$  represents the effect of therapy on tumor cell kill:

$$R_{RT} = 1 - S, \quad (5a)$$

$$R_{CHR} = \xi(1 - S), \quad (5b)$$

where  $R_{RT}$  and  $R_{CHR}$  quantify the effect of stand-alone radiotherapy and CHR treatments respectively, and  $\xi$  is a radiosensitivity parameter at position  $x$  and time  $t$  [4]. The above set of equations returns a one-way coupled system that can be solved sequentially.

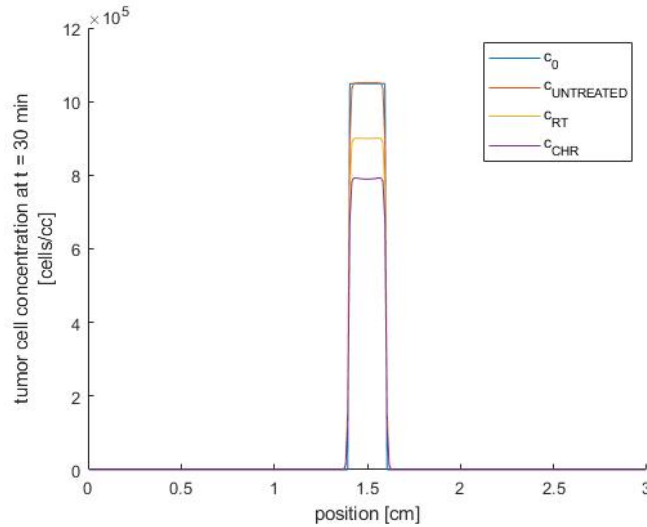


Figure 1: Cell Concentration Comparison

We will develop a highly precise benchmark set of solutions for the coupled equations (Eqs. 1 to 5) presented above. Our preliminary result presents a comparison of tumor response to different treatment scenarios in Fig. 1. This result is precise up to five digits. We will employ sophisticated convergence acceleration techniques including the Wynn-epsilon algorithm [9] to develop our benchmark for the full presentation and subsequent paper.

## ACKNOWLEDGMENTS

J. K. Patel and R. Vasques acknowledge support under award number NRC-HQ-84-15-G-0024 from the Nuclear Regulatory Commission. The statements, findings, conclusions, and recommendations are those of the authors and do not necessarily reflect the view of the U.S. Nuclear Regulatory Commission.

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