

THE SOLUTION OF THE NEUTRON POINT KINETICS EQUATION WITH STOCHASTIC EXTENSION: AN ANALYSIS OF TWO MOMENTS

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ABSTRACT

The neutron point kinetics equation, which models the time-dependent behavior of nuclear reactors, is often used to understand the dynamics of nuclear reactor operations. It consists of a system of coupled differential equations that models the interaction between (i) the neutron population; and (ii) the concentration of the delayed neutron precursors, which are radioactive isotopes formed in the fission process that decay through neutron emission. These equations are deterministic in nature, and therefore can provide only average values of the modeled populations. However, the actual dynamical process is stochastic: the neutron density and the delayed neutron precursor concentrations vary randomly with time. To address this stochastic behavior, Hayes and Allen have generalized the standard deterministic point kinetics equation. They derived a system of stochastic differential equations that can accurately model the random behavior of the neutron density and the precursor concentrations in a point reactor. Due to the stiffness of these equations, this system was numerically implemented using a stochastic piecewise constant approximation method (Stochastic PCA). Here, we present a study of the influence of stochastic fluctuations on the results of the neutron point kinetics equation. We reproduce the stochastic formulation introduced by Hayes and Allen and compute Monte Carlo numerical results for examples with constant and time-dependent reactivity, comparing these results with stochastic and deterministic methods found in the literature. Moreover, we introduce a modified version of the stochastic method to obtain a non-stiff solution, analogue to a previously derived deterministic approach.

1. INTRODUCTION

The neutron point kinetics equation, which simulates the time-dependent behavior of nuclear reactors [1, 2, 3, 4], is a system of coupled differential equations that models the interaction of the neutron population with the concentration of the delayed neutron precursors. Such equations, as expected by its deterministic nature, provide only average values of the modeled neutron and precursors populations. However, certain applications require a stochastic approach of this system to attain an accurate solution.

There are several methods in the current literature that can perform such an endeavor. They differ from each other in the way stochastic fluctuations are introduced in the system, as well as in the choice of the solution procedure.

When considering the insertion of the fluctuations in the system, one can find cases of (i) insertions in the dependent variables; and (ii) insertions in the physical parameters. Examples of insertions in the dependent variables include cases in which the fluctuations observed in the neutron density are caused by the reactivity as a stationary Gaussian function [5]; and cases in which a stochastic non-white reactivity input was used [6]. Literature concerning fluctuations inserted in the reactivity function may be found in references [7, 8, 9]. These references address cases with non-linear neutron point kinetics, which is not the focus of the present work. Regarding the insertion of the fluctuations in the remaining time-dependent variables, Hayes and Allen [2] have generalized the standard deterministic point kinetics equation, deriving a system of stochastic differential equations that model the random behavior of the neutron density and the precursor concentrations in a point reactor. Due to stiffness, the authors of [2] implemented this system numerically using a stochastic and piecewise constant approximation method (Stochastic PCA). Numerical approaches of this same method were also presented by Saha Ray [10].

The different methods of solution include the use of Taylor power series and Euler-Maruyama approach in [10]; integral transforms into discrete time steps in [7]; linearization and Bourret approximations in [6]; Monte Carlo in [2, 11]; and others [7, 12, 13]. It is noteworthy that, with the exception of cases modeled without or with only one group of neutron precursors, the stiff character of the problem remains present in the stochastic approach.

In this paper we present an approach to obtain a non-stiff solution for the stochastic formulation of the neutron point kinetics equation. Specifically, this approach allows the calculation of the neutron and precursor densities at any time of interest (without the need of using progressive time steps), similar to the work presented in [14]. This is attained by the combination of the Decomposition Method [14, 15], the stochastic formulation derived in [2], and the use of the Monte Carlo Method [2, 10, 14]. The initial steps for this solution were already presented in [16], where we obtained a non-stiff solution for the case of constant reactivity. In this work we reproduce the stochastic formulation introduced in [2] and compute numerical Monte Carlo results for examples with constant and time-dependent reactivity, comparing these results with stochastic and deterministic methods found in the literature [2, 10, 14]. We also present the analysis of the first four statistical moments of the obtained solution.

