Thermal stresses arise when there are temperature differences or where materials having different thermal expansion characteristics are joined to one another – cylinder head cover made of mineral-filled PA 66 for a six-cylinder VVT petrol engine from BMW

(photo: DuPont)

Estimating Thermal Stresses Using Single-Point Data

Part Design. By means of a new method of estimation thermal stresses in unreinforced thermoplastic parts on heating and cooling can be calculated simply and in good agreement with measured values. The method requires only the modulus of elasticity and the coefficient of linear thermal expansion from databases and an empirically determined influencing factor as a measure of the effect of temperature and material characteristics.

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Thermal stresses in a component arise because thermal expansion caused by changes in temperature is externally or internally inhibited or even prevented. In the case of uniaxial inhibitions of thermal expansion as a result of temperature change the following known relationship

$$\sigma_\vartheta \approx -E \cdot \alpha \cdot (\vartheta_2 - \vartheta_1) = -E \cdot \alpha \cdot \Delta\vartheta \qquad (1)$$

is usually used to calculate the thermal stresses. In this the modulus of elasticity E and the coefficient of linear thermal expansion α are constant characteristic values for the material in question. On account of the fact that thermal stresses in viscoelastic materials are already relaxing while building up and because their characteristic values are temperature-dependent, such a simple calculation cannot be used for plastics. This would yield totally unrealistic results. Oberbach [1] had already pointed this out and shown on the basis of his own experimental results that thermal stresses on heating and cooling over the same temperature range proceed differently and are of different magnitudes (Fig. 1). These insights are still regarded as state of the art today [2, 3].

Accordingly, for the calculation of thermal stresses in plastics only approximation models have been put forward so far [4–7], but none have become established in practice. A recently published approach

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takes account in relatively simple manner of both the viscoelastic behavior and the dependence of the characteristics of the material on temperature [8]. The simulation model operates with different physical property values for the materials that are obtained from databases or handbooks and so it is assumed that these data are available. In view of the many gaps in documentation, unfortunately also in the Campus database [9], there is interest in calculation methods which make do with even less materials data.

Objective: An Expression of Acceptable Accuracy

Against this background attempts were made to develop an approximation expression with which the thermal stresses in thermoplastics in the event of externally inhibited thermal expansion can be estimated using simple computational means but with acceptable accuracy within a sufficiently wide temperature range. In particular this should be possible with a minimum of characteristic materials data which, furthermore, would have to be widely available; thus primarily with single-point data from the Campus materials database [9].

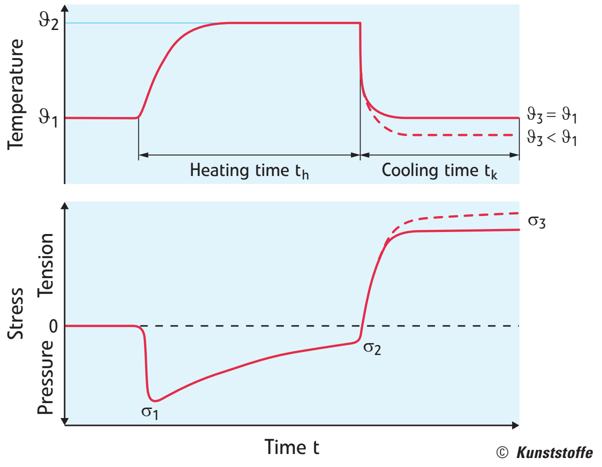
Stress Variation on Temperature Change

Fig. 1. Schematic curve of temperature versus stress in the alternating temperature test [1]

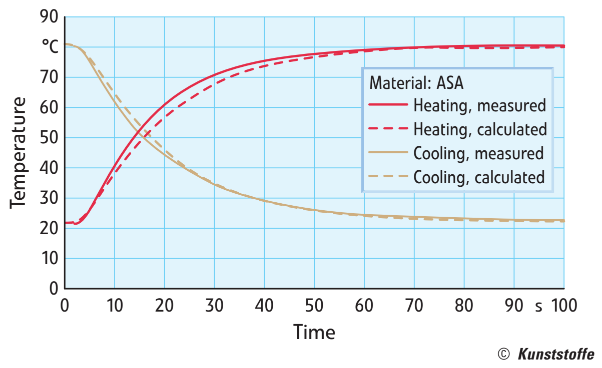
Temperature Variations in the Center of the Test Item

