



Dezyne École College

Bachelor of Computer Applications (BCA)

First Year – 2nd Semester Discrete Math

Probable Question for Discrete Math

Sets

Q1: Define a set. Give two examples of finite and infinite sets.

Q2: What is the power set of $\{a, b, c\}$? How many subsets does it contain?

Q3: State the difference between a subset and a proper subset.

Q4: Find $A \cup B$, $A \cap B$, and $A - B$ if $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$.

Q5: What is the cardinality of the set $\{x \in \mathbb{Z} \mid -3 \leq x \leq 4\}$?

Q6: Prove that $A \subseteq B$ if and only if $A \cup B = B$.

Q7: Show that $A - (B \cup C) = (A - B) \cap (A - C)$ using set identities.

Q8: If $A = \{x \in \mathbb{N} \mid x < 10\}$, how many subsets of A contain at least one even number?

Q9: Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$. Find:

a) $A \cup B$

b) $A \cap B$

c) $A - B$

d) $B - A$

e) $A \times B$

Q10: Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$. Find $A^c \cup B^c$.

Q11: Explain the concept of recursive definition of a set with an example involving even numbers.

Q13: If $A = \{1, 2\}$, $B = \{a, b\}$, write the elements of $A \times B$ and $B \times A$. Are they equal? Why or why not?

Q14: Prove that $(A \cup B)' = A' \cap B'$ using a Venn diagram.

Functions

- Q1: Define a function. How does it differ from a relation?
- Q2: Give an example of a one-to-one (injective) function from $\mathbb{Z}$ to $\mathbb{Z}$.
- Q3: What is the domain and codomain of the function $f(x) = x^2$, $x \in \mathbb{R}$?
- Q4: Let $f(x) = 2x + 1$, $g(x) = x^2$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.
- Q5: Prove that the composition of two bijective functions is bijective.
- Q6: Define inverse of a function. Show that if f and f^{-1} are inverses, then $f(f^{-1}(x)) = x$.
- Q7: Prove that a function $f: A \rightarrow B$ is invertible if and only if it is bijective.
- Q8: Give an example of a function that is injective but not surjective. Explain why.
- Q9: If $f: \mathbb{N} \rightarrow \mathbb{N}$ is defined by $f(n) = n+5$, is f injective, surjective, or bijective?
- Q10: Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 3x+2$. Find its inverse and verify your answer.
- Q11: Define a polynomial function. Give an example.
- Q12: What is the absolute value function? Sketch its graph.
- Q13: Define floor and ceiling functions with examples for $x = 3.7$ and $x = -2.1$
- Q14: Find the composition of functions: If $f(x)=x^2+1$ and $g(x)=2x$ then find $f(g(x))$ and $g(f(x))$
- Q15: Show that the function $f(x)=x^2$ is not one-to-one.
- Q16: Define and give examples of:
- a) One-to-one function
 - b) Onto function
 - c) Bijective function
- Q17: Find the domain and range of the function $f(x)=\sqrt{x-1}$.
- Q18: Let $A=\mathbb{N}$ and $B=\mathbb{Z}$ Show that A and B are countably infinite sets.

Relations

Q1: Define a binary relation with an example.

Q2: Construct the Boolean matrix for the relation $R = \{(1,2), (2,1), (3,3)\}$ on the set $A = \{1, 2, 3\}$.

Q3: State and explain reflexive, symmetric, and transitive properties of a relation.

Q4: Given a relation R on $A = \{1, 2, 3\}$ with $R = \{(1,2), (2,3)\}$

(i) Represent R using a Boolean matrix.

(ii) Find R^2 using Boolean matrix multiplication.

Q5: Find the adjacency matrix for the relation $R = \{(a,b), (b,c), (c,a)\}$ on $A = \{a, b, c\}$.

Q6: Check whether the relation represented by the following Boolean matrix is symmetric and reflexive:

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q7: Apply Warshall's Algorithm to the matrix and find the transitive closure.:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Q8: Let $R = \{(1,2), (2,1)\}$, $S = \{(1,1), (2,2)\}$ on $A = \{1, 2\}$. Find: $R \cup S$, $R \cap S$, and $R \circ S$.

Q9: Determine whether $R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ is an equivalence relation. Justify.