To this end, in Section 2 we present a brief review on the key aspects of the Decomposition Method and the stochastic formulation that were used to derive the non-stiff stochastic solution. In Section 3 we present the non-stiff solution method. In Section 4 we present some of our new findings, and review previous ones. In Section 5 we summarize our results and present our conclusions.

2. A BRIEF REVIEW

In this section we present a short review on the aspects of the non-stiff deterministic solution given in [14], followed by a direct presentation of the stochastic model derived in [2].

2.1. The Deterministic Solution

The neutron point kinetics equation with time-dependent reactivity is written as

$$\begin{aligned}\frac{d}{dt}n(t) &= \frac{\rho(t) - \beta}{\Lambda}n(t) + \sum_{i=1}^6 \lambda_i C_i(t), & n(0) &= n_0, \\ \frac{d}{dt}C_i(t) &= \frac{\beta_i}{\Lambda}n(t) - \lambda_i C_i(t), & C_i(0) &= \frac{\beta_i n_0}{\Lambda \lambda_i},\end{aligned}\quad (1)$$

for $i \in [1, 6]$. Here, $n(t)$ is the neutron density; $\rho(t)$ is the time-dependent reactivity; $C_i(t)$ is the density of the i -th delayed neutron precursor group; λ_i is the decay constant for a specific group i ; Λ represents the neutron mean generation time; and β_i represents the delayed-neutron fraction in a specific group i , such that the total fraction of delayed neutrons is given by $\beta = \sum_{i=1}^6 \beta_i$.

The solution for this system is obtained using a recursive scheme, with finite recursive depth R . The truncation index is determined by an exponential convergence criterion (for details see [14, 15]) evaluated after each recursion step. To this end, we write the neutron population $n(t) = \sum_{j=0}^R n_j(t)$ and the precursors concentration $C_i(t) = \sum_{j=0}^R C_{i,j}(t)$ in terms of the solution from a recursion initialization ($j = 0$) and the respective correction terms ($j > 0$) for an appropriate $R \in \mathbb{N}$. Additionally, reactivity is expressed as $\rho(t) = \rho_0 + \rho_1(t)$, with ρ_0 being a constant value and $\rho_1(t)$ a generic time-dependent function.

For convenience we introduce the matrices:

$$\Xi = \begin{bmatrix} \frac{\rho_1(t)}{\Lambda} & \{\lambda_i\} \\ \left\{ \frac{\beta_i}{\Lambda} \right\}^T & 0 \end{bmatrix}, \quad (2)$$

$$\mathbf{Y}_j(t) = (n_j(t), C_{1,j}(t), C_{2,j}(t), C_{3,j}(t), C_{4,j}(t), C_{5,j}(t), C_{6,j}(t))^T, \quad (3)$$

and

$$\Omega = \text{diag} \left(\frac{\rho_0 - \beta}{\Lambda}, -\lambda_1, -\lambda_2, -\lambda_3, -\lambda_4, -\lambda_5, -\lambda_6 \right), \quad (4)$$

which allows us to cast the original problem (1) in matrix form:

$$\frac{d\mathbf{Y}(t)}{dt} - \Omega \mathbf{Y}(t) = \Xi(t) \mathbf{Y}(t). \quad (5)$$

Formally, Eq. (5) may be solved by the Laplace transform:

$$\mathbf{Y}(t) = \exp(\Omega t) \mathbf{Y}(0) + \int_0^t \exp(\Omega \tau) \Xi(t - \tau) \mathbf{Y}(\tau) d\tau. \quad (6)$$

Observe that the integral in this equation contains the time-dependent reactivity and the decay constants that cause the stiffness of the problem. The integral equation (6) can be decomposed into a set of equations [17] that allow to determine closed form solutions by recursion (see also [14]):

$$\frac{d\mathbf{Y}_0(t)}{dt} - \Omega\mathbf{Y}_0(t) = 0, \quad \frac{d\mathbf{Y}_i(t)}{dt} - \Omega\mathbf{Y}_i(t) = \Xi(t)\mathbf{Y}_{i-1}(t). \quad (7)$$

Since the Laplace transform is a linear operation and the equation itself is linear, the solution by Laplace transform can be employed directly in the recursion formalism, maintaining the exact solution in the limit $R \rightarrow \infty$.

