UNIT 9 TIME VALUE OF MONEY

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9.0 INTRODUCTION

The time value of money (TVM) states that a sum of money is worth more now than the same sum will be at a future date because of its earnings potential. A sum of money in the hand has greater value than the same sum to be paid in the future. Investors prefer to receive money today rather than the same amount of money in the future because once the money is invested, it grows over time. For example, money deposited into a savings account earns interest. Over a period of time, the interest is added to the principal, earning more interest. This is the power of compounding interest. If the money is not invested, it loses it value over time. If you do not invest for 2-3 years, you will lose the additional money it could have earned over that period if invested. It will have even less buying power when you retrieve it because inflation has reduced its value. The notion that money has time value is one of the most basic concepts of investment analysis. For any productive asset it's value will depend upon the future cash flows associated with that particular asset. In order to assess the adequacy of cash flows one of the important parameters is to assess the time value of the cash flows viz., Rs.100 received after one year would not be the same as Rs.100 received after two years. There are several reasons to account for this difference based on the timing of the cash flows, some of which are as follows:

- there is a general preference for current consumption to future consumption,
- capital (savings) can be employed to generate positive returns,
- due to inflation purchasing power of money decreases over time, and
- future cash flows are uncertain.

Translating the current value of money into its equivalent future value is referred to as compounding. Translating a future cash flow or value into its equivalent value in a prior period is referred to as discounting. This Unit deals with basic mathematical techniques used in compounding and discounting.

9.1 OBJECTIVES

After going through this unit, you should be able to:

- understand the time value of money,
- understand what gives money its time value,
- understand the methods of calculating present and future value, and
- understand the use of present value technique in financial decisions.

9.2 DETERMINING THE FUTURE VALUE

Process of calculating future value of money from present value is classified as compounding. Compounding is the process in which earning of interest either from capital gains or interest, are reinvested to generate additional earnings over time. Let us discuss this concept through example.

suppose that you deposit Rs.1000 with a bank which pays 10 per cent interest compounded annually for a period of 3 years. The deposit will grow as follows:

First Year	Principal at the beginning. Interest for the year (1000x.10) Total amount	Rs. 1000 100 1100
Second Year	Principal at the beginning. Interest for the year (1100x.10). Total Amount	1100 110 1210
Third Year	Principal at the beginning. Interest for the year (1210x.10) Total Amount	1210 121 1321

To get the future value from present value for a one year period

$$FV = PV + (PV \times k)$$

where PV = Present Value

k = Interest rate

$$FV = PV(1+k)$$

Similarly for a two year period

FV = PV	+ (PV × k)	+ (PV × k)	+ (PV × k × k)
Principal amount	First period interest on principal	Second period interest on the principal	Second periods interest on the first periods interest

$$FV = PV+PVk+PVk+PVk^2$$
$$= PV+2PVk+PVk^2$$
$$= PV\ (1+2k+K^2) = PV\ (1+k)^2$$

Thus, the future value of amount after n periods is (9.1)

$$FV = PV\,(1+k)^n$$

where FV = Future value n years hence
PV = Cash today (present value)
k = Interest rate par year in percentage
n = number of years for which compounding is done

Equation (9.1) is the basic equation for compounding analysis. The factor $(1+k)^n$ is referred to as the compounding factor or the future value interest factor (FVIFk,n). Published tables are available showing the value of $(1+k)^n$ for various combinations of k and n. One such table is given in appendix A of this unit.

Example 9.1 Find out the future value of Rs.1000 compounded annually for 10 years at an interest rate of 10%.

Solution: The future value 10 years hence would be

$$FV = PV\,(1+k)^n$$
$$FV = 1{,}000\,(1+.10)^{10}$$
$$= 1000 \times (1.10)^{10}$$
$$= 1000\,(2.5937)$$
$$= 2593.7$$

The appreciation in present value of an amount can also be expressed in terms of return. A return is the income on investment over each period divided by the amount of investment at the beginning of the period. From the above example the arithmetic average return would be (2593.7 −1000)/1000=159.37% over the ten year period or 15.937% per year. The main drawback of using arithmetic average is that it ignores the process of compounding. To overcome this, the correct method is to use geometric average return to calculate overage annual return.