Fig. 2. Comparison of measured and calculated temperature variations in the center of the test item made of ASA (grade: Luran S 757 R, manufacturer: BASF) [8]

Thermal stresses in the case of externally constrained deformation are made up in principle of two components: their mean value over the entire cross section of the specimen or part and local deviations from this value. The mean value is the quotient of the force building up under external deformation inhibition and the cross sectional area. A particular temperature can be assigned to it. The local stress deviations which overall balance out across the cross section are caused by different temperatures within the cross section, especially in the case of rapid heating or cooling. Due to this internal inhibition uniaxial or multiaxial stress states arise that are not easily apprehended by analytical means, especially in the case of viscoelastic deformation behavior. At best this can be done using more complex methods of calculation, e.g. FEM when using a suitable law of the material.

In the task set here the part is not at a constant temperature but rather undergoes a more or less rapid temperature change depending on the heat transfer conditions (Fig. 2). During both heating and cooling the thermal stresses setting in increase in magnitude as the rate of temperature change per time unit rises. Probably the most extreme conditions occur when there is very rapid temperature change, chiefly in the event of shock-like contact heating by means of a medium allowing rapid heat transfer, such as water for instance. Of greatest interest in practice are the maximum thermal stresses; thus the envisioned solution is directed at the maxima of the curves of thermal stress versus time during heating and during cooling (Fig. 3). However, since the local stress values cannot be calculated in elemental fashion the solution must be limited to said mean value. But this can be regarded as the upper limit for the thermal stresses actually occurring.

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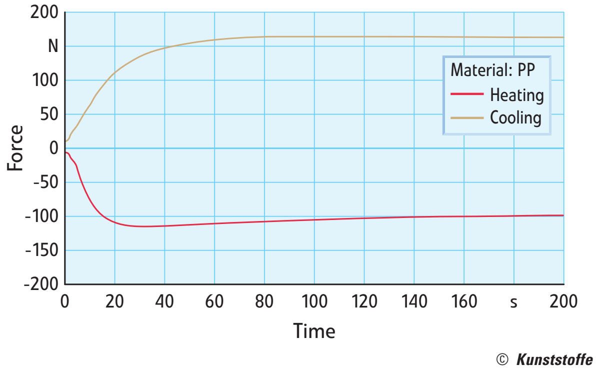
Force Variations on Temperature Change

Fig. 3. Measured force variations on heating and cooling by a temperature interval of 60°C with reference to the example of PP (grade: BC 245 MO, manufacturer: Borealis) [10]

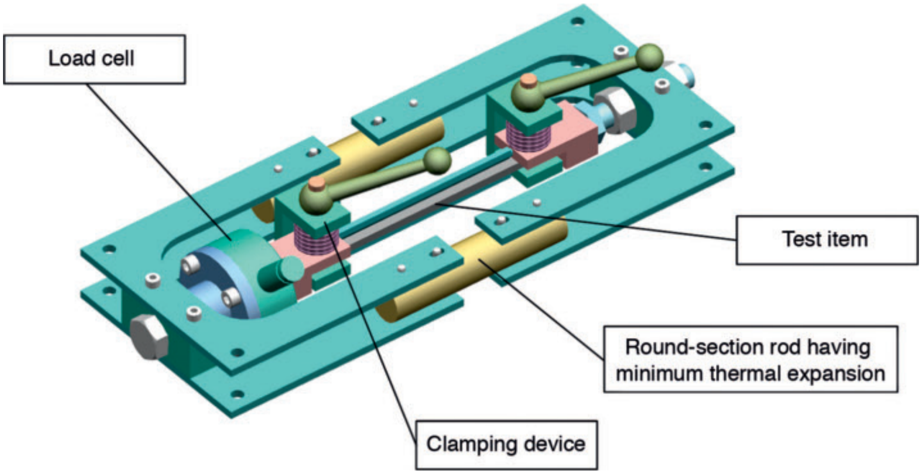

Fig. 4. Test setup for the experimental determination of thermal stresses [10]