2.2. The Stochastic Formulation

The formulation postulated by Hayes in [2] describes the variation of the population and can be interpreted as a balance between deaths, births and transformations of neutrons in the system. These events have its probabilities determined by the physical parameters of the model, such as the total and partial delayed neutron fraction; the fraction of delayed neutrons of each precursor group; the decay constant of each group; and the average number of neutrons produced in each fission. Based on this concept Hayes [2] was able to express the expectation values and probabilities of each event. Assuming a time interval small enough such that only one event occurs, one obtains an Itô system of differential equations, given in its most general form by:

$$\frac{d}{dt} [\mathcal{Y}(t)] = \hat{\mathbf{A}}\mathcal{Y}(t) + \mathcal{Q}(t) + \hat{\mathbf{B}}^{\frac{1}{2}} \frac{d\mathbf{W}}{dt}, \quad (8)$$

$$\frac{d}{dt} \begin{bmatrix} n(t) \\ c_1(t) \\ c_2(t) \\ \vdots \\ c_6(t) \end{bmatrix} = \hat{\mathbf{A}} \begin{bmatrix} n(t) \\ c_1(t) \\ c_2(t) \\ \vdots \\ c_6(t) \end{bmatrix} + \begin{bmatrix} q(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \hat{\mathbf{B}}^{\frac{1}{2}} \frac{d}{dt} \begin{bmatrix} W_0 \\ W_1 \\ W_2 \\ \vdots \\ W_6 \end{bmatrix}, \quad (9)$$

where

$$\hat{\mathbf{A}} = \begin{bmatrix} \frac{\rho-\beta}{\Lambda} & \lambda_1 & \lambda_2 & \dots & \lambda_6 \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \dots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \frac{\beta_6}{\Lambda} & 0 & \dots & 0 & -\lambda_6 \end{bmatrix} \quad (10)$$

and

$$\hat{\mathbf{B}} = \begin{bmatrix} \zeta & a_1 & a_2 & \dots & a_6 \\ a_1 & b_{1,1} & b_{2,3} & \dots & b_{2,6} \\ a_2 & b_{3,2} & b_{2,2} & \dots & b_{3,6} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ a_6 & b_{1,2} & \dots & b_{5,6} & b_{6,6} \end{bmatrix}, \quad (11)$$

in which

$$\zeta = \gamma n(t) \sum_{i=1}^6 \lambda_i c_i(t), \quad \gamma = \frac{-1 - (\rho_0 + \rho(t)) + 2\beta + (1 - \beta)^2 \nu}{\Lambda},$$

$$a_i = \frac{\beta_i}{\Lambda} (-1 + (1 - \beta)) n(t) - \lambda_i c_i(t),$$

and

$$b_{i,j} = \frac{\beta_{i-1} \beta_{j-1} \nu}{\Lambda} n(t) + \delta_{ij} \left(\frac{\beta_i^2 \nu}{\Lambda} n(t) + \lambda_i c_i(t) \right).$$

The last term in equation (9) contains a vector with Wiener processes.

3. THE PROPOSED COMBINATION

It is noteworthy that the matrix $\hat{\mathbf{B}}$ depends on both the neutron populations and the delayed neutron concentrations. Therefore, we propose that the solution of this problem can be obtained by a double decomposition, as discussed next. In the first stage the solution of the purely deterministic problem [14] is obtained by the processes already described in the section 2.1.. Once the solution \mathcal{Y} is known, the matrix $\hat{\mathbf{B}}$ can be determined for a sequence of discrete time steps so that its components are constant in each time step. In order to obtain the square root of the matrix $\hat{\mathbf{B}}$ one may use its symmetric nature and resort to diagonalization.