Rearranging the equation 9.1 we get

$$k = n\sqrt{\frac{FV}{PV}} - 1 \qquad (2.2)$$

using the values from example 9.1

$$= \sqrt[10]{\frac{2593.7}{1{,}000}} - 1$$
$$= \left(\frac{2593.7}{1000}\right)^{1/10} - 1$$
$$= 1.10 - 1$$
$$= .10 = 10\%$$

9.2.1 Shorter Compounding Period

So far in our discussion we have assumed that the compounding is done annually, now let us consider the case where compounding is done more frequently. In this case the equation (9.1) is modified to factor in the change of frequency of compounding.

$$FV_n = PV\,(1 + \frac{k}{m})^{m \times n} \qquad (9.3)$$

where FV_n = Future value after n years

PV = Present Value
K = nominal annual rate of interest
m = Frequency of compounding done during a year
n = number of years for which compounding is done.

If the interest is payable semiannually frequency of compounding is 2, if it is payable monthly frequency is 12, if it is payable weekly frequency is 52 and so on.

Example 9.2 Calculate the future value of Rs.5000 at the end of 6 years, if nominal interest rate is 12 per cent and the interest is payable quarterly (frequency = 4)

Solution:

$$FV_n = PV\ (1+\frac{k}{m})^{m\times n}$$

$$FV_6 = 5000\,(1+\frac{.12}{4})^{6\times 4}$$

$$= 5000\,(1+.03)^{24}$$

$$= 5000\times 2.0328$$

$$= 10{,}164$$

The future value of Rs.5000 after 6 years on the basis of quarterly compounding would be Rs.10 164 whereas in case of semi-annual and annual compounding the future value would be–

$$FV_6 = 5000\,(1+\frac{.12}{2})^{6\times 2}$$

$$= 5000\,(1+06)^{12}$$

$$= 5000\times 2.0122$$

$$= 10{,}061$$

$$FV_6 = 5000\ (1+.12)^6$$

$$= 5000\ (1.9738\ )$$

$$= 9868$$

This difference in future value is due to the fact that interest on interest has been calculated.

9.2.2 Effective vs. Nominal Rates

In the above example we have seen how the future value changes with the change in frequency of compounding. In order to understand the relationship between effective and nominal rate let us calculate the future value of Rs.1000 at the interest rate of 12 per cent when the compounding is done annually, semiannually, quarterly and monthly.

$$FV = 1000\,(1+.12)^1$$
$$= 1120$$

$$FV = 1000\,(1+\frac{.12}{2})^2$$
$$= 1000\,(1.06)^2$$
$$= 1000(1.1236)$$
$$= 1123.6$$

$$FV = 1000\,(1+\frac{.12}{4})^4$$
$$1000 = (1.03)^4$$
$$1000 = (1.1255)$$
$$= 1125.5$$

$$FV = 1000\,(1+\frac{.12}{12})^{12}$$
$$= 1000\,(1.01)^{12}$$
$$= 1000(1.1268)$$
$$= 1126.8$$

From the above calculations we can see that Rs.1000 grows to Rs.1120, Rs.1123.6, Rs.1125.5 and Rs.1126.8 although the rate of interest and time period are the same. In the above case 12.36, 12.55 and 12.68 are known as effective rate of interest. The relationship between the effective and nominal rate of interest is given by

$$r = (1 + \frac{k}{m})^m - 1 \qquad (9.4)$$

where r = effective rate of interest
k = nominal rate of interest
m = frequency of compounding per year

Based on the above stated example the effective interest rate is calculated as follows:

a) Effective interest rate for monthly compounding

$$r = (1+\frac{.12}{12})^{12} - 1$$
$$= (1.01)^{12} - 1$$
$$= 1.1268 - 1$$
$$= .1268 = 12.68$$

b) Effective interest rate for quarterly compounding

$$r = (1+\frac{12}{4})^4 - 1$$
$$r = (1.03)^4 - 1$$
$$r = 1.1255 - 1 = .1255$$
$$= 12.55\%$$

c) Similarly the effective interest rate for semi-annual compounding is

$$r = (1 + \frac{12}{2})^2 - 1$$
$$r = (1.06)^2 - 1$$
$$r = 1.1236 - 1 = .1236 = 12.36$$

Doubling Period

One of the first and the most common questions regarding an investment alternative is the time period required to double the investment. One obvious way is to refer to the table of compound factor from which this period can be calculated. For example the doubling period at 3%, 4%, 5%, 6%, 7%, 8%, 9%, 10%, 12% would be approximately 23 years, 18 years, 14 years, 12 years, 10 years, 9 years, 8 years, 7 years, and 6 years respectively.