Comparative Experimental Data

To demonstrate the usefulness of the results it was possible in this project to draw upon extensive measured data from earlier investigations [8, 10]. In doing so the thermal stresses were determined experimentally using an experimental setup specially devised for this purpose. This consists essentially of a test frame (Fig. 4) with load cell for mounting the shouldered test bars, a water bath for each initial and final temperature and a circulating pump for ensuring heat transfer by forced convection. By suitable choice of geometry and materials the experimental error due to independent thermal expansion of the test frame is limited to about 1 %. Measurements were carried out in accordance with current standards over the temperature range of 23 °C (room temperature) to 80 °C with intermediate values of 40 °C and 60 °C. The thermal stress determined experimentally in this way is the quotient of the force arising due to inhibition of deformation and the cross sectional area and so is a mean value for the entire cross section of the test item.

A selection of widely used unreinforced commodity or engineering plastics was investigated: four semicrystalline (PE-HD, PP, PBT, POM) and four amorphous (ABS, ASA, PC, SB) thermoplastics, as well as a PBT GF30 in order to explore the effect of reinforcing fibers. Inherently interesting materials such as polyamide 6 and polyamide 66 were not included in the study on account of their strong tendency to absorb water.

Solution Approach

The approach to a solution starts from relationship (1) and deliberately uses the values for the modulus of elasticity E and coefficient of linear thermal expansion α determined at room temperature since almost without exception they can be found in databases. It also makes use of a long known method in which the time- and temperature-dependence of the rigidity of a material are each expressed approximately by a separate influencing factor [11, 12]. Here, however, it is attempted to combine both effects in a single factor δ. This makes the computational model particularly simple and is justified in that the maximum stress under this shock-inducing temperature change is attained after approximately one to three minutes depending on the material and thickness of the part (Fig. 2). Thus, this influencing factor is primarily dependent on temperature while time-dependence is of secondary importance. It is possible to describe this temperature-dependence by a power function having a base value δ_0 and a temperature-dependent exponent since this has proved to be highly suitable for the creep modulus [13].

Result: A Simple Estimation Expression

These considerations result in the estimation expression:

$$\begin{aligned}\sigma_\vartheta &\approx -E\cdot\alpha\cdot\delta\cdot\Delta\vartheta \\ &= -E\cdot\alpha\cdot\delta_0^{\left(\frac{\vartheta}{\vartheta_R}-1\right)}\cdot\left(\vartheta-\vartheta_R\right)\end{aligned} \qquad (2)$$

for the thermal stress when the reference temperature υ_R is changed to any desired temperature υ. If the temperature changes from the initial value υ_1 to the final temperature υ_2 then using the Boltzmann su-

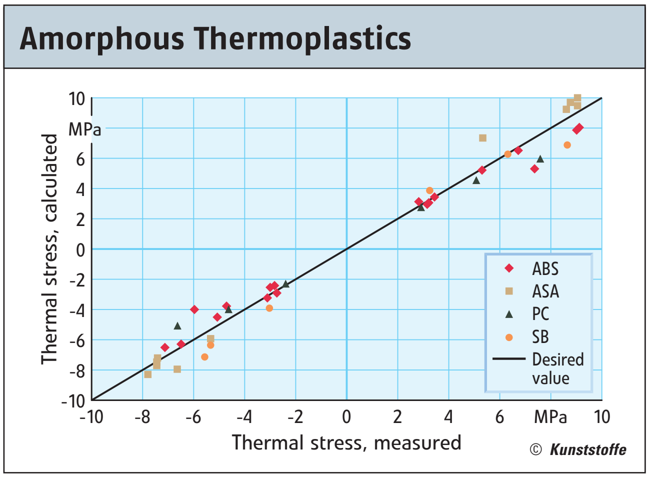

Fig. 5. Amorphous thermoplastics: comparison of calculated and measured thermal stresses with δ_0 = 0.80 for heating and δ_0 = 0.88 for cooling [10]