When considering a specific time interval the function $\mathcal{Q}(t)$ is known and it is possible to apply an analogue decomposition scheme to the one used for the deterministic model with the recursion initialization and recursive scheme:

$$\frac{d\mathcal{Y}_0(t)}{dt} - \Omega \mathcal{Y}_0(t) = 0 \quad \text{and} \quad \frac{d\mathcal{Y}_i(t)}{dt} - \Omega \mathcal{Y}_i(t) = \Xi(t) \mathcal{Y}_{i-1}(t) + \mathcal{F}, \quad (1)$$

where $\mathcal{F} = \mathcal{Q}(t) + \hat{\mathbf{B}}^{\frac{1}{2}} \frac{d\mathbf{W}}{dt} = \text{constants}$ known in each time step.

4. NUMERICAL RESULTS

4.1. Results for Constant Reactivities

We first tackle cases with constant reactivities in order to compare the proposed method with results already established in the literature. We consider sub-critical, critical, and super-critical constant reactivities.

4.1.1. Constant sub-critical reactivity

This case consists in a problem of constant sub-critical reactivity, considering only one group of precursors. Note that for one precursor group the stiffness of the problem disappears. The physical parameters are $\Lambda_1 = 0.1$, $\beta_1 = 0.005$, $\nu = 2.5$, $q = 200$, $\Lambda = \frac{2}{3}$, and $\rho = -\frac{1}{3}$, with initial conditions $\mathbf{x}(0) = [400, 300]^T$ and a simulated time of two seconds.

Table 1: Results considering one group of precursors and constant sub-critical reactivity

	Monte Carlo	Stochastic PCA [2]	Euler-Maruyama Approximation [10]	Deterministic Model.[15]	Proposed Method
$E(n(2))$	400.032	395.32	412.23	412.13	402.350
$\sigma(n(2))$	27.311	29.411	34.391	–	28.610
$E(c_1(2))$	300.01	300.67	315.96	315.93	305.84
$\sigma(c_1(2))$	7.807	8.3564	8.2656	–	7.924

A total of 1000 histories were accumulated in order to achieve a statistical error smaller than 0.05% with 95% confidence according to the Central Limit Theorem. The proposed method achieved similar results to Monte Carlo and Stochastic PCA methods; however, we observe that the Euler-Maruyama Approximation yields higher values that approach the deterministic solution.

4.1.2. Constant critical and super-critical reactivities

This case considers a stiff equation system with six precursor groups. The physical parameter set was drawn from reference [6]: $\beta = [0.007]$, $\nu = 2.5$, $\Lambda = 0.00002$, $q = 0$, $m = 6$,

$$\lambda_i = [0.0127, 0.0317, 0.115, 0.311, 1.4, 3.87]^T,$$

$$\beta_i = [0.000266, 0.001491, 0.001316, 0.002849, 0.000896, 0.000182]^T,$$

and initial condition

$$\mathbf{x}(0) = 100 \left[1, \frac{\beta_1}{\lambda_1 t}, \frac{\beta_2}{\lambda_2 t}, \frac{\beta_3}{\lambda_3 t}, \frac{\beta_4}{\lambda_4 t}, \frac{\beta_5}{\lambda_5 t}, \frac{\beta_6}{\lambda_6 t} \right]^T.$$

Table 2: Results for critical reactivity, $\rho = 0.003$

	Monte Carlo	Stochastic PCA [2]	Euler-Maruyama Approximation [10]	Deterministic Model.[15]	Proposed Method
$E(n(0.1))$	183.04	186.31	208.6	200.005	187.05
$\sigma(n(0.1))$	168.79	164.16	255.95	–	167.83
$E(\sum_{i=1}^6 c_i(0.1))$	4.478×10^5	4.491×10^5	4.498×10^5	4.497×10^5	4.488×10^5
$\sigma(\sum_{i=1}^6 c_i(0.1))$	1495.72	1917.2	1233.38	–	1475.55