Q10: Draw the directed graph for the relation $R = \{(1,1), (1,2), (2,3), (3,1)\}$ and write its Boolean matrix.

Q11: Prove or disprove: If a relation is symmetric and transitive, then it must be reflexive.

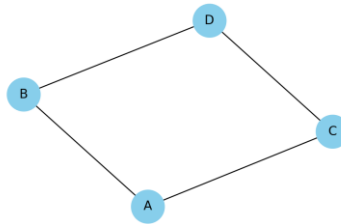
Q12: Show with an example that the composition of two transitive relations may not be transitive.

Proof Methods

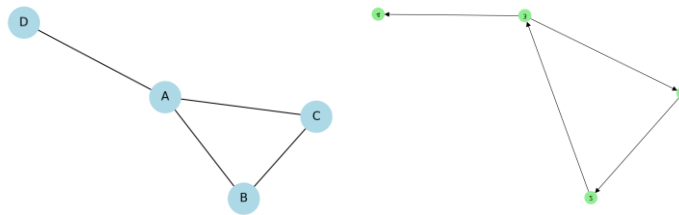
- Q1: Define vacuous proof with an example.
- Q2: What is a counterexample? Use it to disprove the statement: "Every even number greater than 2 is a prime."
- Q3: State the Division Algorithm and explain its terms with an example.
- Q4: Prove the implication: "If n is even, then n^2 is even" using direct proof.
- Q5: Explain the difference between constructive and non-constructive proofs with one example of each.
- Q6: Prove: If $a|b$ and $b|c$, then $a|c$. Use direct proof.
- Q7: Using proof by contrapositive, prove: "If n^2 is odd, then n is odd."
- Q8: Use proof by contradiction to prove: square root of 2 is irrational.
- Q9: State the Principle of Mathematical Induction (PMI). Explain each step briefly.
- Q10: What is the Second Principle of Mathematical Induction?
- Q11: State the Fundamental Theorem of Arithmetic. What does it say about prime factorization?
- Q12: Use PMI to prove: $1/1 \cdot 2 + 1/2 \cdot 3 + 1/3 \cdot 4 + \dots + 1/n(n+1) = n/(n+1)$, for all $n \in \mathbb{N}$
- Q13: Show that for all $n \geq 1$: $1^3 + 2^3 + 3^3 + \dots + n^3 = [n(n+1)/2]^2$
- Q14: Prove that: $n^5 - n$ is divisible by 30 for all n in $\mathbb{N}$
- Q15: Prove that $11^n - 4^n$ is divisible by 7 for all $n \geq 1$
- Q16: Prove by induction: $1 + 2 + 4 + \dots + 2^n - 1 = 2^n - 1$
- Q17: Define partial correctness in the context of algorithms. How is it different from total correctness?
- Q18: What is a loop invariant? Why is it important in proving the correctness of algorithms?
- Q19: Identify a suitable loop invariant for the linear and binary search, selection sort and bubble sort algorithm.
- Q20: Prove the partial correctness of the linear and binary search, selection sort and bubble sort algorithm using a loop invariant.

Graph Theory

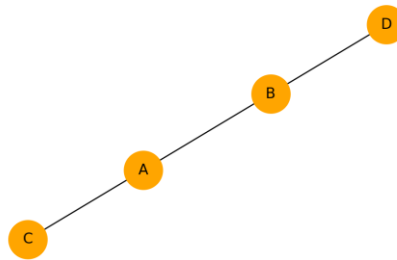
Q1: Define a simple graph with an example diagram.



Q2: What is the degree of a vertex also find in degree and out degree? Explain with a diagram.

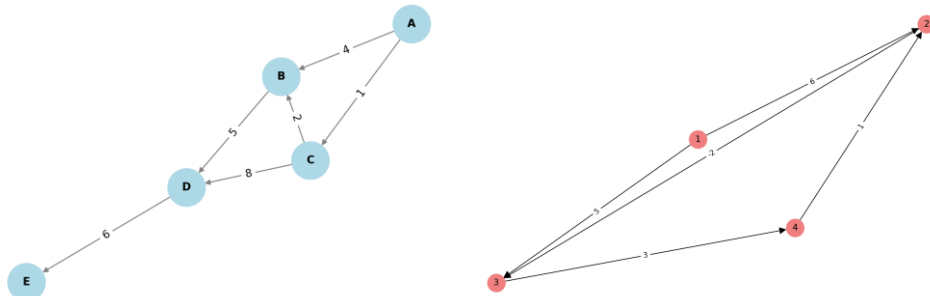


Q3: Represent the following graph using an adjacency matrix and also find if the graph below is bipartite or not.



