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B. E. (Third Semester) Examination, ~~2020~~ APR-MAY 2022

(New Scheme)

(CSE Engg. Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

***Note** : Attempt all questions. Part (a) of each question is compulsory and carrying 2 marks each and attempt two questions from (b), (c) and (d) carrying 7 marks each.*

1. (a) Define Euler's formula. 2

(b) For a function $f(x) = |x|, -\pi < x < \pi$ obtain a fourier series. Deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\ldots=\pi^2/8 .$$ 7

(c) Obtain the Fourier expansion of $x \sin x$ as a cosine series in $(0,\pi)$. Hence show that

$$\frac{1}{1\cdot 3}-\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}-\ldots.\infty=\frac{\pi-2}{4}$$ 7

(d) The following values of y give the displacment in inches of a certain machine part for the rotation x of the flywheel. Expand y in terms of a fourier series : 7

x :	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$
y :	0	9·2	14·4	17·8	17·3	11·7

2. (a) Write the condition for the existence of Laplace Transform. 2

(b) Find the Laplace transform of :

(i) $f(t)=|t-1|+|t+1|\ t\geq 0$ 4

(ii) $f(t)=1-et/t$ or $\left(\frac{1'-et}{t}\right)$ 3

(c) Applying convolution theorem to evoluate :

(i) $L^{-1}\left(\frac{S}{\left(S^2+a^2\right)^2}\right)$ 3

(ii) $\tan^{-1}(2/5)$ find the inverse transform. 4

(d) Solve $ty''+2y'+ty=\cos t$ given that $y(0)=1$. 7

3. (a) Define analytic function. 2

(b) If $f(z)$ is a regular function of z, prove that : 7

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2=4|f'(z)|^2$$

(c) Evaluate using Cauchy's integral formulae : 7

(i) $\oint_C \frac{\sin^2 z}{(z-\pi/6)^3}\, dz$, where C is the circle $|z|=1$.

(ii) $\oint_C \frac{z}{(z^2-3z+2)}\, dz$, where C is $|z-2| = ½$

(d). Apply the calculus of residues to prove that : 7

$$\int_0^{2\pi} \frac{d\theta}{1-2p\sin\theta+p^2} = 2\pi/1-p^2 \; (0<p<1)$$

4. (a) From the partial differential equation. 2

$$(x-a)^2+(y-b)^2 = z^2 \cot^2 \alpha$$

(b) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x=0$ and $z=0$ when y is an odd multiple of $\pi/2$. 7

(c) Solve equation :

$$(D^2 - DD' - 2D'^2)z = (y-1)e^x$$ 7

(d) Using the method of separation of variables, solve

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$. 7

5. (a) The mean and standard deviation of the Binomial distribution with n observation and probability of success 'P' and failure 'q' are. 2

(b) A random variable x has the following probability function :

Variable of x :	−2	−1	0	1	2	3
$P(x)$:	0·1	k	0·2	$2k$	0·3	k

Find the value of k and calculate mean and variance. 7

(c) In sampling enalrge number of parts manufactured by a machine. The mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least

3 defective parts. 7

(d) The mean inside diameter of a sample of 200 washers produced by a machine is 5·02 mm and the standard deviation is 0·05 mm. The purpose for which these washer are intended allows a maximum tolerance in the diameter of 4·96 mm to 5·08 mm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed. 7