Table 3: Results for super-critical reactivity, $\rho = 0.007$

	Monte Carlo	Stochastic PCA [2]	Euler-Maruyama Approximation [10]	Deterministic Model.[15]	Proposed Method
$E(n(0.001))$	135.66	134.55	139.568	139.61	135.86
$\sigma(n(0.001))$	93.376	91.242	92.042	–	93.210
$E(\sum_{i=1}^6 c_i(0.001))$	4.464×10^5	4.694×10^5	4.463×10^5	4.463×10^5	4.463×10^5
$\sigma(\sum_{i=1}^6 c_i(0.001))$	16.226	19.444	6.071	–	17.845

Once again, the proposed method presents results almost identical to Monte Carlo and Stochastic PCA Methods. It is noteworthy that, in spite of high deviations, the statistical

error is smaller than 0.05% (with 95% confidence). To achieve this criterion, a total of 1000 histories were collected for the case with critical reactivity $\rho = 0.003$, while for the case with super-critical $\rho = 0.007$ a total of 5000 histories were accumulated.

4.2. Reactivities with Time Dependence

Here we present results of typical cases for reactivities with time dependence: (i) a linear variation of reactivity with time; and (ii) a periodical one, i.e., represented by a sinusoidal time dependence.

4.2.1. Linear reactivity

In this example, reactivity is described as a linear function of time $\rho(t) = at$, where the rate of reactivity insertion a whose unit is $[\$/s]$ is a constant. The density is calculated for two different values of a . Parameters are described in table 4 for a thermal reactor [3] for $\Lambda = 0.00001s$.

Table 4: Nuclear parameters for a thermal reactor

Delayed Neutron Fraction by Precursor Group	Decay Constant by Precursor Group
$\beta_i \cdot 10^{-3}$	$\lambda_i (s^{-1})$
0,246	0,0127
1,363	0,0317
1,203	0,115
2,605	0,311
0,819	1,40
0,167	3,87
$\beta = 6,4 \cdot 10^{-3}$	

Table 5 presents the results obtained with the proposed method and compares them to the Decomposition Method (DM), validated in [15] by the methods GAEM [18] and by the Padè-type approach.

Table 5: Neutrons Density

a	t(s)	Neutrons Density	
		DM	Proposed method
0,25	0,25	1,069651	1,069542
	0,50	1,156849	1,156695
	0,75	1,265493	1,265332
	1	1,401838	1,401982

Figure 1 shows the average behavior of the stochastic solution compared to the deterministic solution, as well as two samples. Comparing the stochastic and deterministic solutions one observes a systematic shift, i.e, the stochastic solution, despite having the same qualitative behavior as the deterministic solution, assumes slightly lower values.

In Figure 2 we present skewness and kurtosis of the linear case, calculated by the method of moments, considering all the accumulated histories ($\approx 10^5$). We can assume based on the results of kurtosis that samples are distributed around the average as expected from a normal distribution. Further skewness indicates that the majority of the histories are symmetric around zero.