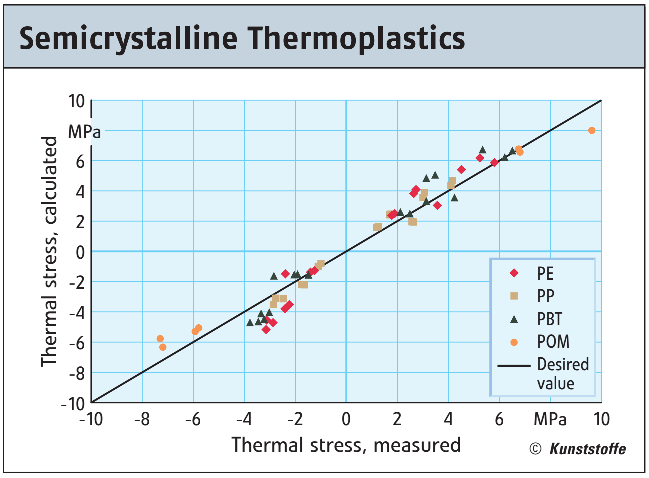

Fig. 6. Semicrystalline thermoplastics: comparison of calculated and measured thermal stresses with δ_0 = 0.61 for heating and δ_0 = 0.71 for cooling [10]

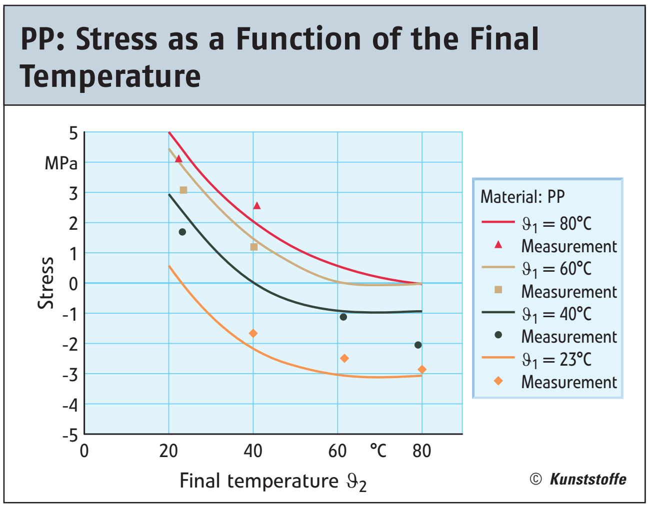
PP: Stress as a Function of the Final Temperature

Fig. 7. Comparison of measured and calculated thermal stresses as a function of the final temperature with reference to the example of PP (grade: BC 245 MO, manufacturer: Borealis) [10]

perposition principle [14] the thermal stress is given by:

$$\sigma_{\vartheta} \approx \sigma_{\vartheta_2} - \sigma_{\vartheta_1} \approx -E \cdot \alpha \cdot \left[\delta_0^{\left(\frac{\vartheta_2}{\vartheta_R}-1\right)} \cdot (\vartheta_2 - \vartheta_R) - \delta_0^{\left(\frac{\vartheta_1}{\vartheta_R}-1\right)} \cdot (\vartheta_1 - \vartheta_R) \right] \quad (3)$$

To use this relationship in practice all that is needed apart from the physical property values E and α is the base variable δ_0 which depends on the material and if need be the thickness d of the part.

In determining the base variable δ_0 from the experimental results by the least squares method characteristic magnitudes emerged for the two groups of amorphous and semicrystalline thermoplastic materials investigated and at the same time there were significant differences between heating and cooling (Table 1) over the temperature range under examination 20°C ≤ υ ≤ 80°C. Comparison of the measured thermal stresses with the calculated values for the temperature ranges 20/40°C, 20/60°C, 20/80°C, 40/60°C and 40/80°C in both directions reveals very good agreement (Figs. 5 and 6). The thickness of the part primarily affects the speed of temperature change. Its effect on the maximum values of the thermal stresses is only about ±10 % and so remains in the range of uncertainty of the approximation and can be left out of consideration in relationship (3) [10].