Q4: Prove that the sum of degrees of all vertices in an undirected graph is twice the number of edges.

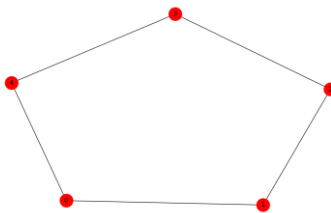
Q5: Using the given graph, apply Dijkstra's algorithm from vertex A to E and for other graph find the shortest path from 1 to 4 and also tell that the given graph are isomorphic or not.



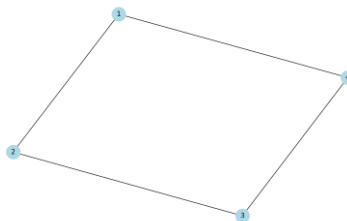
Q6: Draw a cycle graph C_5 and identify its properties.

Q7: Prove that a connected graph has an Euler circuit if and only if all vertices have even degree.

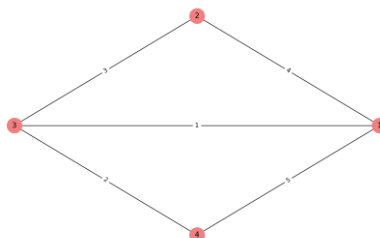
Q8: Determine the given graph is bipartite



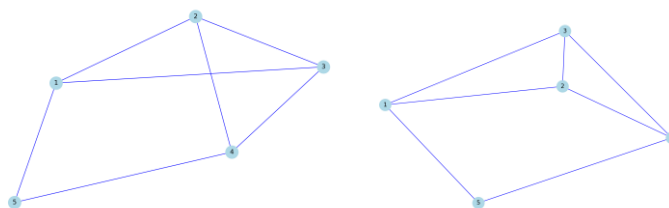
Q9: Compute the incidence matrix of the given graph



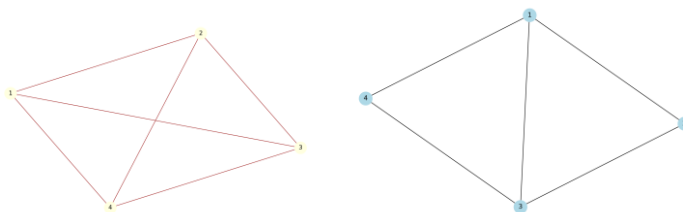
Q10: Find the minimum spanning tree of the weighted graph below using suitable theorem



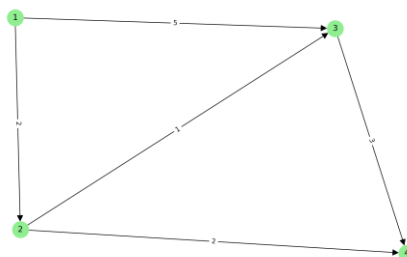
Q11: Find a Hamiltonian cycle in the graph below, if it exists. Show the path.



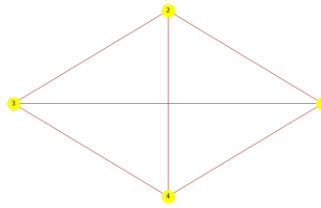
Q12: Find the number of spanning trees for the graph shown using Kirchhoff's matrix-tree theorem.



Q13: Using Dijkstra's algorithm, find the shortest path from vertex 1 to vertex 4 in the weighted graph below. Show all steps.



Q14: Prove whether the graph below is Eulerian or not. If Eulerian, find the Eulerian circuit.



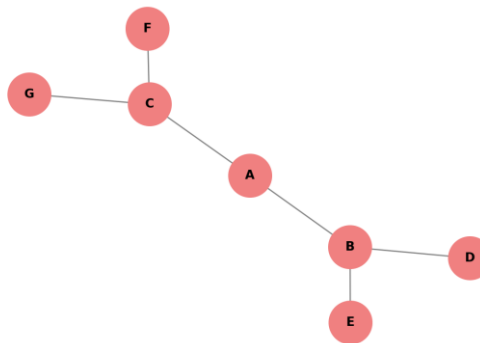
Trees

Q1: Define a tree and give an example with a diagram.