If one is not inclined to use future value interest factor tables there is an alternative, known as rule of 72. According to this rule of thumb the doubling period is obtained by dividing 72 by the interest rate. For example, at the interest rate of 8% the approximate time for doubling an amount would be 72/8 = 9 years.

A much more accurate rule of thumb is rule of 69. As per this rule the doubling period is equal to

$$.35 + \frac{69}{\text{Interest rate}}$$

Using this rule the doubling period for an amount fetching 10 percent and 15 percent interest would be as follows.

$$.35 + \frac{69}{10} = .35 + 6.9 = 7.25 \text{ years}$$
$$.35 + \frac{69}{15} = .35 + 4.6 = 4.95 \text{ years}$$

9.2.3 Continuous Compounding

The extreme frequency of compounding is continuous compounding where the interest is compounded instantaneously. The factor for continuous compounding for one year is eAPR where e is 2.71828 the base of the natural logarithm. The future value of an amount that is compounded for n years is

$$FV = PV \text{ x } e^{kn}$$

Where k is annual percentage rate and e^{kn} is the compound factor.

Example 9.3: Find the future value of Rs.1000 compounded continuously for 5 year at the interest rate of 12% per year and contrast it with annual compounding.

$$\text{Solution: } FV_{\bar{5}} = PVe^{N(APR)}$$
$$= 1000 \times 2.71828$$
$$= 1000 \times 2.71828^{.60}$$
$$= 1000 \times 1.82212$$
$$= 1822.12$$

$$FV_{\bar{5}} = PV(1+k)^n$$
$$= 1000(1+.12)^5$$
$$= 1000(1.7623)$$
$$= 1762.3$$

From this example you can very well see the effects of extreme frequency of compounding.

So far in our discussion we have assumed that the interest rate is going to remain the same over the life of the investment, but now a days we are witnessing an increased volatility in interest rates as a result of which the financial instruments are designed in a way that interest rates are benchmarked to a particular variable and with the change in that variable the interest rates also change accordingly.

In such cases the Future Value is calculated through this equation.

$$FV_n = PV(1+k_1)(1+k_2)(1+k_3)+\ldots \quad (1+k_n) \tag{9.5}$$

Where k_n is the interest rate for period n.

Example 9.4: Consider a Rs.50, 000 investment in a one year fixed deposit and rolled over annually for the next two years. The interest rate for the first year is 5% annually and the expected interest rate for the next two years are 6% and 6.5% respectively calculate the future value of the investment after 3 years and the average annual interest rate.

Solution:

$$FV = PV\ (1+k_1)(1+k_2)(1+k_3)$$
$$= 50{,}000(1+.05)(1+.06)(1+.065)$$
$$= 59{,}267.25$$

Average annual interest rate

$$\frac{.05+.06+.065}{3}$$
$$=.58333 \text{ (wrong)}$$

By now we know the values of FV, PV, and n. The average annual interest rate would be

$$k = \sqrt[n]{\frac{FV}{PV}}$$

$$k = \sqrt[3]{\frac{59267.25}{50{,}000}} = \sqrt[3]{1.185345} = 5.8315\%$$

This is also equivalent to

$$k = \sqrt[3]{(1+.05)(1+.06)(1+.065)} - 1$$
$$= 5.8315$$

☞ Check Your Progress 1

1. Calculate the compound value of Rs. 1000, interest rate being 12% per annum, if compounded annually, semi annually, quarterly and monthly for 2 years.

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2. Calculate the future value of Rs. 1000 deposited initially, if the interest is 12% compounded annually for the next five years.

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3. Mr. X bought a share 15 years ago for Rs. 10, the present value of which is Rs. 27.60. Compute the compound growth rate in the price of the share.

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4. **State whether the following statements are True of False.**

a) Finding the present value is simply the reverse of compounding.
b) The present value interest factor (PVIF) is the reciprocal of the future value interest factor (FVIF).
d) if the discount rate decreases, the present value of a given future amount decreases.
e) The present value interest factor for a dollar on hand today is 0.

9.3 ANNUITY

An annuity is defined as stream of uniform period cash flows. The payment of life insurance premium by the policyholder to the insurance company is an example of an annuity. Similarly, deposits in a recurring bank account are also an annuity.

Depending on the timing of the cash flows annuities are classified as:

a) Regular Annuity or Deferred Annuity

b) Annuity Due.

The regular annuity or the deferred annuities are those annuities in which the cash flow occur at the end of each period. In case of an annuity due the cash flow occurs at the beginning of the period.