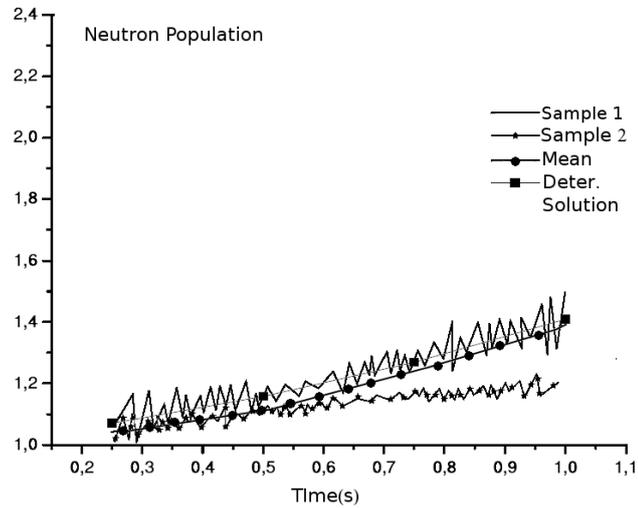


Figure 1: Samples and Mean for a linear type reactivity

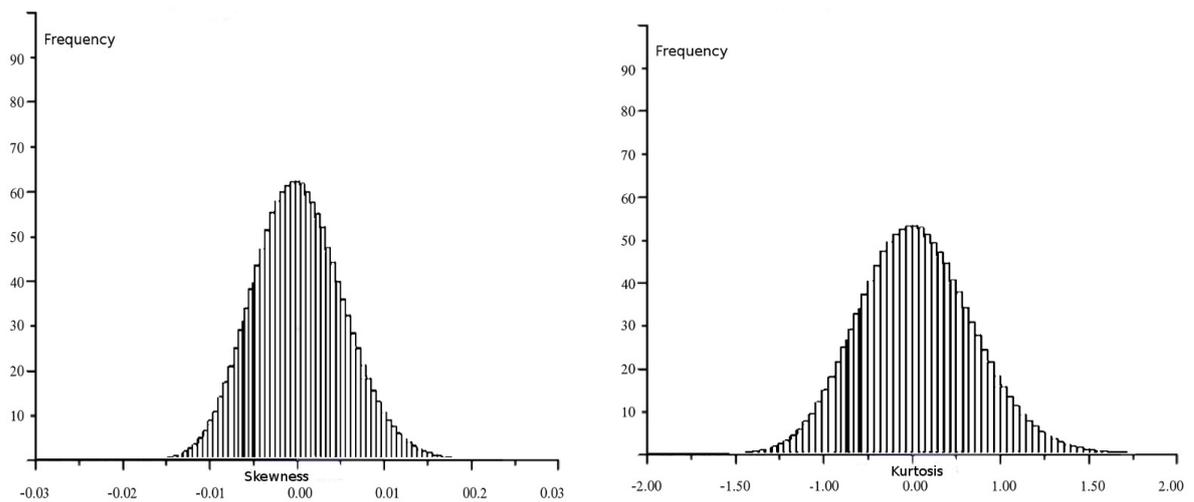


Figure 2: Skewness and Kurtosis for a linear type reactivity

4.2.2. Sinusoidal reactivity

Finally, we simulate a case with reactivity of sinusoidal type $\rho(t) = \rho_0 \sin(t)$, with $\rho_0 = 0.00073$ and $\Lambda = 0.00003s$. Nuclear parameters are given in table 4 [3]. Figure 3 shows the average behavior of the stochastic solution compared to the deterministic solution as well as to a sample. Comparing the stochastic and deterministic solutions one sees that the deterministic solution represents the average of the stochastic approach. Moreover, the individual sample oscillates around the deterministic curve.