Q2: How many edges are there in a tree with 10 vertices? Prove your answer.

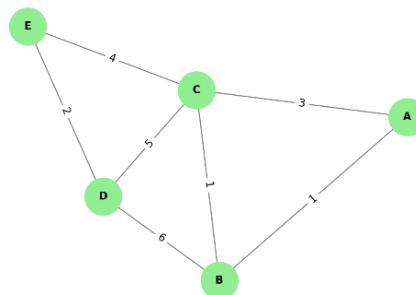
Q3: What is a spanning tree? Illustrate with a diagram.

Q4: Perform DFS and BFS traversal.



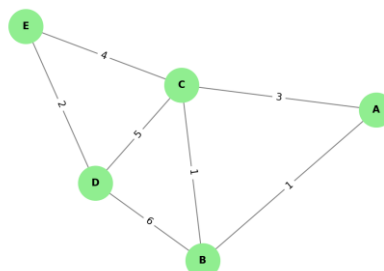
Q5: Differentiate between DFS and BFS using an example tree.

Q6: Use BFS and DFS to construct a spanning tree from the given graph.



Q7: Prove that a connected graph with no cycles is a tree.

Q8: Use Kruskal's algorithm to find the minimum spanning tree of the graph provided.



Q9: Prove that a tree with n vertices has exactly $n-1$ edges.

Propositional Logic

- Q1: Define a proposition. Give 2 examples of a proposition and 2 that are not.
- Q2: Construct the truth table for the logical expression: $p \vee q$.
- Q3: State the difference between a tautology and a contradiction. Give examples.
- Q4: Write the converse, inverse, and contrapositive of the implication: "If it rains, then the ground gets wet."
- Q5: Determine whether the following compound statement is true or false:
"2 + 2 = 4 and 5 < 3"
- Q6: Construct a truth table for the expression: $(p \wedge \neg q) \rightarrow r$.
- Q7: Verify the logical equivalence: $p \rightarrow q \equiv \neg p \vee q$ using a truth table.
- Q8: Prove the statement is a tautology using a truth table: $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
- Q9: Prove the following using laws of logic (without truth table): $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- Q10: Define DNF, CNF, PDNF and PCNF with suitable examples.
- Q11: Convert the expression $(p \wedge q) \vee r$ into its Disjunctive Normal Form (DNF).
- Q12: Write the CNF of the logical statement $\neg(p \vee q)$.
- Q13: Find the PDNF of the expression: $(p \rightarrow q) \vee \neg r$
- Q14: Find the PCNF of the expression: $(p \vee q) \rightarrow r$
- Q15: Convert the following formula into both PDNF and PCNF: $\neg(p \leftrightarrow q) \vee r$
- Q16: Construct the truth table for the compound statement $(p \wedge \neg q) \vee r$ and use it to determine both PDNF and PCNF.

Total No. of Pages: 02

[Total No. of Questions: 18]



2445

Bachelor of Computer Applications (BCA)
(Part-I) (Semester-II)
Examination, 2024
BCA - 202
Discrete Mathematics

Duration of Examination: 3 Hours**परीक्षा की अवधि: 3 घण्टा****Max. Marks: 70****पूर्णांक: 70****Instructions to the Candidates:****परीक्षार्थी के लिए निर्देश:-****Note:- The question paper is divided into 02 Parts : Part- A & Part B.****Part-A**

Part-A Will consist of 10 compulsory questions. Answer to each question shall be limited up to 50 words. Each question will carry 02 marks. Total 20 Marks.

(Marks-10x2=20)**Part-B**

Part-B Will consist of 10 questions. Student will have to answer 05 questions, selecting At least one questions from each unit. The answer to each question shall be limited to 400 words. Each question carries 10 Marks. Total 50 Marks.