Example 9.5: Suppose Mr. Ram deposits Rs. 10,000 annually in a bank for 5 years, at 10 per cent compound interest rate. Calculate the value of this series of deposits at the end of five years assuming that (i) each deposit occurs at the end of the year (ii) each deposit occurs at the beginning of the year.

Solution: The future value of regular annuity will be

Rs. $1000\ (1.10)^4 + 1000\ (1.10)^3 + 1000\ (1.10)^2 + 1000\ (1.10) + 1000$

$= 6105.$

The future value of an annuity due will be

Rs. 1000 $(1.10)^5$ + 1000 $(1.10)^4$ +1000 $(1.10)^3$+1000 $(1.10)^2$ +1000 (1.10)

= Rs 1000 (1.611) + 1000 (1.4641) + 1000 (1.331) + 1000 (1.21)+1000 (1.10)

= Rs. 6716.

In the above example you have seen the difference in future value of a regular annuity and annuity due. This difference in value is due to the timing of cash flow. In case of regular annuity the last cash flow does not earn any interest, whereas in the case of annuity due, the cash flows earns an interest for one period.

Formula

In general terms the future value of an annuity (regular annuity) is given by the following formula:

$$FVA_n = A(1+k)^{n-1} + A(1+k)^{n-2} + \ldots + A \qquad (9.6)$$

$$= A\sum_{t=1}^{n}(1+k)^{n-t}$$

$$= A\left[\frac{(1+k)^n - 1}{k}\right]$$

Future value of an annuity due

$$FVA_{n(due)}\ A(1+k)^n + A(1+k)^{n-1} + \ldots + A(1+k)$$

$$FVA_{n(due)} = A\sum_{t=1}^{n}(1+k)^{n-t+1} \qquad (9.7)$$

$$= A\left[\frac{(1+k)^n - 1}{k}\right](1+k)$$

Where FVA_n = Future value of an annuity which has a duration of n periods

A = Constant periodic cash flow

k = Interest rate per period

n = duration of the annuity

The term $\left[\frac{(1+k)^n - 1}{k}\right]$ is referred to as the future value interest factor for an annuity ($FVIFA_{k,n}$). The value of this factor for several combinations of k and n are given in the appendix at the end of this unit.

Present Value of an Uneven Series

In real life cash flows occurring over a period of time are often uneven. For example, the dividends declared by the companies will vary from year to year, similarly payment of interest on loans will vary if the interest is charged on a floating rate basis. The present value of a cash flow stream is calculated with the help of the following formula:

$$PV_n = \frac{A_1}{(1+K)} + \frac{A_2}{(1+k)^2} + \ldots\ldots\ldots + \frac{A_n}{(1+k)^n} = \sum_{t=1}^{n}\frac{A_t}{(1+k)^t} \qquad (9.8)$$

Where

PV_n = present value of a cash flow stream

A_t = cash flow occurring at the end of the year

k = discount rate

n = duration of the cash flow stream

Shorter Discounting Periods

Sometimes cash flows may have to be discounted more frequently than once a year-semi-annually, quarterly, monthly or daily. The result of this is two fold (i) the number of periods increases (ii) the discount rate applicable per period decreases. The formula for calculating the present value in case of shorter discounting period is

$$PV = FV_n \left[\frac{1}{1+k/m}\right]^{n/m} \quad (9.9)$$

Where m = number of times per year discounting is done.

Example 9.6: Calculate the present value of Rs. 10,000 to be received at the end of 4 years. The discount rate is 10 percent and discounting is done quarterly.

Solution:

$$\begin{aligned} PV &= FV_4 \times \text{PVIF } k/m,\ m \times n \\ &= 10{,}000 \times \text{PVIF } 3\%,\ 16 \\ &= 10{,}000 \times 0.623 \\ &= \text{Rs. } 6230 \end{aligned}$$

Determining the Present Value

In the previous sections we have discussed the computation of the future value, now let us work the process in reverse. Let us suppose you have won a lottery ticket worth Rs. 1000 and this Rs. 1000 is payable after three years. You must be interested in knowing the present value of Rs. 1000. If the interest rate is 10 per cent, the present value can be calculated by discounting Rs. 1000 to the present point of time as follows.

$$\text{Value three years hence} = \text{Rs. } 1000 \left(\frac{1}{1.10}\right)$$

$$\text{Value one years hence} = \text{Rs. } 1000 \left(\frac{1}{1.10}\right)\left(\frac{1}{1.10}\right)$$

$$\text{Value now (Present Value)} = \text{Rs. } 1000 \left(\frac{1}{1.10}\right)\left(\frac{1}{1.10}\right)\left(\frac{1}{1.10}\right)$$