Group of materials	Numerical value of factor δ_0	
	Heating	Cooling
Amorphous thermoplastics	0.80	0.88
Semicrystalline thermoplastics	0.61	0.71

Table 1. Base value δ_0 of the temperature-dependent influencing factor for the heating and cooling of amorphous and semicrystalline thermoplastics (reference temperature υ_R = 23°C)

An analogous situation is found for the influence of reinforcing fibers. This is explained by the fact that they reduce the coefficient of thermal expansion and simultaneously raise the modulus of elasticity. Thus the values found for δ_0 can also serve as at least an upper limit for fiber-reinforced thermoplastics, the estimate then lying on the safe side.

A certain constraint on the validity of (3) emerges from the requirement that on heating pressure stresses calculated with negative mathematical sign always arise so that the large expression in brackets must always be positive. This has the consequence that for the base variable δ_0 there is a lower limit on heating which further depends on the initial temperature υ_1 and the reference temperature υ_R. For υ_R = 23°C this limit lies at δ_0 = 0.66. Of the values determined here (Table 1) only the group of semicrystalline thermoplastics for which δ_0 = 0.61 lies in the critical domain, but this only has a negative effect on heating from υ_1 ≥ 60°C (Fig. 7) – a situation that should occur relatively rarely. In contrast there are no problems with the other values.

Conclusion

The relationship (3) presented here makes it possible to estimate thermal stresses in components composed of unreinforced thermoplastics on heating or cooling over the temperature range 20°C ≤ υ ≤ 80°C in good agreement with measured values (Figs. 7 and 8). All that is needed for this purpose is the modulus of elasticity E and the coefficient of linear thermal expansion α in the form of Campus single-point characteristic values measured at room temperature together with an empirically determined base variable δ_0 of the influencing factor δ that is characteristic of the material. How well thermal stresses might be calculable outside the investigated range of 20°C to 80°C remains open. It was generally observed that on approaching the heat distortion temperature HDT/A agreement between calculated and measured values declines appreciably.

It is characteristic of this method of estimation that the base variable δ_0 of the influencing factor has not been determined separately for each material but rather for each chosen group, i.e. separately for amorphous and semicrystalline thermoplastics. This allows the rapid and near-realistic estimation of the thermal stresses. More precise calculations would require correspondingly more detailed ▶

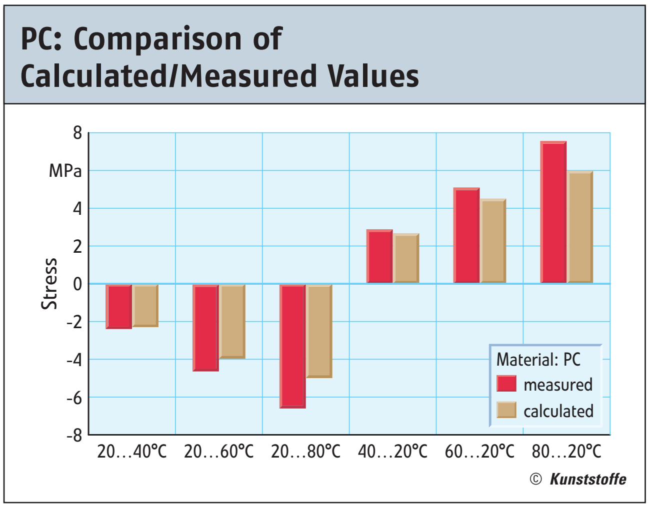
PC: Comparison of Calculated/Measured Values

Fig. 8. Usefulness of the estimation method: measured and calculated thermal stresses in PC (grade: Makrolon 2805, manufacturer: Bayer) compared [10]

and specially measured physical properties data or functions.

The thermal stress on the contact heating of plastic test items in water with forced convection determined in this way may be regarded as the upper limit. Heat transfer to the plastic in the case of free convection with air as contact medium takes place substantially more slowly and as a result the thermal stresses turn out to be smaller due to the greater effect of relaxation. The uniaxial or multiaxial stress states arising on account of different temperatures due to mutual inhibition within the cross section cannot be captured directly by the estimation expression (3). However, given knowledge of the local temperature differences with respect to the instantaneous mean value they could be estimated by applying the Boltzmann superposition principle [14].

In view of the results obtained it may be concluded overall that despite said limitations the expression presented here is suitable for the realistic estimation of thermal stresses in thermoplastics. ■

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The full list of references can be downloaded from the internet at www.kunststoffe-international.com/A002

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