(Marks-5x10=50)**Part-A**

- 1- If $A = \{2, 3, 4, 5\}$ and $B = \{0, 1, 2, 3\}$ find $A \cap B$
- 2- If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ and $C = \{0, 2, 3\}$, find $(A \cap B) \times C$
- 3- Define Tautology.
- 4- Define Scalar matrix with example.
- 5- If $A = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$, find $2A + 3B$.
- 6- Give an example of relation which is symmetric but neither reflexive nor anti symmetric nor transitive.
- 7- If X and Y are the two finite sets, such that $n(X \cup Y) = 36$, $n(X) = 20$, $n(Y) = 28$, then find $n(X \cap Y)$
- 8- Given two sets $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, find $A \cup B$ and $A \cap B$. Explain these operations represent in the context of set theory.
- 9- Define what it means for a relation R on a set A to be symmetric. Give an example of a symmetric relation on the set $A = \{1, 2, 3\}$
- 10- What is the degree of a vertex in a graph? If a simple graph has 5 vertices and 4 edges, what is the maximum degree a vertex can have in this graph? Explain your reasoning.

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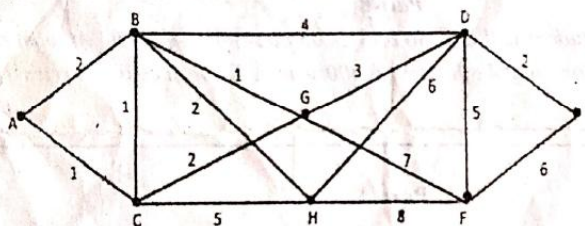
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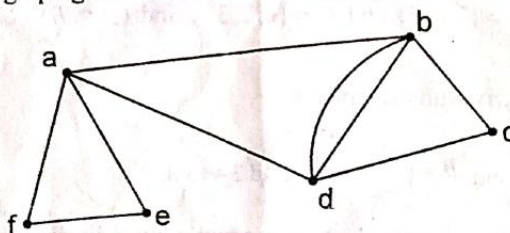
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Part-B

- 11- (a) Construct the truth table for $(q \wedge (P \rightarrow Q) \rightarrow P)$
 (b) Define What an Euler path and an Euler circuit are in a graph. Given a graph G with vertices of degrees 3, 3, 2, 2, 1 and 1, determine whether G contains an Euler Path or circuit. Justify your answer based on the properties of Euler paths and circuits.
- 12- Prove that for any natural number n, the sum of the first n odd numbers is n^2 . Use the principle of mathematical induction to support your argument.
- 13- (a) Show that $\sqrt{2}$ is irrational.
 (b) Prove that $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$.
- 14- Use Mathematical Induction to show that $1+2+2^2+2^3+\dots+2^n=2^{n+1}-1$
- 15- Prove that n^3-n is divisible by 3 for $n \geq 1$.
- 16- (a) Apply Dijkstra's Algorithm OR Improved version of Dijkstra's Algorithm to find a shortest path from A to F. Also write steps wherever possible.



In the pseudograph given below either describe an Eulerian circuit or explain why no Eulerian circuit exists.



- 17- (a) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $n/2$, then prove that G has Hamilton cycle.
 (b) If G is self complementary graph, then prove that G has $n \equiv 0$ (or) $1 \pmod{4}$ vertices.
- 18- (a) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic?

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- (b) Prove that the number of odd degree vertices in any graph is even.

Roll No.....

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B.C.A. (Part I) EXAMINATION, 2018

BCA-105

DISCRETE MATHEMATICS

Time allowed : Three hours

Maximum marks : 50

Part-A (Compulsory) {Marks : 15}

Answer all ten questions (50 words each). Each question carries $1\frac{1}{2}$ marks.

Part-B (Compulsory) {Marks : 15}

Answer all five questions (50 words each). Each question carries 3 marks.

Part-C {Marks : 20}

Answer any three questions (400 words each). There will be interanl choice in each question. Three questions of 7, 7 & 6 marks.