Formula

Compounding translates a value at one point in time into a value at some future point in time. The opposite process translates future value into present value. Discounting translates a value back in time. From the basic valuation equation

$$FV = PV\,(1+k)^n$$

Dividing both the sides by $(1+k)^n$ we get

$$PV = FV\left[\frac{1}{(1+k)}\right]^n \quad (9.10)$$

The factor $\left[\frac{1}{(1+k)}\right]^n$ is called the discounting factor or the present value interest factor [$PVIF_{k,n}$]

Example 9.7: Calculate the present value of Rs. 1000 receivable 6 years hence if the discount rate is 10 per cent.

Solution: The present value is calculated as follows:

$$
\begin{aligned}
PV_{kn} &= FV_n \times PVIF_{k,n} \\
&= 1{,}000 \times (0.5645) \\
&= 564.5
\end{aligned}
$$

Example 9.8: Suppose you are receiving an amount of Rs.5000 twice in a year for next five years once at the beginning of the year and the other amount of Rs. 5000 at the end of the year, which you deposit in the bank which pays an interest of 12 percent. Calculate the value of the deposit at the end of the fifth year.

Solution: In this problem we have to calculate the future value of two annuities of Rs.5000 having duration of five years. The first annuity is an annuity due and the second annuity is regular annuity, therefore the value of the deposit at the end of five year would be

$$
\begin{aligned}
&FVA_n + FVA_{n(due)} \\
&= A\left[\frac{(1+k)^n - 1}{k}\right] + A\left[\frac{(1+k)^n - 1}{k}\right](1+k) \\
&= A\,(FVIFA_{12,5}) + A\,(FVIFA_{12,5})\,(1+k) \\
&= 5000\,(6.353) + 5000\,(6.353)(1.12) \\
&= 31{,}765 + 35{,}577 \\
&= 67336
\end{aligned}
$$

The value of deposit at the end of the fifth year is Rs. 67,342.

Sinking Fund Factor

Suppose you are interested in knowing how much should be saved regularly over a period of time so that at the end of the period you have a specified amount. To answer this question let us manipulate the equation

$$FVA_n = A\left[\frac{(1+k)^n - 1}{k}\right]$$

which shows the relationship between FVA_n, A, k, and

$$A = \left[\frac{k}{(1+k)^n - 1}\right]^{FVA_n} \qquad (9.11)$$

Equation 9.11 helps in answering this question. The periodic deposit is simply A and is obtained by dividing FVAn by $FVIFA_{k,n}$. In eq 9.11 $\left[\frac{k}{(1+k)^n - 1}\right]$ is the inverse of $FVIFA_{k,n}$ and is called the sinking fund factor.

Example 9.9: How much should you save annually so as to accumulate Rs. 20, 00,000 by the end of 10 years, if the saving earns an interest of 12 per cent?

Solution:

$$A = FVA_n \left[\frac{k}{(1+k)^n - 1} \right]$$

$$= Rs.20{,}000 \times \frac{1}{FVIFA_{12\%,10}}$$

$$= Rs.20{,}00000 \times \frac{1}{17.548}$$

$$= 1{,}11{,}400$$

Present value of an annuity

Let us suppose you expect to receive Rs.2000 annually for the next three years. This receipt of Rs.2000 is equally divided. One part viz., Rs.1000 is received at the beginning of the year and the remaining Rs.1000 is received at the end of the year. We are interested in knowing the present value when the discount rate is10 per cent. The cash flows stated above are of two types which are similar to regular annuity and annuity due. The present value of this cash flow is found out as follows:

a) Present value of Rs.1000 received at the end of each year for three years (Regular annuity).

$$Rs.\,1000(\frac{1}{1.10}) + Rs.1000(\frac{1}{1.10})^2 + Rs.1000(\frac{1}{1.10})^3$$

$$1000 \times 09091 + 1000 \times 08264 + 1000 \times 0.7513$$

$$Rs.2479.$$

b) Present value of Rs.1000 received at the beginning of each year for three year (annuity due)

$$Rs.1000 + Rs1000(\frac{1}{1.10}) + Rs.1000(\frac{1}{1.10})^2$$

$$= 1000 + 1000 \times 0.9091 + 1000 \times 08264$$

$$= Rs.2735$$

The present value of this annuity is Rs. 2479+Rs.2735 = Rs. 5214.

