Printed Pages – 6 Roll No. : ....

A000112(014)

B.Tech. (First Semester) Examination Nov.-Dec. 2021

(AIC Scheme)

(AEI, Bio Tech, Chem., Civil, CSE, Elect., EEE, EI, Et&T, IT, Mech., Mining, Metallurgy, Mechatronics, Production, Automobile, Agriculture, Plastic)

MATHEMATICS-I

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : *All units are compulsory. Solve any **two** parts from (b), (c) and (d). Part (a) is compulsory.*

Unit-I

1. (a) Compute $\Gamma 3\cdot 5$ and $\Gamma -1/2$. 4

(b) If n, is a positive integer, show that

$$\int_0^1 x^n (\log x)^n \, dx = \frac{(-1)^n \cdot n!}{(m+1)^{n+1}}, \quad m > -1$$

Hence Evaluate

$$\int_0^1 x^5 (\log x)^3 \, dx$$ 8

(c) Given $\int_0^\infty \frac{x^{n-1} dx}{1+x} = \frac{\pi}{\sin n\pi}$

Show that

$$\Gamma n \, \Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

Hence evaluate

$$\int_0^\infty \frac{dy}{1+y^4}.$$ 8

(d) Find the volume of the solid generated by revolving the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. About

(i) the major axis

(ii) the minor the axis 8

Unit-II

2. (a) Verify Rolle's theorem for

$f(x) = e^x (\sin x - \cos x)$ in $(\pi/4, 5\pi/4)$ 4

(b) Find the maximum and minimum value of

$$f(x) = 2x^3 - 21x^2 + 36x - 20 \quad 8$$

(c) Use Taylor's series to prove that

$$\tan^{-1}(x+h) = \tan^{-1}x + (h\sin z)$$

$$\frac{\sin z}{1} - (h\sin z)^2 - \frac{\sin 2z}{2} \quad 8$$

(d) Expand by Maclaurin's the function $\log(1+\sin x)$. 8

Unit-III

3. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$. 4

(b) Find the fourier expansion for $f(x)$ if

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\infty = \frac{\pi^2}{8}. \quad 8$$

(c) Expand

$$f(x) = \begin{cases} 1/4 - x, & 0 < x < 1/2 \\ x - 3/4, & 1/2 < x\ 1 \end{cases}$$

as a fourier series of sine terms. 8

(d) Given

$$f(x) = \begin{cases} -x+1, & \text{for } -\pi \le x \le x\ 0 \\ x+1, & \text{for } 0 \le x \le \pi \end{cases}$$

Is the function even or odd? Find the fourier series for $f(x)$ and deduce the values of

$1/1^2 + 1/3^2 + 1/5^2 \ldots$ 8

Unit-IV

4. (a) P. T. for $z = x^3 + y^3 - 3axy$ $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$. 4

(b) The temperature T at any point (x, y, z) in space is given by $T = 400\,xyz^2$. Find the highest temperature the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ 8

(c) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x\log z - y^2 = -4$ at $(-1, 2, 1)$. 8

(d) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)^{-2}$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -9\left(x+y+z\right)^{-2}$$ 8

Unit-V

5. (a) Find the sum and product of the eigen values of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$ 4

(b) Investigate the values of λ and μ. So that the equation

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$x + 2y + \lambda z = \mu$. have

(i) no solution

(ii) a unique solution

(iii) infinite no,. of solutions. 8

(c) Find all Eigen values and Eigen vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$ 8

(d) Find the matrix represented by the polynomial

$$A^8 - 5A^7 + 7A^6 - 3A^5$$

$$+ A^4 - 5A^3 + 8A^2 - 2A + I = 0$$

where matrix is

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}.$$ 8