Printed Pages – 6 Roll No. : ....

A000112(014)

B.Tech. (First Semester) Examination April-May 2022

(AIC Scheme)

(AEI, Bio Tech, Chem., Civil, CSE, Elect., EEE, EI, Et & T, IT, Mech., Mining, Metallurgy, Mechatronics, Production, Automobile Agriculture, Plastic Branch,
CSE (AIML) & Lateral Diploma)

MATHEMATICS-I

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Attempt all questions. Part (a) is compulsory in each unit and carries 4 marks. Attempt any ***two*** *parts from (b), (c) and (d) of each unit which carry 8 marks each.*

Unit-I

1. (a) (i) Define Beta and Gamma functions.

(ii) Area bounded by the curve $y = f(x)$ the x-axis and the ordinates $x = a$, $x = b$ is ....

(b) If $I_n = \int_0^{\frac{\pi}{2}} x \cos^n x \, dx \ (n > 1)$, prove that

$$I_n = \frac{n-1}{n} I_{n-2} - \frac{1}{n^2}.$$ Hence evaluate I_4.

(c) Given

$$\cancel{\int_0^9 \frac{x^{n-1}}{1+x} dx = \frac{\pi}{\sin nz}} \qquad \int_0^\infty \frac{x^{n-1}}{1+x} dx = \frac{\pi}{\sin n\pi}$$

Show that

$$\Gamma n \, \Gamma(1-n) = \frac{\pi}{\sin n\pi}.$$

Hence evaluate

$$\cancel{\int_0^9 \frac{dy}{1+y^4}}. \qquad \int_0^\infty \frac{dy}{1+y^4}$$

(d) Find the volume of the solid generated by revolving the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

About : (i) the major axis

(ii) the minor axis

Unit-II

2. (a) Find the values of a, b and c So that

$$\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2.$$

(b) Expand $e^{a \sin^{-1} n}$ in ascending powers of x using Maclaurin's theorem.

(c) Use Taylor's series to prove that $\tan^{-1}(x + n) =$ h

x

$$\tan^{-1} n + (h\sin z) \frac{\sin z}{1} - (h \sin z)^2 \frac{\sin 2z}{2} + (h \sin z)^3 \frac{\sin 3z}{3} + \ldots\ldots.$$

Where $z = \cot^{-1} x$.

(d) Find the point on the curve $y^2 = 2x$, which is nearest to the point (1, –4).

Unit-III

3. (a) (i) Define Dirichlet's conditions for Fourier Series.

(ii) Write the DeAlembert's ratio test.

(b) Expand the function $f(x) = x \sin x$ as a Fourier Series in the ~~Interval~~ interval $-\pi \le x \le \pi$.

(c) Expand

$$f(x) = \begin{cases} \frac{1}{4} - x, & \text{if } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{if } \frac{1}{2} < x < 1 \end{cases}$$

as the Fourier Series of sine terms.

(d) Prove that :

$$\log_e \left(1 + \frac{1}{n}\right)^n = \left[1 - \frac{1}{2(n+1)} - \frac{1}{2.3(n+1)^2} - \frac{1}{3.4(n+1)^3} \cdots\cdots\cdots \infty\right]$$

Unit-IV

4. (a) Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the

direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

(b) If $x^x y^y z^z = c$, show that at $x = y = z$,

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}.$$

(c) In a plane triangle. Find the maximum value of ~~cos4, cosB, cosC~~. cosA cosB cosC.

(d) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$.

Unit-V

5. (a) (i) Write the statement of Cayley Hamilton Theorem.

(ii) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$, then eigen values of A^{-1} are ....

(b) For what values of k the equations

$$x+y+z=1$$

$$2x+y+4z=k$$

$$4x+y+10z=k^2$$

have a solution and solve them completely in each case.

(c) Find the Eigen roots and Eigen vectors of the matrix

$$A=\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}.$$

(d) Find the matrix represented by the polynomial $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I=0$ where matrix is

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$