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Part-A (Compulsory)

1. Check that the given statements are proposition or not :

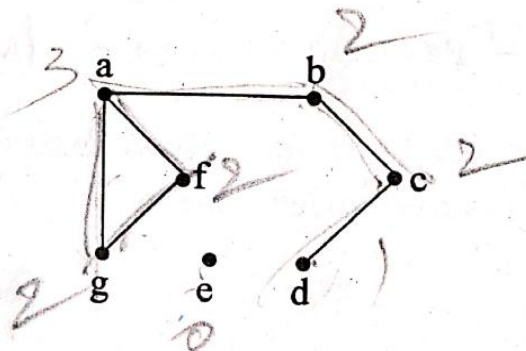
- (i) Open the door
- (ii) Temp on venus in 800°C
- (iii) $x + 4 \geq 8$

2. Define complete bipartite graph.

3. Write fundamental theorem of arithmetic.

4. Define partition of set.

5. What is degree of a graph ? Find degree of each vertex in the following graph :



6. Check the validity of the following argument :

"if the moon is made of green cheese, then cows jump

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over it. The moon is made of green cheese. Therefore, cows jump over that moon".

7. Find power set of $A = \{a, b, c\}$.
8. Differentiate floor and ceiling function.
9. Define bijective function with example.
10. What is weighted graph ? Give example.

Part-B (Compulsory)

11. Prove De-Morgan's law using Venn diagram.
12. Draw a graph with five vertices a, b, c, d, e such that $\deg(a) = 3, \deg(b) = 2, \deg(c) = 2, \deg(d) = 3, \deg(e) = 2$ where a and b are adjacent to e .
13. If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 $B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$
and $U = \{1, 2, \dots, 20\}$
then find the following :

(i) A^c

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(ii) $(A - B)^c$

(iii) $(A \oplus B)^c$

(iv) $(A \cup B)^c - \cup$

14. Proof by direct method.

"If n is odd integer then n^2 is odd integer".

15. Let p and q be the propositions

p : Today is Monday.

q : Grass is dry.

Express each of these propositions as an English sentence :

(a) $\sim p \wedge \sim q$

(b) $\sim p \vee (p \wedge q)$

(c) $\sim p \rightarrow \sim q$

Part-C

16. (a) Find converse, inverse and contrapositive of the following statement :

"If you drive more than 400 miles, then you will need to buy petrol."

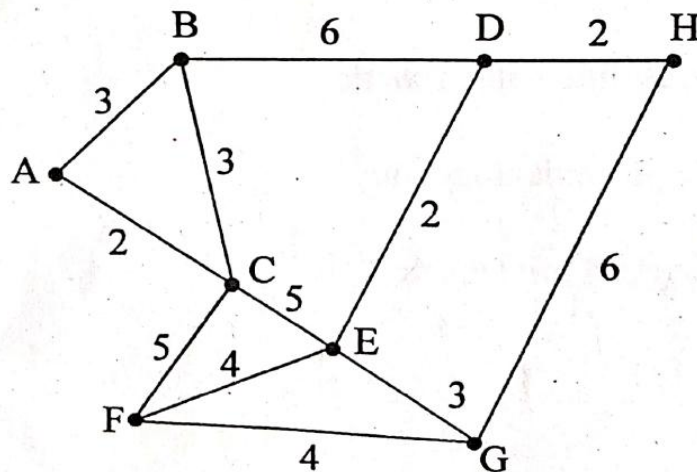
- (b) Prove that the following statements are logically equivalent :

$$p \vee (q \wedge r) \text{ and } (p \vee q) \wedge (p \vee r)$$

Or

Define logical connectives with suitable examples.

17. Find the minimum spanning tree using Kruskal's algo.



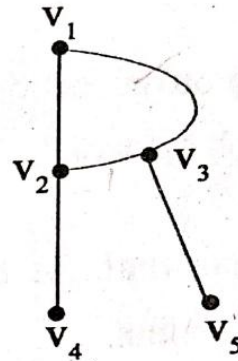
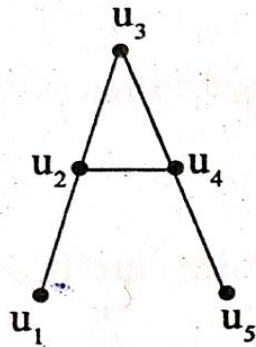
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Or

Prove that the following graphs are isomorphic to each other :



18. Using Mathematical induction, prove that :

$$2 + 5 + 8 + \dots + (3n - 1) = \frac{n(3n + 1)}{2}$$

Or

Write short notes on :

(i) Absolute Value function

(ii) Exponential function

(iii) Logarithmic function