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Boundary Conditions for the 1-D Non-Classical Transport Equation

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1. Introduction

A non-classical theory has been recently introduced [1] in order to model non-classical particle transport taking place in certain random media. Non-classical transport arises when the probability density function for a particle's distance to collision is not given by an exponential. Assuming that scattering is isotropic, the non-classical transport equation in rod geometry with no internal source is written as

$$\frac{\partial \psi^{\pm}}{\partial s}(x,s) \pm \frac{\partial \psi^{\pm}}{\partial x}(x,s) + \Sigma_t(s)\psi^{\pm}(x,s) = \delta(s)\frac{c}{2}\int_0^{\infty} \Sigma_t(s')[\psi^{+}(x,s') + \psi^{-}(x,s')]ds'. \tag{1}$$

Here, the path-length s is the distance traveled by the particle since its previous interaction, $\psi^{\pm}$ represents the angular flux in the directions ± 1, c is the scattering ratio, and $\Sigma_t(s)ds$ represents the probability (ensemble-averaged over all physical realizations) that a particle scattered or born at any point x will experience a collision between $x \pm s$ and $x \pm (s + ds)$.

In this work we investigate for the first time the inclusion of boundary conditions in the non-classical transport equation. This investigation is performed in a homogenized 1-D random periodic system [?] consisting of alternating solid and void layers. Specifically, we address the issue of the assigned value of s in Eq. (1) for particles that enter the system from the exterior. We consider different choices of parameters, providing numerical results for the non-classical equation and comparing these results against "benchmark" numerical results, as well as with the atomic mix and the Levermore-Pomraning models.

2. Numerical Results

In order to solve Eq. (1) in the finite ($0 \leq x \leq X$) solid-void 1-D random periodic system given in [?], we use the identity

$$\Sigma_t(s) = \frac{p(s)}{1 - \int_0^s p(s')ds'} \tag{2}$$

together with the analytically obtained path-length distribution function

$$p(s) = \begin{cases} \frac{\Sigma_{t1}}{\ell}\left[(2n+1)\ell - s\right] e^{-\Sigma_{t1}(s-n\ell)}, & \text{if } 2n\ell \leq s \leq (2n+1)\ell \\ \frac{\Sigma_{t1}}{\ell}\left[s - (2n+1)\ell\right] e^{-\Sigma_{t1}[s-(n+1)\ell]}, & \text{if } (2n+1)\ell \leq s \leq 2(n+1)\ell \end{cases} \tag{3}$$

for $n = 0, 1, 2, \ldots$. Here, Σ_{t1} is the total cross section in the solid material and ℓ is the thickness of each layer in the 1-D system.

For the test problems in this paper we define fixed values of Σ_{t1} and ℓ and present numerical results for different choices of the scattering ratio c. We assume that the incoming fluxes at the boundaries are given by

$$\psi^{+}(x = 0) = 2; \qquad \text{and} \qquad \psi^{-}(x = X) = 0. \tag{4}$$

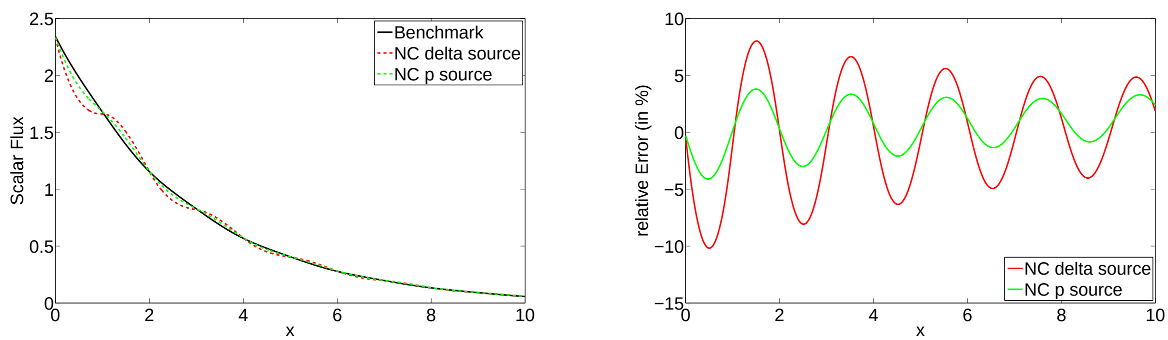
The first approach to model boundary conditions for Eq. (1) is the most intuitively appealing: we simply define that all particles enter the system with the same value of s. The obvious choice in this case is $s = 0$, as follows:

$$\psi^+(0, s) = 2\delta(s); \qquad \text{and} \qquad \psi^-(X, s) = 0. \tag{5}$$

The second approach assumes that particles enter the system after having already traveled different distances s. One appealing choice consists of using $p(s)$ itself to define the boundary conditions:

$$\psi^+(0, s) = 2p(s); \qquad \text{and} \qquad \psi^-(X, s) = 0. \tag{6}$$

Figure 1 shows numerical results and relative (%) errors obtained for a problem with choice of parameters given by $\Sigma_{t1} = \ell = 1$, $c = 0.5$, and $X = 10$. We see that the assumption that all particles enter the system with the value $s = 0$ does not yield an accurate solution, being clearly inferior to the second approach.



(a) Ensemble-averaged scalar fluxes

(b) Relative (%) errors

Figure 1: Comparison of the ensemble-averaged scalar fluxes and relative (%) errors of the different approaches

The full version of this work will investigate the accuracy of different approaches to model the boundary conditions in this homogenized 1-D random periodic system. We will present solutions for several different choices of parameters, as well as include comparisons with classical models such as Atomic Mix and Levermore-Pomraning.

References

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