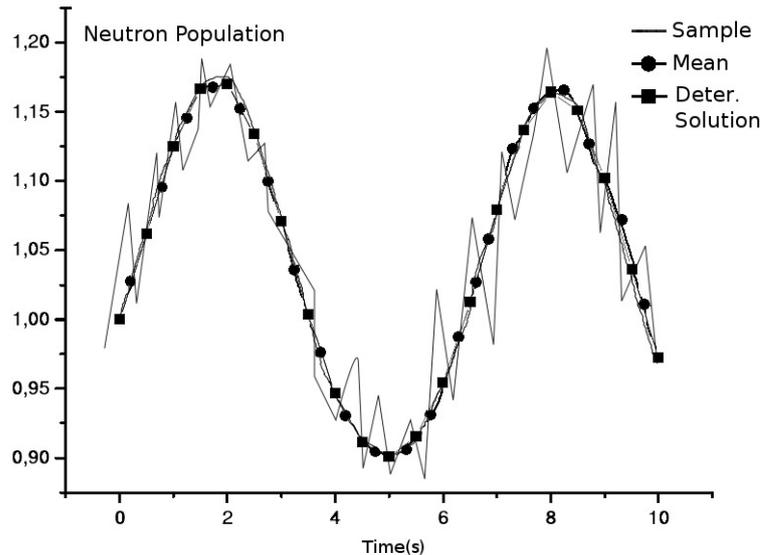


Figure 3: Samples and Mean for a sinusoidal type reactivity

In Figure 4 the skewness and kurtosis of the sinusoidal are presented, calculated by the method of moments considering all the accumulated histories ($\approx 10^6$). We can assume based on the results of kurtosis that samples are distributed around the average as expected from a normal distribution. However, skewness shows a slightly asymmetric behavior.

5. CONCLUSIONS

In this work we have presented a brief review of the method to obtain a non-stiff solution of the neutron point kinetics equation, as well as a stochastic formulation of the same system. We propose a combination of these methods in order to obtain a non-stiff solution for the problem with six precursor groups and time dependent reactivities. The stochastic characteristics are analyzed using four moments of the distribution: mean, variance, skewness, and kurtosis.

Starting with the non-stiff approach and the analytical representation of its solution, one can add a stochastic term and obtain a solution to any desired interval. The solution

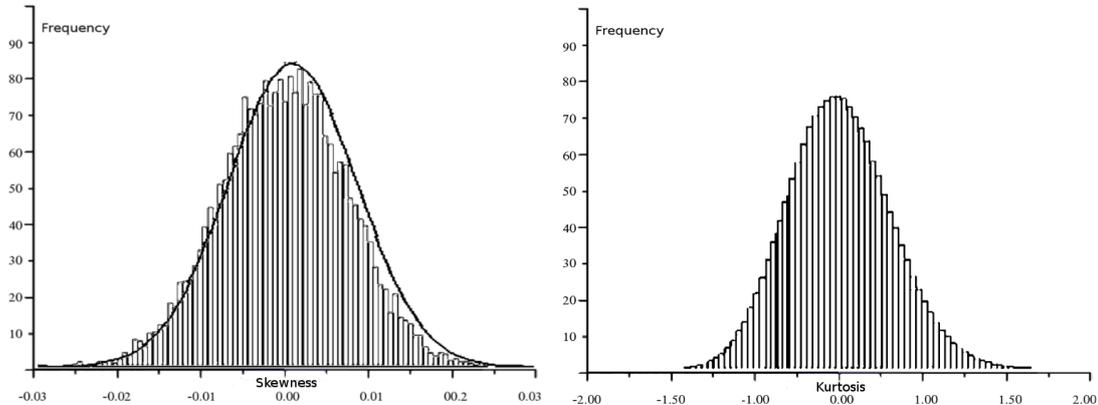


Figure 4: Skewness and Kurtosis for a sinusoidal type reactivity

is generated through successive iterations in each time interval by the use of a recursive system with a stochastic term as a source term.

In order to validate the proposed method we present a comparison with other results in the literature, showing that the method is in agreement with those already established. In the current literature the stochastic problem is solved considering two precursor groups and only constant reactivities, using numerical techniques to obtain such solutions; however, these solutions are limited to feasible time intervals due to the stiffness inherent to the problem.

The mean values follow the behavior of the deterministic model; however, when individual realizations are taken into account they present considerable variations around the mean value of the ensemble [16]. While the fluctuations in the population $n(t)$ are significant, the ones in the precursor concentration are negligible. The skewness being systematically around zero confirms agreement of the mean value with the deterministic model, and the kurtosis indicates a distribution close to a homogeneous one.

The analyzed moments still depend on the size of the sample set and on the frequency with which the stochastic fluctuation are applied. In principle, an adjustment such as variance reduction and its consequences for higher moments could yield more realistic results. To obtain such results independent of the sample size or frequency of application of the stochastic fluctuations it would be necessary to find a reference scale. This still

needs to be identified, as well as a necessary ingredient to mimic reactor fluctuations, which is a topic for future work.

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