Formula

In general terms the present value of a regular annuity may be expressed as follows:

$$PVN_n = \frac{A}{(1+k)} + \frac{A}{(1+k)^2 +} ......+ \frac{A}{(1+k)^n}$$

$$= A\left[\frac{1}{1+k} + \frac{1}{(1+k)^2} + ...\frac{1}{(1+k)^n}\right]$$

$$= A\left[\frac{(1+k)^n - 1}{k(1+k)^n}\right]$$

In case of annuity due

$$PVA_{n(due)} = A\left[\frac{(1+k)^n - 1}{k(1+k)^n}\right](1+k) \qquad (9.12)$$

where PVA_n = Present value of an annuity which has a duration of n periods

A = Constant periodic flows

k = discount rate

Capital Recovery Factor

Equation 9.12 shows the relationship between PVA_n, A, K and n. Manipulating it a bit:

We get

$$A = PVA_n\left[\frac{k(1+k)^n}{(1+k)^n - 1}\right] \qquad (9.13)$$

$\left[\frac{k(1+k)^n}{(1+k)^n - 1}\right]$ in equation 2.13 is inverse of $PVIFA_{k,n}$ and is called the capital recovery factor.

Example 9.10: Suppose you receive a cash bonus of Rs.1, 00,000 which you deposit in a bank which pays 10 percent annual interest. How much can you withdraw annually for a period of 10 years?

From eq.9.13

$$A = PVA_n \times \frac{1}{PVIFA_{10\%}10}$$

$$A = \frac{1,00,000}{6.145}$$

$$A = 16,273$$

Present value of perpetuity:

A perpetuity is an annuity of an infinite duration

$$PVA_\infty = A\left[\frac{1}{(1+k)} + \frac{1}{(1+k)^2} + ... + \frac{1}{(1+k)}\infty\right]$$

$$PVA_\infty = A \times PVIFA_{k,\infty}$$

Where PVA_∞ = Present value of a perpetuity

A = Constant annual payment

$PVIFA_{k,\infty}$ = Present value interest factor for perpetuity

The value of $PVIFA_{k,\infty}$ is

$$\sum_{t=1}^{\infty} \frac{1}{(1+k)^t} = \frac{1}{k}$$

The present value interest factor of an annuity of infinite duration (perpetuity) is simply 1 divided by interest rate (expressed in decimal form. The present value of an annuity is equal to the constant annual payment divided by the interest rate, for example, the present value of perpetuity of Rs.20, 000 if the interest rate is 10%, is Rs. 2, 00,000.

☞ Check Your Progress 2

1. Calculate the present value of Rs. 600 (a) received one year from now (b) received at the end of five years (c) received at the end of fifteen years. Assume a 5% time preference rate.

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2. Mr. Ram is borrowing Rs. 50,000 to buy a motorcycle. If he pays equal installments for 25 years and 4% interest on the outstanding balance, what is the amount of installment? What will be amount of the instalment if quarterly payments are requested to be made?

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3. A bank has offered to pay you an annuity of Rs. 1,800 for 10 years if you invest Rs. 12,000 today. What rate of return would you earn?

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4. a) An annuity due has periodic payments that are at the…….of each payment interval.

   b) A special type of annuity for which payments continue forever is called a……

   c) The time from today until the start of the annuity is referred to as……..

   d) The present value of a deferred annuity is the discounted value of the …………at the beginning of the period of deferment.

   e) The difference in the present value of an ordinary perpetuity and perpetuity due is the amt. of……….

Derivation of Formulas

i) **Future Value of an Annuity**

Future value of an annuity is

$$FVA_n = A(1+k)^{n-1} + A(1+k)^{n-2} + ...A(1+k) + A \qquad (a1)$$

Multiplying both sides of the equation a1 by $(1+k)$ gives.

$$(FVA_n)(1+k) = A(1+k)^n + A(1+k)^{n-1} + ...A(1+k)^2 + A(1+k) \qquad (a2)$$

Subtracting eq. (a1) from eq. (a2) yields

$$FVA_n k = A\left[\frac{(1+k)^n - 1}{k}\right] \qquad (a3)$$

Dividing both sides of eq. (a3) by k yields

$$FVA_n = A\left[\frac{(1+k)^n - 1}{k}\right]$$

ii) **Present Value of an Annuity**

The present value of an annuity is

$$PVA_n k = A(1+k)^{-1} + A(1+k)^{-2} + ...+ A(1+k)^{-n} \qquad (a4)$$

Multiplying both sides of eq (a4) by $(1+k)$ gives:

$$PVA_n (1+k) = A + A(1+k)^{-1} + ...+ A\ (1+k)^{-n+1} \qquad (a5)$$

Subtracting eq (a4) from eq (a5) yields:

$$PVa_n\ k = A\left[1-(1+k)^{-n}\right] = A\left[\frac{(1+k)^n - 1}{k(1+k)^n}\right] \qquad (a6)$$

