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# A000112(014)

## B.Tech. (First & Second Semester) Examination April-May 2023

**(New AICTE Scheme)**

**(Common to all Branches) (P1 Group)**

## MATHEMATICS - I

***Time Allowed : Three hours***

***Maximum Marks : 100***

***Minimum Pass Marks : 35***

***Note :*** *All questions are to be attempted. Part (a) is compulsory in each unit and carries 4 marks. Attempt any **two** parts from (b), (c) and (d) of each unit which carry 8 marks each.*

### Unit-I

1. (a) (i) If $f(x) = f(2a - x)$ then $\int_0^{2a} f(x)dx$ is equal to .......

(ii) The value of $\int_0^{\infty} e^{-x^2} dx =$ ................

(b) If $n$ is a positive integer, show that

$$\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}, \; m > -1$$

(c) Find the volume of a sphere of radius $a$.

(d) Find a reduction formula for $\int e^{ax} \sin^n x \, dx$.

Hence evaluate

$$\int e^x \sin^3 x \, dx.$$

**Unit-II**

2. (a) Evaluate :

$$Lt_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}.$$

(b) Use Taylor's series to prove that

$$\tan^{-1}(x+h) = \tan^{-1} x + (h \sin z)$$





$$\frac{\sin z}{1} - (h \sin z)^2 - \frac{\sin 2z}{2} + (h \sin z)^3, \frac{\sin 3z}{3} - \ldots\ldots$$

where $z = \cot^{-1} x$.

(c) Show that the right circular cylinder of given surface (including the ends) and maximum volume is such that its height is equal to the diamter of the base.

(d) Prove that :

$$\text{if } (0 < a < b < 1), \frac{b-1}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$$

Hence show that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$$

**Unit-III**

3. (a) (i) The series :

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots\ldots$$

converges if ..................

(ii) If $\sum u_n$ is a convergent series of positive terms, then $Lt_{n\to\infty}$ is

(b) Test for the converence the series :

$$1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\cdots\cdots\cdots\infty$$

(c) Test the convergece of the series

$$\sum_{n=1}^{\infty}\frac{n!}{\left(n^n\right)^2}$$

(d) Obtain the fourier expansion of $x\sin x$ as cosine series in $(0, \pi)$.

**Unit-IV**

4. (a) (i) If $u=x^y$, then find $\frac{\partial u}{\partial x}$ & $\frac{\partial u}{\partial y}$.

(ii) If $z=x^3+y^3$, then $\frac{\partial^2 u}{\partial x\partial y}$ is equal to ......





(b) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum.

(c) What is the directional derivative of $\phi=xy^2+yz^3$ at the point $(2, -1, 1)$ in the direction of the normals to the surface

$$x\log z-y^2=-4$$

at (–1, 2, 1).

(d) Show that :

$$div\left(grad\ r^n\right)=n(n+1)r^{n-2}$$

where $r^2=x^2+y^2+z^2$.

**Unit-V**

5. (a) (i) If $A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$,

then eigen value of $A^{-1}$ are .............

(ii) Define rank of a matrix.

(b) For the matrix $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & -1 & -1\end{bmatrix}$ find non-singular matrices $P$ and $Q$ such that PAQ is in the normal form. Hence find the rank of $A$.

(c) Investigate the value of $\lambda$ and $\mu$ so that the equation :

$$\begin{aligned}2x+3y+5z&=9\\7x+3y-2z&=8\\2x+3y+\lambda z&=\mu\end{aligned}$$

having

(i) no solution

(ii) a unique solution and

(iii) an infinite number of solutions.

(d) Find the matrix represented by the polynomial

$$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+1$$





where matrix

$$A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}.$$