Dividing both the sides of eq (a6) by k results in:

$$PVA_n = A\left[\frac{(1+k)^n - 1}{k(1+k)^n}\right]$$

iii) **Present Value of a Perpetuity**

$$PVA_\infty = A(1+k)^{-1} + A(1+k)^{-2} + ...+ A(1+k)^{\infty+1} + A(1+k)^{\infty} \qquad (a7)$$

Multiplying both the sides of eq (a7) by (1+k) gives:

$$PVA_\infty = (1+k) = A(1+k) + A(1+k)^{-1} + ...A(1+k)^{-\infty+2} + A(1+k)^{-\infty+1} \qquad (a8)$$

Subtracting equation (a7) from equation (a8) gives:

$$PVA_\infty k = A[1-(1+k)^{\infty}]$$

$$\text{As } (1+k)^{-\infty} \to o \text{ eq. (a8) becomes:}$$

$$PVA_\infty k = A$$

$$\Rightarrow PVA_\infty = \frac{A}{k} \qquad (a9)$$

iv) **Continuous Compounding**

In Section 9.2.2 we had established a relationship between the effective and nominal rate of interest where compounding occurs n times a year which is as follows:

$$r = (1 + \frac{k}{m})^{m} - 1 \qquad \text{(a10)}$$

Rearranging equation a10, it can be expressed as

$$r = [(1 + \frac{k}{m/k})^{m/k}]^{k} - 1 \qquad \text{(a11)}$$

Let us substitute m/k by x om eq (a11)

$$r = [(1 + \frac{1}{x})^{k}] - 1 \qquad \text{(a12)}$$

In continuous compounding $m \to \infty$ which implies $x \to \infty$ in eq (a12)

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e = 2.71828...$$

From equation (a12) results in

$$R = e^{k} - 1$$

$$\Rightarrow (r + 1) = e^{k}$$

Thus the future value of an amount when continuous compounding is done is as follows:

$$FV_n = PV \times e^{km} \qquad \text{(a13)}$$

v) **Continuous Discounting**

From equation (a12)

$$PV = FV_n \times e^{-km}$$

9.4 SUMMARY

Individuals generally prefer possession of cash right now or in the present moment rather than the same amount at some time in the future. This time preference is basically due to the following reasons: (a) uncertainty of cash flows (b) preference for current consumption (c) availability of investment opportunities. In case an investor opts to receive cash in future s/he would demand a risk premium over and above the risk free rate as compensation for time to account for the uncertaininty of cash flows. Compounding and discounting are techniques to facilitate the comparison of cash flows occurring at different time periods. In compounding future value of cash flows at a given interest rate at the end of a given period of time are cash flows at a given interest rate at the beginning of a given period of time is found out. An annuity is a series of periodic cash flows of equal amount. Perpetuity is an annuity of infinite duration. *Table 9.1* depicts the various formulas used for discounting and compounding.

9.5 KEY WORDS

An annuity is a contract between you and an insurance company in which you make a lump-sum payment or series of payments and, in return, receive regular disbursements, beginning either immediately or at some point in the future.

Regular annuity or the deferred annuities are those annuities in which the cash flow occurs at the end of each period.

Annuity due in annuity due, the cash flow occurs at the beginning of the period.

The time value of money (TVM) states that a sum of money is worth more now than the same sum to be received at a future date because of its earnings potential.

9.6 ANSWER TO CHECK YOUR PROGRESS

☞ Check Your Progress 1

1. i) Annual Compounding Rs. 1,254.
   ii) Half year Compounding Rs. 1,262.
   iii) Quarterly Compounding Rs. 1,267.
   iv) Monthly Compounding Rs. 1, 270.

2. Rs. 1,806

3. 7%

4. a) T, b) T, c) F, d) F

☞ Check Your Progress 2

1. a) Rs. 571.20 b) 470.50 c) 288.60
2. Equal yearly instalment = Rs. 3200.61

   Equal quarterly instalment = Rs. 793.28
3. 8.15%
4. a) beginning, b) perpetuity, c) Period of deferment, d) Periodic payments, e) one payment

Table 9.1: Summary of Discounting and Compounding Formulas

Purpose compound value of a lump sum	Given PV Present Value	Calculate FV_n Future value n years hence	Formula $FV_n = PV\,(1+k)^n$
Doubling Period Compound value of a lump sum with shorter compounding period	Interest Rate PV and frequency of compounding (m)	Time Required to double an amount Future value after n year (FV_n)	$0.35 + \frac{69}{\text{Interest Rate}}$ $FV_n = PV\,(1 + \frac{k}{m})^{m \times n}$
Relationship between effective and nominal rate	Nominal interest rate (K) and frequency of compounding (m)	Effective interest rate (R)	$r = (1 + \frac{k}{m})^m - 1$
Present value of a single amount	Future value (FV_n)	Present Value (PV)	$PV_n = FV_n\,(\frac{1}{1+k})^n$
Future value of a regular a	Constant periodic cash flow (A) interest rate (k) and duration (n)	Further value of a regular annuity (FVA_n)	$FVA_n = A[\frac{(1+k)^n - 1}{k}]$

Future value of a annuity due	Constant periodic cash flow (A) interest rate (k) and duration (n)	Future value of an annuity due FVA_n (due)	$FVA_{n(due)} = A[\frac{(1+k)^n - 1}{k}](1+k)$
Present value of a regular annuity	Constant periodic cash flow (A) interest rate (k) and duration (n)	Present value of a regular annuity PVA_n	$PVA_n = A\left[\frac{(1+k)^n - 1}{k(1+k)^n}\right]$
Present value of an annuity due	Constant periodic cash flow (A) interest rate (k) and duration (n)	Present value of an annuity due PVA_n (due)	$PVA_{n(due)} = A[\frac{(1+k)^n - 1}{k(1+k)^n}](1+k)$
Present value of a perpetuity	Constant cash flows (A) and interest rate (k)	Present value of an perpetuity PVA_∞	$PVA_\infty = \frac{A}{K}$

9.7 SELF-ASSESSMENT QUESTIONS/EXERCISES

1) Explain the mechanism of calculating present value of cash flows giving suitable examples.
2) "The finance manager should take into consideration the time value of money in order to take correct financial decisions." Elucidate.
3) Examine the various techniques employed to adjust the time value of money.
4) "Cash flows of different periods in absolute terms are incomparable". Discuss.

Exercises

5) Calculate the present value of the following cash flows assuming a discount rate of 8%

Year	Cash Flows
1	Rs. 10,000
2	Rs. 20,000
3	Rs. 10,000
4	Rs. 5,000

(**Ans**. Rs. 38,015)

6) Mr. A has to receive Rs. 5.000 per year for 6 years Calculate the present value of the annuity assuming that he can earn interest on his investment at 12% p.a.

[**Ans**. Rs. 20,555)

7) A company needs Rs. 10, 00,000 after 5 years from now for replacement of its fixed assets. It has established a Sinking Fund for the purpose. The investments are to be made at the end of each year. What annual payment must be made to ensure the needed Rs. 10, 00,000 after 5 years? Assume 10% interest per year on investments.

[**Ans**. Rs.1.63, 800.16]

8) Determine the amount of equal payment to be made for a loan of Rs. 2, 00,000 taken for a period of 4 years at 10% rate of interest.

[**Ans.** 63,091.48]

9) If you deposit Rs. 10,000 today at 12 per cent rate of interest, in how many years will this amount grow to Rs. 80,000? Work out this problem by using the Rule of 72. Do not use the Compound Factor Tables.

(**Ans.**18 Yrs)

10) At the time of his retirement Mr. X is given a choice between two alternatives: (1) an annual pension of Rs. 10,000 as long as he lives, and (i) a lump sum payment of Rs. 60,000. If Mr. X expects to live for 15 years and rate of interest is 15 percent, which alternative should he select?

[**Ans**. Lump sum, because the present value of Rs. 10,000 received for 15 years at percent rate of interest is Rs. 58470 which is less than Rs. 60,000]

11) MR Rajan is to receive Rs.5.000 after five years from now. His time preference for money is 10% p.a. Calculate its present value, if the discount factor is 0.621.

(**Ans**. Rs. 3,105)

12) A 10.payment annuity of Rs 5000 will begin 7 years hence. What is the value of this annuity now if the discount rate is 12%?

[**Ans**. R. 14,315]